Soft contribution to exclusive Drell-Yan process

$$\pi^- p \rightarrow \ell^+ \ell^- n$$

Kazuhiro Tanaka (Juntendo U/KEK)

High momentum beam line at J-PARC

- Primary beam (proton)

$E = 30\text{GeV}\ (\rightarrow 50\text{GeV?})$

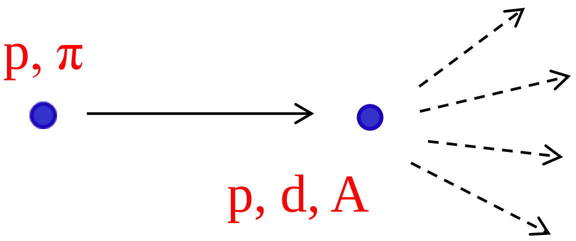
$L = 10^{35}\,\text{cm}^{-2}\text{s}^{-1}$

$\leftrightarrow$ PANDA (anti-proton)

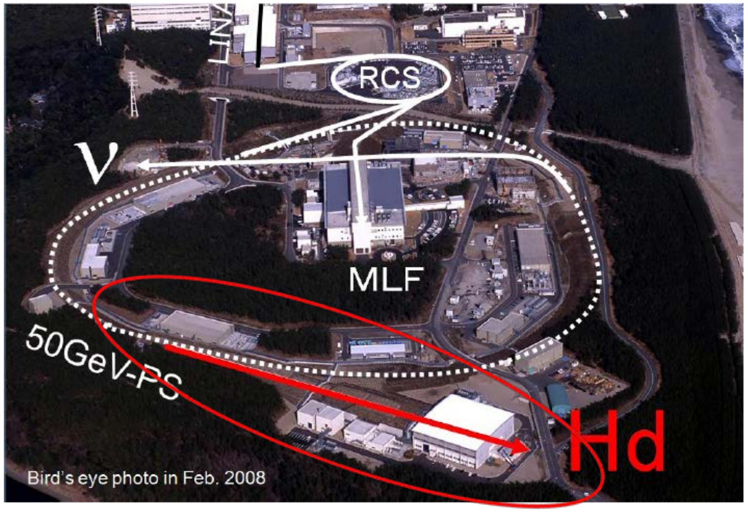
$E \leq 15\text{GeV}\ ,\ \ L = 10^{32}\,\text{cm}^{-2}\text{s}^{-1}$

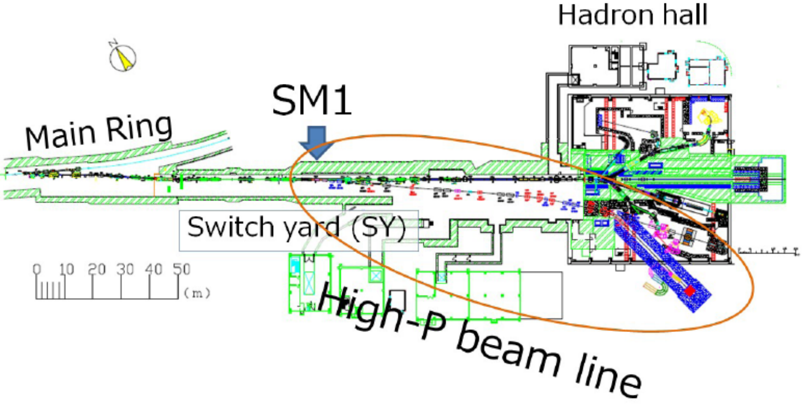
- Secondary beam (pion)

E = 15-20GeV



Hadron Facility at J-PARC





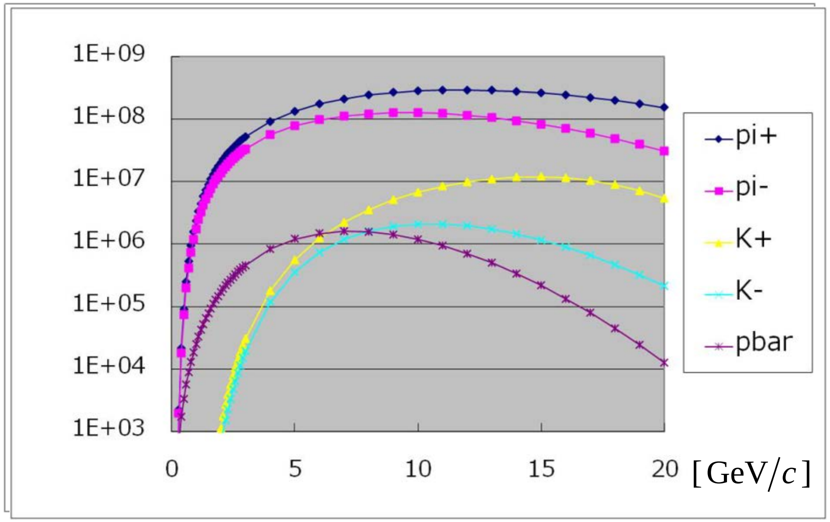
High-momentum beamline

- 30 GeV proton
- ~15-20 GeV unseparated (mainly pions)

Unseparated Secondary Beam Intensity

beam loss limit @ SM1:15kW

(limited by the thickness of the tunnel wall)



0° extraction angle

high intensity

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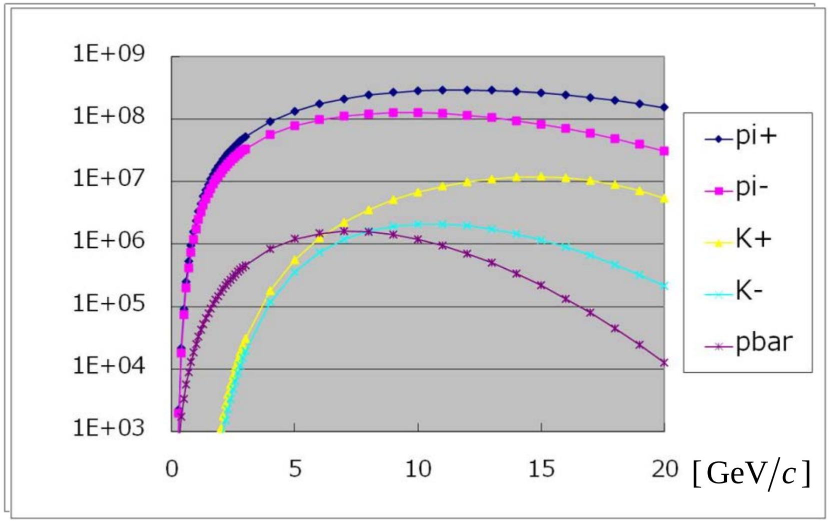


0° extraction angle

high intensity

not too high energy $d\sigma \sim 1/s^a$

best suited to study meson-induced hard exclusive processes

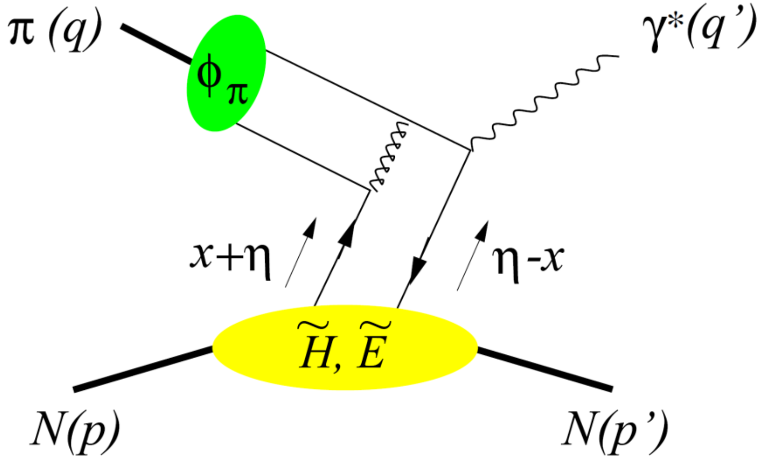
Exclusive lepton pair production in πN scattering

$$\pi^- p \to \gamma^* n \to \mu^+ \mu^- n$$

Berger, Diehl, Pire, PLB523(2001)265

"exclusive limit of DY"

small $t = (q - q')^2$



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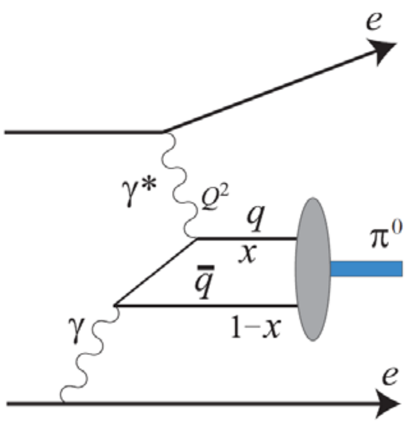

@Belle, Babar

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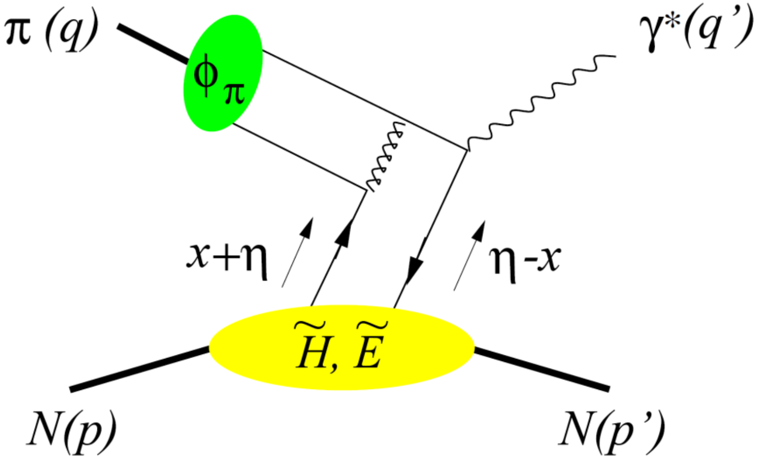

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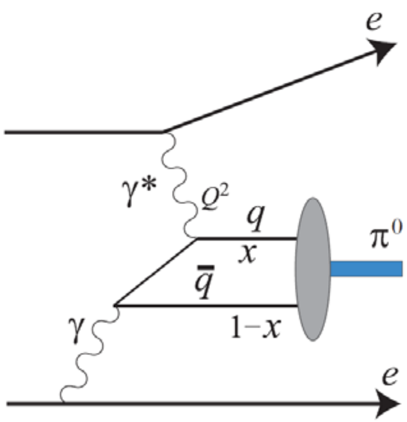

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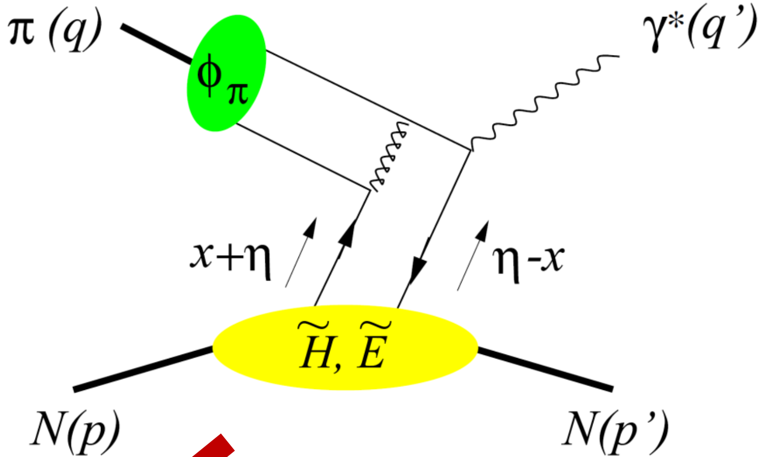

$t \to 0$

$\Delta q(x)$

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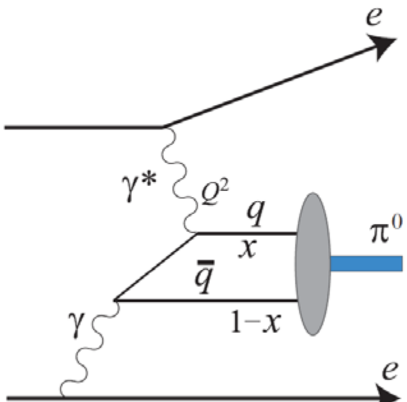

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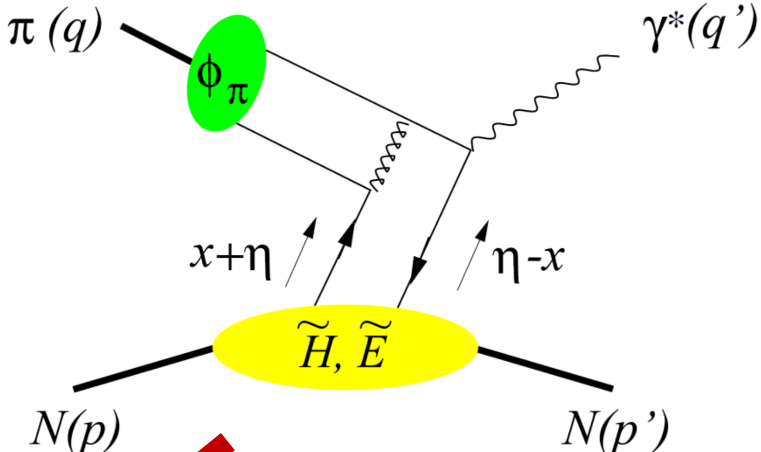

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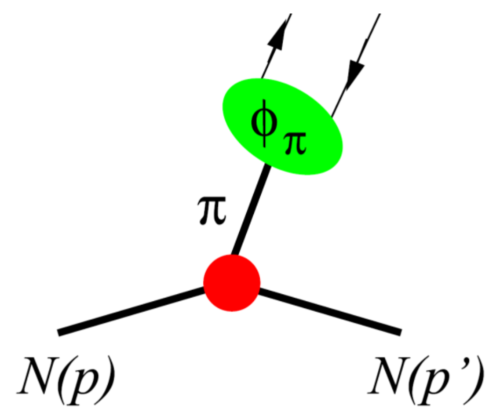
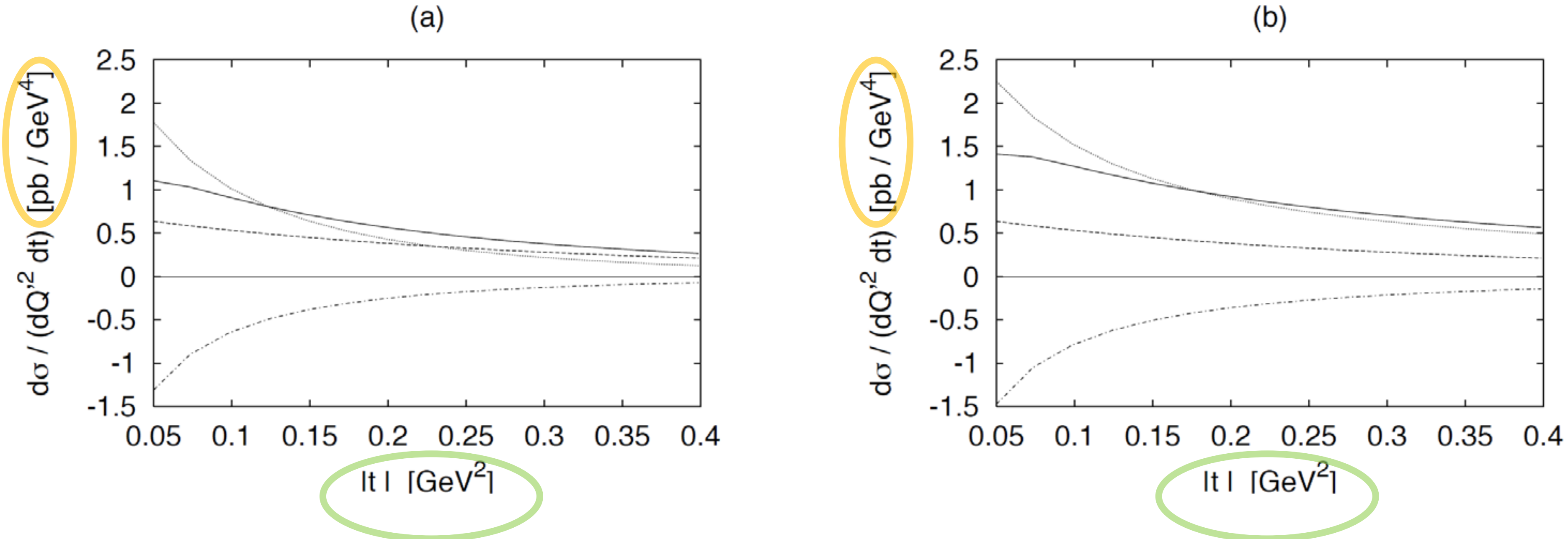


LO Estimates

Bjorken variable $\tau = \frac{Q'^2}{s - M^2}$

Berger, Diehl, Pire, PLB523(2001)265

$Q'^2 = 5 GeV^2 \qquad \tau = 0.2$



(dashed) $= |\tilde{\mathcal{H}}|^2$; **(dash-dotted)** $=$ **Re**$(\tilde{\mathcal{H}}^* \tilde{\mathcal{E}})$; **(dotted)** $= |\tilde{\mathcal{E}}|^2$

$$\frac{d\sigma}{dQ'^2 dt}(\pi^- p \to \gamma^* n) = \frac{4\pi\alpha_{em}^2}{27}\frac{\tau^2}{Q'^8} f_\pi^2 \left[(1-\eta^2)|\widetilde{\mathcal{H}}^{du}|^2 - 2\eta^2 \mathrm{Re}\left(\widetilde{\mathcal{H}}^{du*}\widetilde{\mathcal{E}}^{du}\right) - \eta^2 \frac{t}{4M^2}|\widetilde{\mathcal{E}}^{du}|^2 \right]$$

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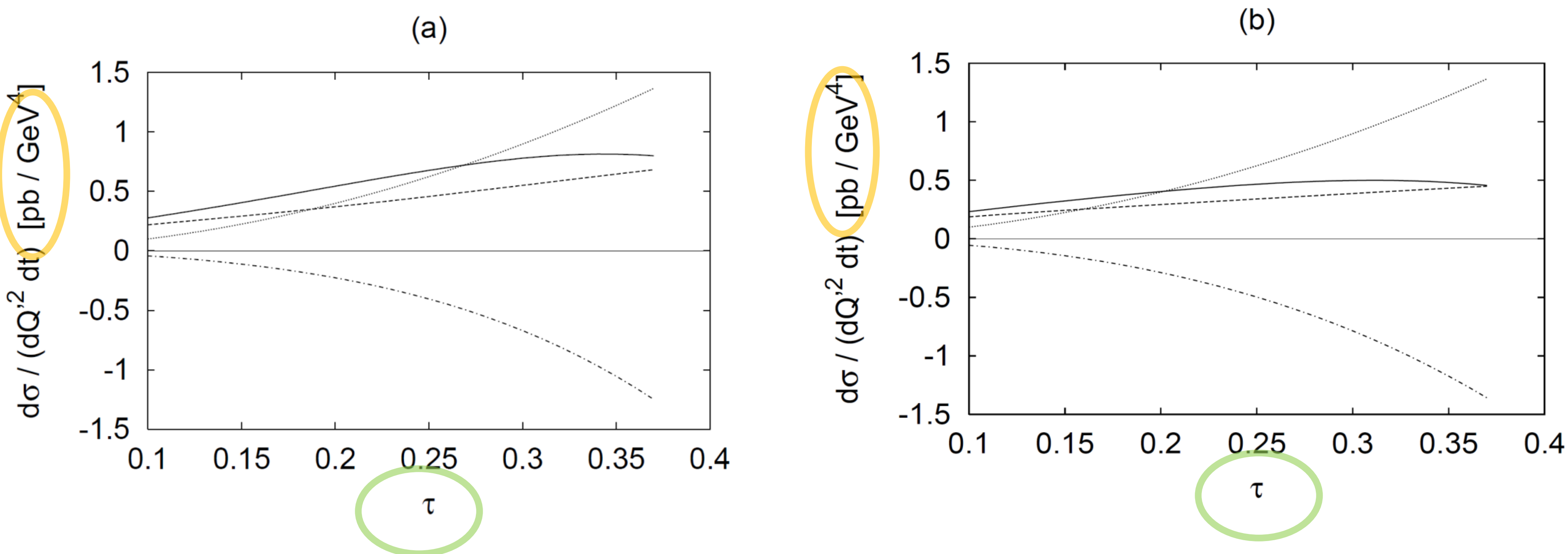

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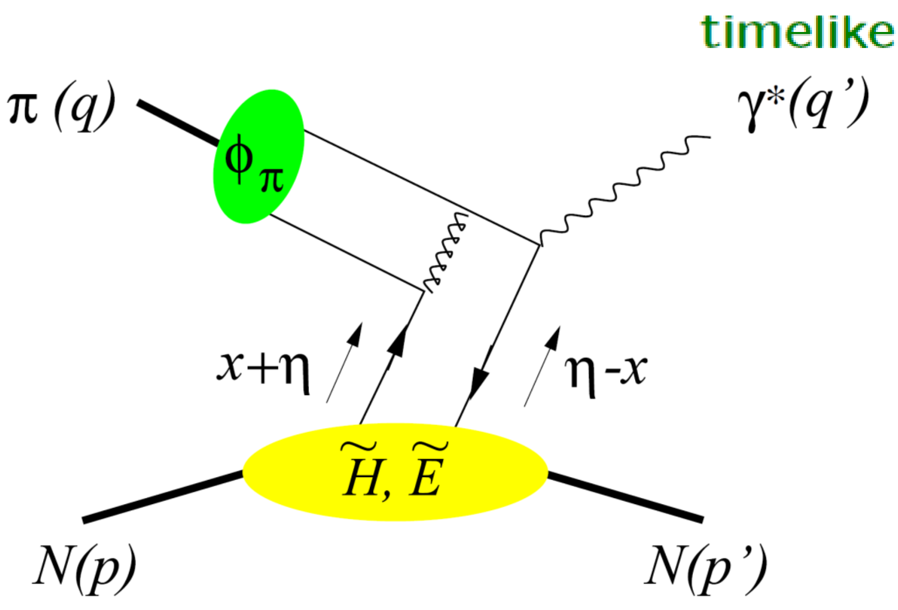
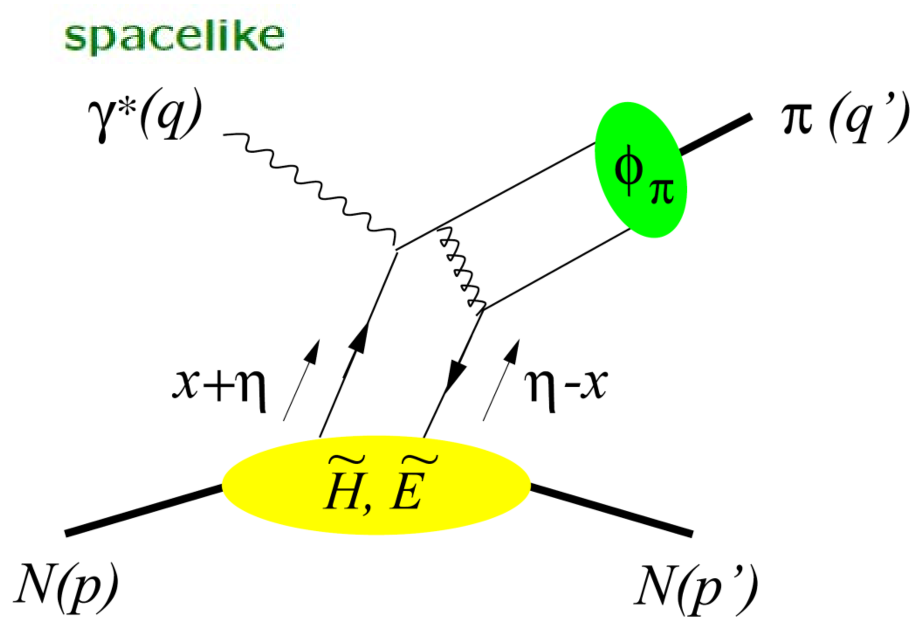
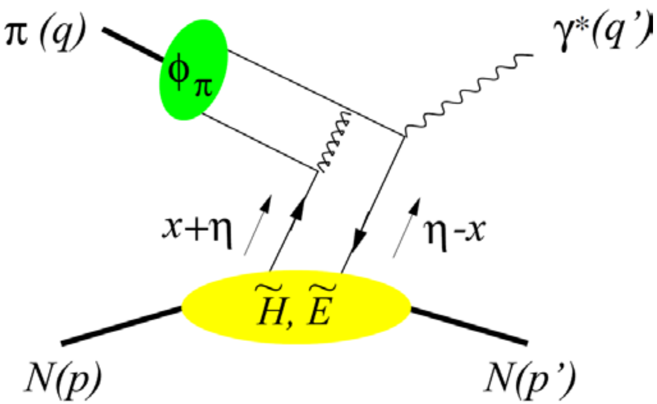
Pion beams reveal $\tilde{H}, \tilde{E}$ Generalized Parton distributions





exDY@J-PARC

DVMP@JLab



Bjorken variable: $\tau = \dfrac{Q'^2}{2p\cdot q}$

Skewness: $\eta = \dfrac{p^+ - p'^+}{p^+ + p'^+}$

Berger, Diehl, Pire, PLB523(2001)

$$\frac{d\sigma}{dQ'^2\,dt\,d(\cos\theta)\,d\varphi} = \frac{\alpha_{em}}{256\,\pi^3}\frac{\tau^2}{Q'^6}\sum_{\lambda',\lambda}|M^{0\lambda',\lambda}|^2\sin^2\theta$$

$$M^{0\lambda',\lambda}(\pi^- p \to \gamma^* n) = -ie\frac{4\pi}{3}\frac{f_\pi}{Q'}\frac{1}{(p+p')^+}\bar{u}(p',\lambda')\left[\gamma^+\gamma_5\,\tilde{\mathcal{H}}^{du}(\eta,t) + \gamma_5\frac{(p'-p)^+}{2M}\,\tilde{\mathcal{E}}^{du}(\eta,t)\right]u(p,\lambda)$$

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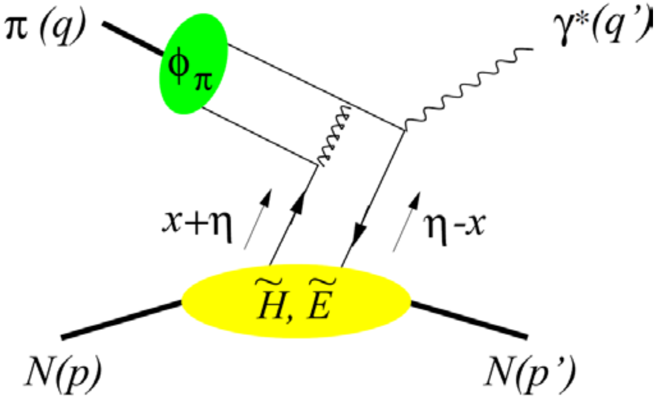

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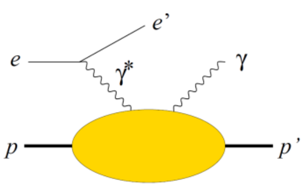

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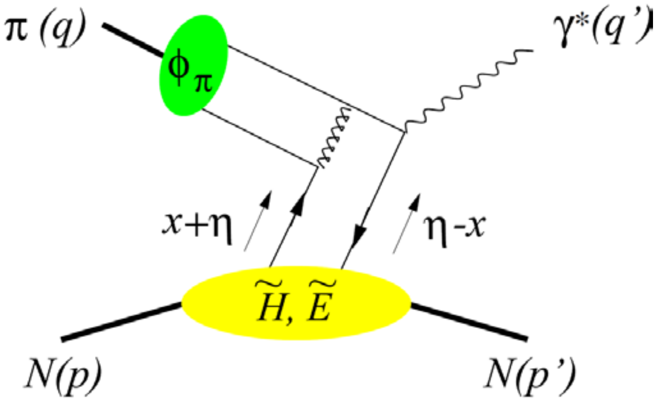
$$J_q = \frac{1}{2}\int_{-1}^{1}dx\,x\left(H^q(x,\eta,0) + E^q(x,\eta,0)\right)$$



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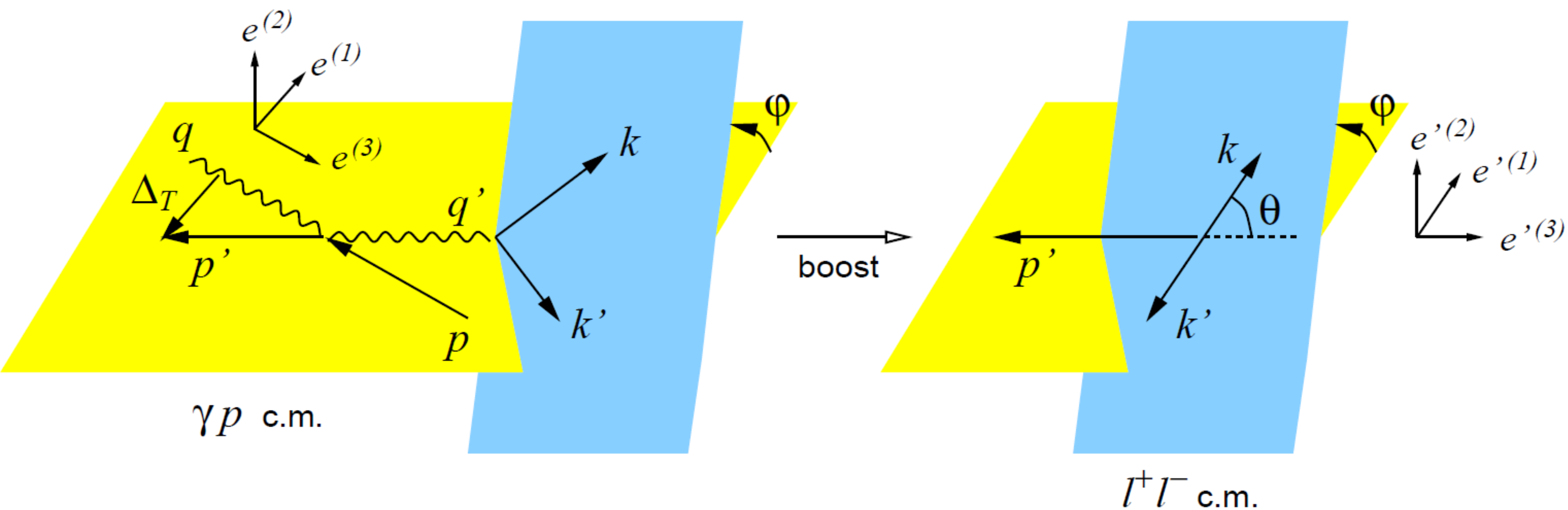

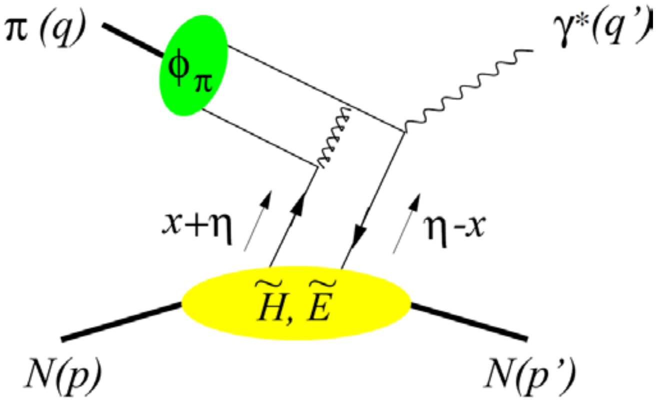

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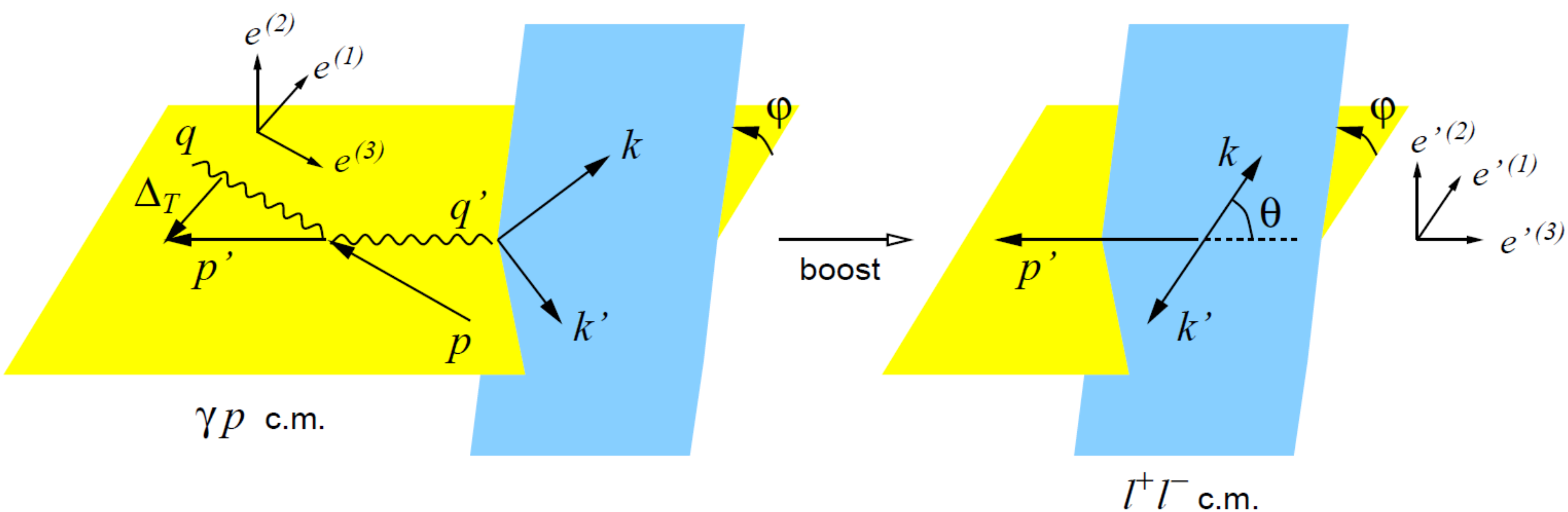

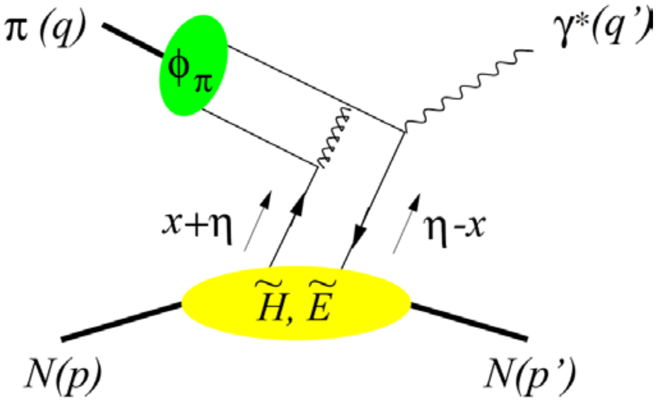

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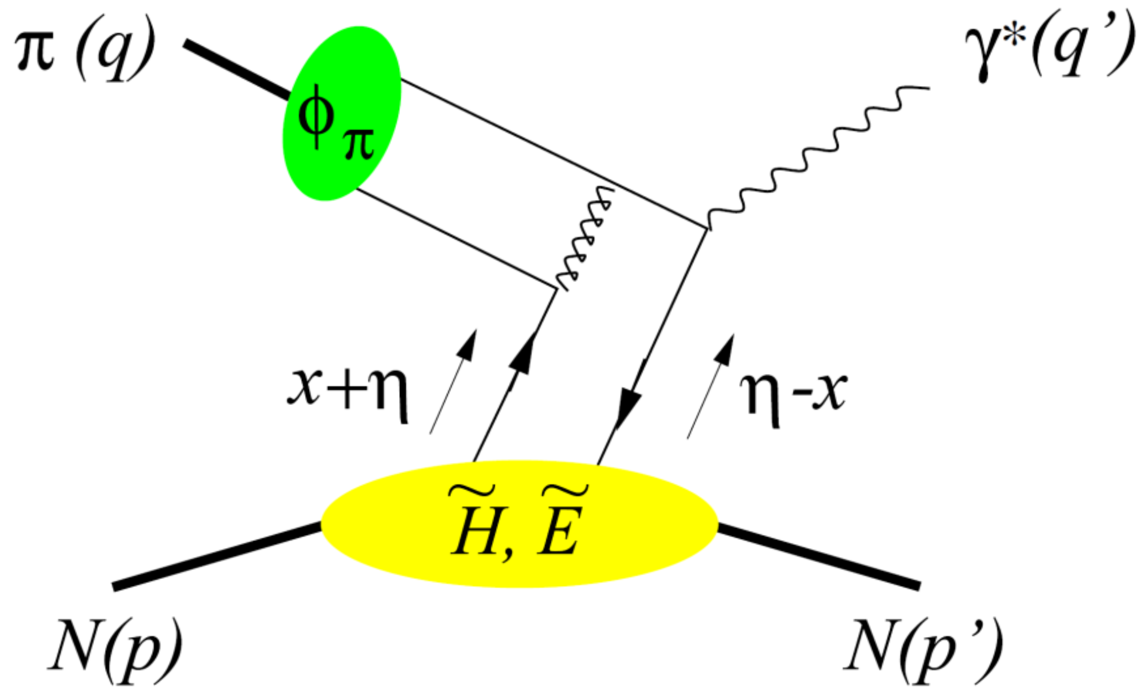
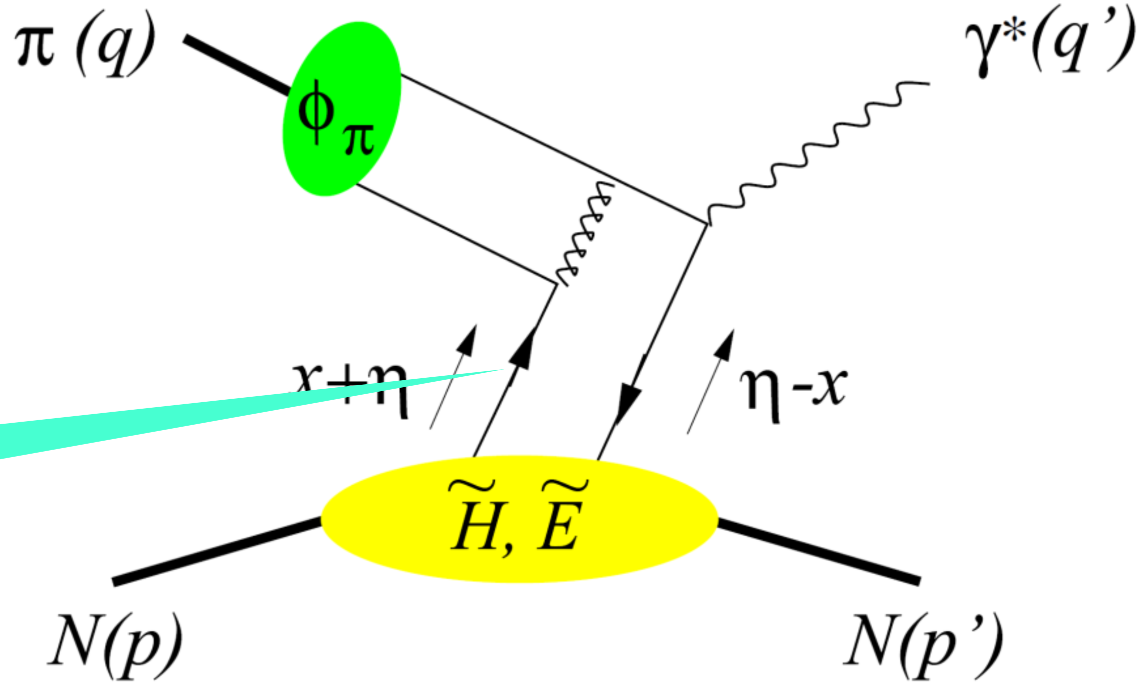
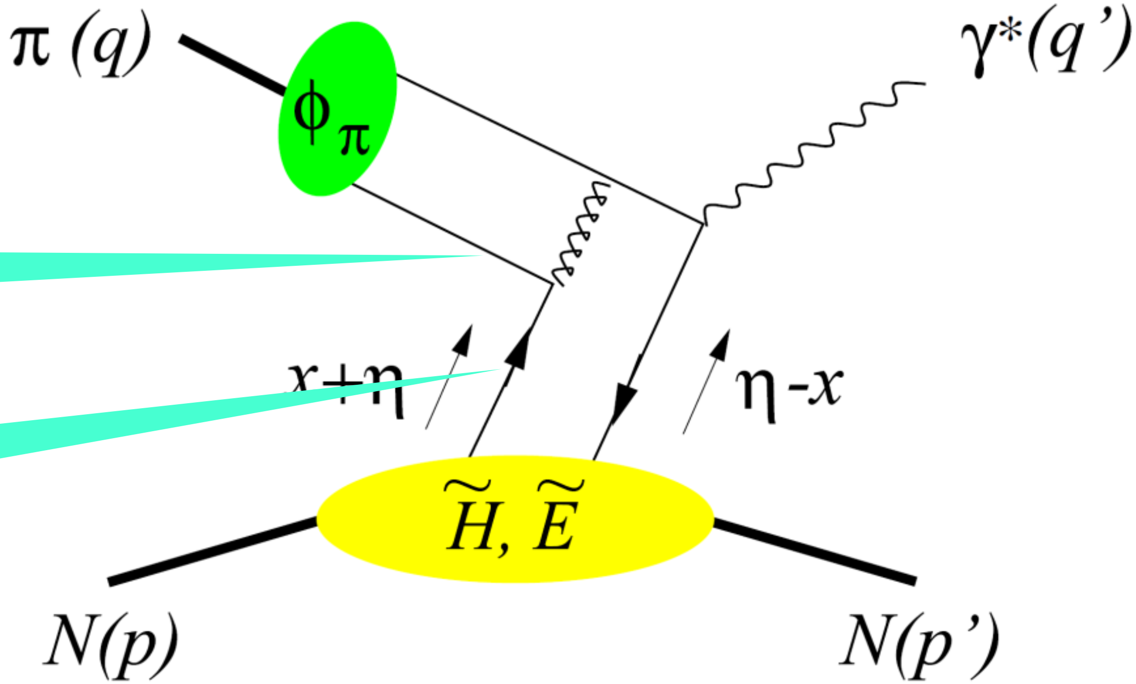


π (q)
φ_π
γ*(q')
x+η
η-x
H̃, Ẽ
N(p)
N(p')



$$\left(x+\eta\right)\bar{P}^{+}$$

$$\bar{P}^{+}=\frac{p^{+}+p'^{+}}{2}$$



uq^{-}

$(x+\eta)\bar{P}^{+}$

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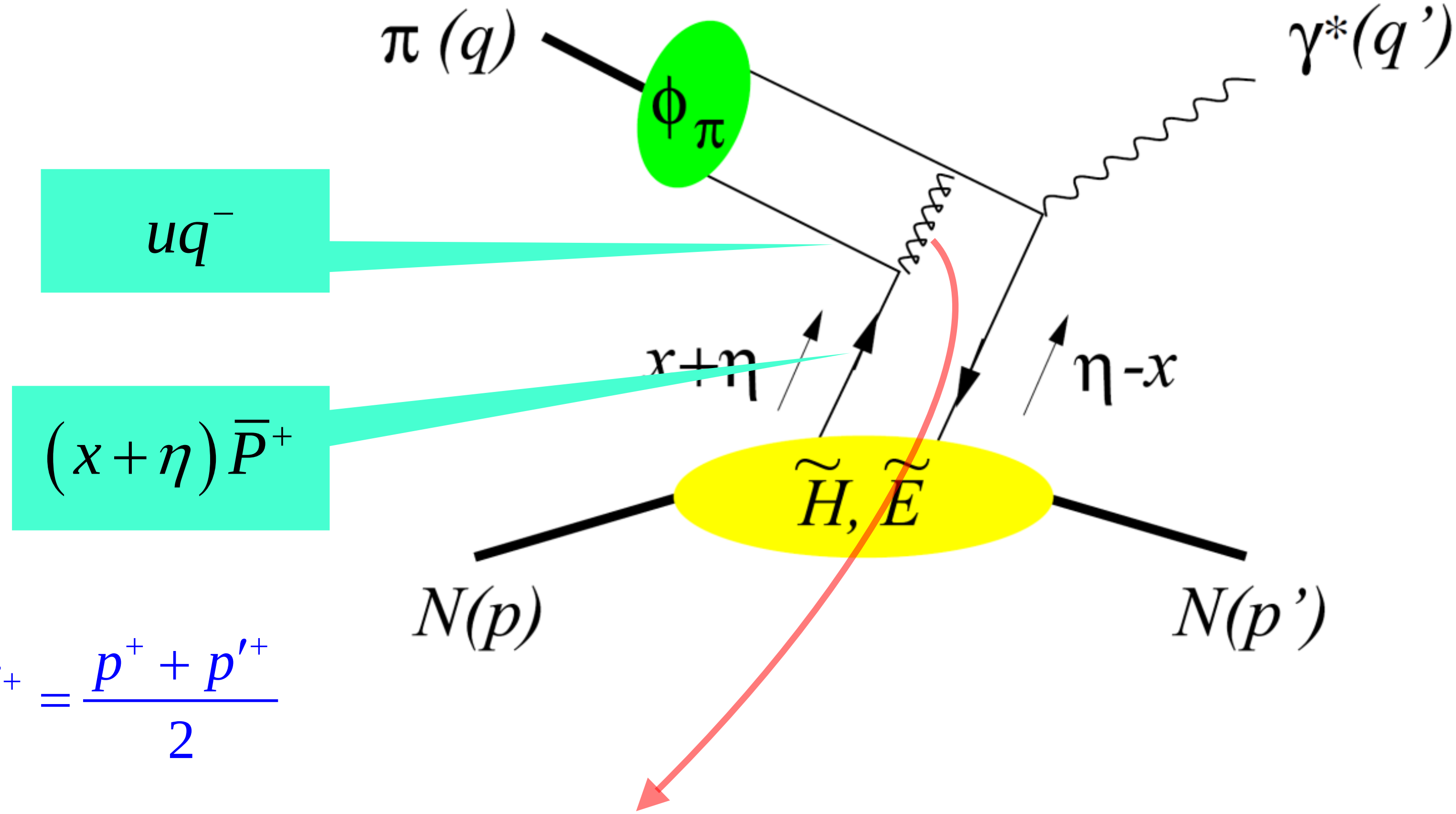
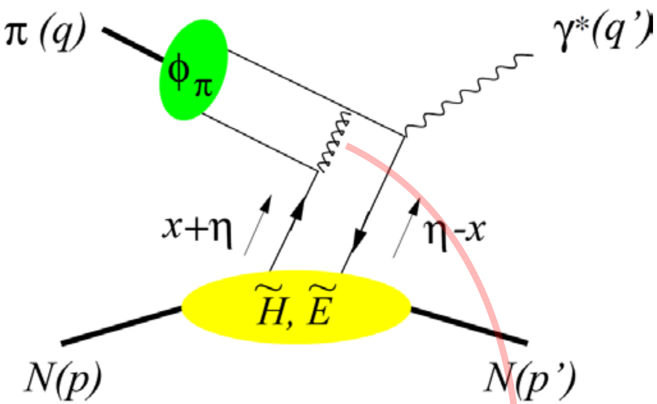


$$\bar{P}^{+} = \frac{p^{+} + p'^{+}}{2}$$

$$\frac{1}{\left[uq + (x+\eta)\bar{P}\right]^{2}} \simeq \frac{1}{u(x+\eta)s}$$



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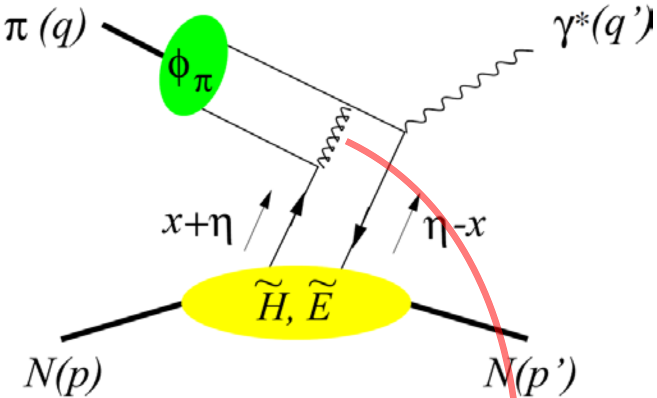

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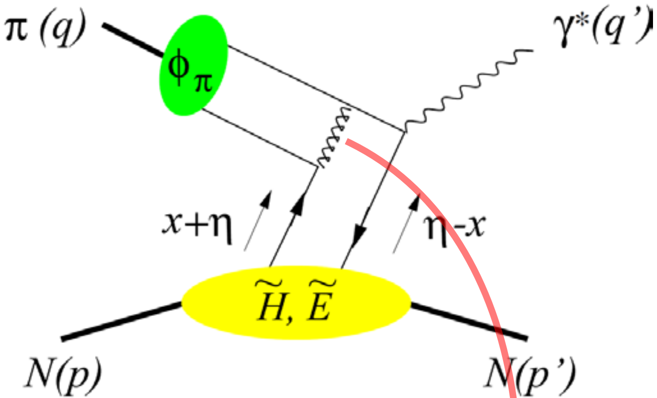
$$\phi_\pi(u) \sim u(1-u)$$



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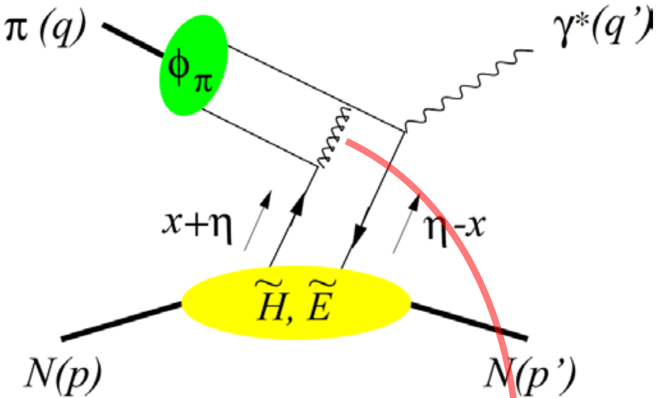
$$M^{0\lambda',\lambda}(\pi^- p\to\gamma^* n) = -ie\frac{4\pi}{3}\frac{f_\pi}{Q'}\frac{1}{(p+p')^+}\bar u(p',\lambda')\left[\gamma^+\gamma_5\,\tilde{\mathcal{H}}^{du}(\eta,t)+\gamma_5\frac{(p'-p)^+}{2M}\,\tilde{\mathcal{E}}^{du}(\eta,t)\right]u(p,\lambda)$$

$$\tilde{\mathcal{H}}^{du}(\eta,t) = \frac{8\alpha_S}{3}\int_0^1 du\,\frac{\phi_\pi(u)}{4u(1-u)}\int_{-1}^{1} dx\left[\frac{e_d}{-\eta-x-i\epsilon}-\frac{e_u}{-\eta+x-i\epsilon}\right]\left[\tilde H^d(x,\eta,t)-\tilde H^u(x,\eta,t)\right]$$

$$\int\frac{dz^-}{2\pi}e^{ix\bar P^+z^-}\langle p'|\bar\psi(-\frac{z^-}{2})\gamma^+\gamma_5\psi(\frac{z^-}{2})|p\rangle = \frac{1}{\bar P^+}\left[\tilde H^q(x,\eta,t)\bar u(p')\gamma^+\gamma_5 u(p)+\tilde E^q(x,\eta,t)\bar u(p')\frac{\gamma_5(p'-p)^+}{2M}u(p)\right]$$

$$\phi_\pi(u)\sim u(1-u)$$

$$M^{\pm1,\lambda;\lambda'}(\pi^- p\to\gamma^* n)\sim\frac{\alpha_s}{Q'^2}\int_0^1 du\,\frac{\phi_p(u)}{u(1-u)}\otimes\frac{1}{(\eta\pm x+i\varepsilon)^2}\otimes\left\{H_T^q(x,\eta,t),\tilde H_T^q(x,\eta,t),E_T^q(x,\eta,t),\tilde E_T^q(x,\eta,t)\right\}$$



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long. photon

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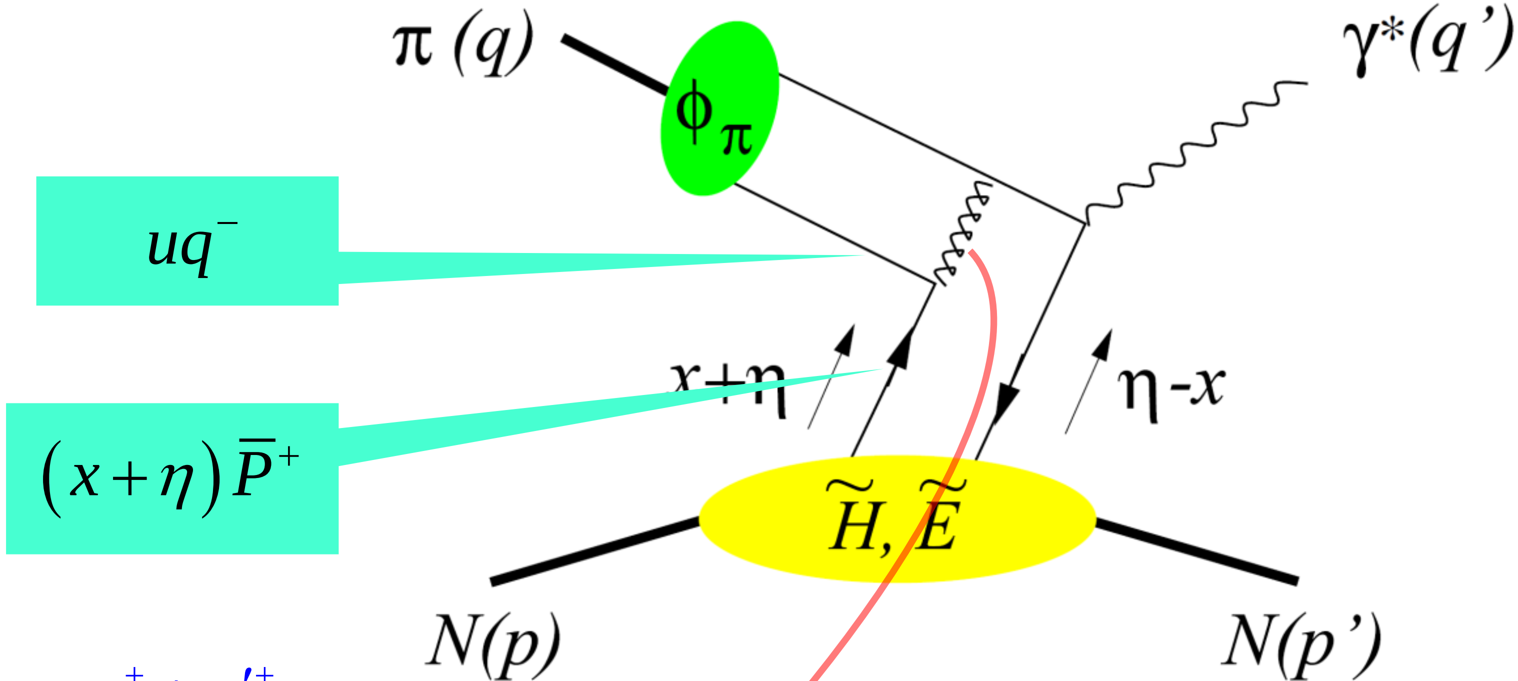
Collinear factorization does not work at twist-3:

- quark $k_\perp$ ("k_T-factorization") Goloskokov, Kroll

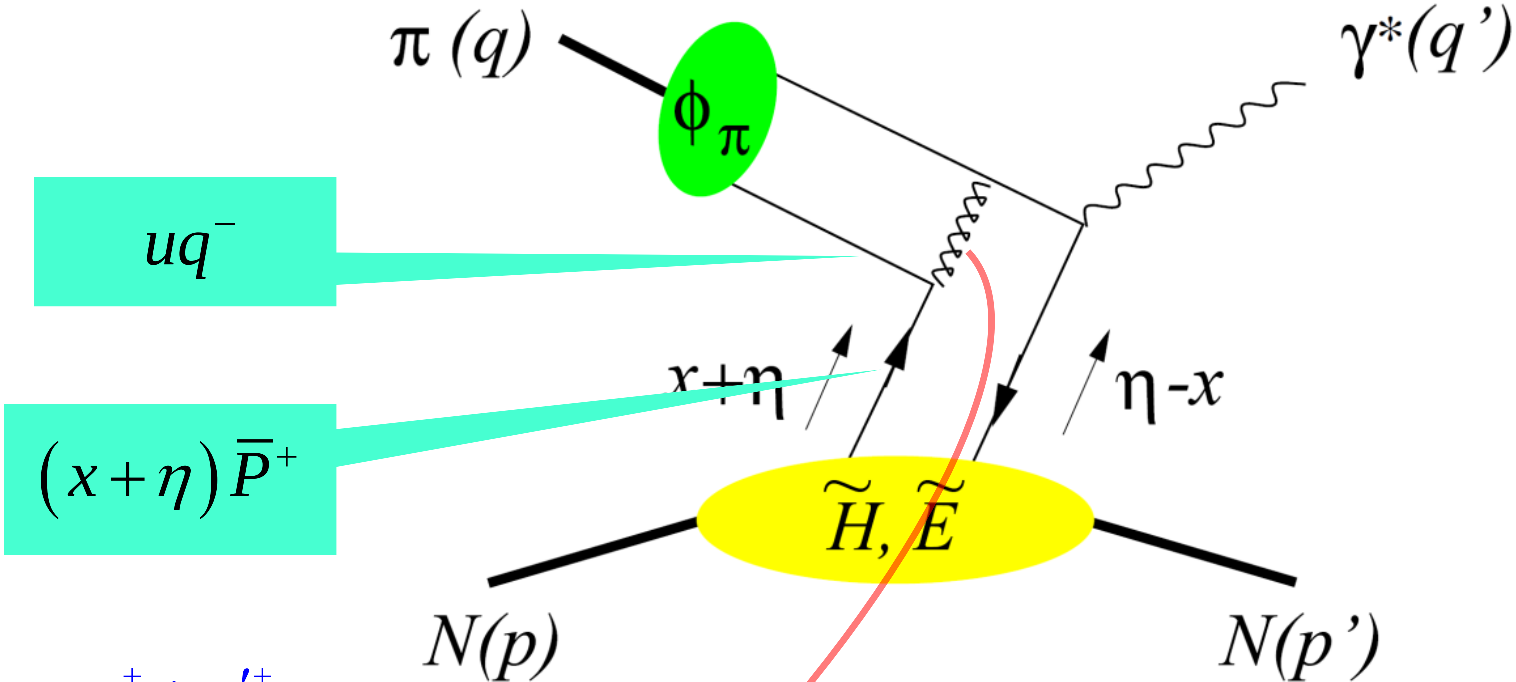
  with Sudakov resummation

  Li, Sterman



$$\bar{P}^{+} = \frac{p^{+} + p'^{+}}{2}$$

$$\frac{1}{\left[uq + (x+\eta)\bar{P}\right]^{2}} \simeq \frac{1}{u(x+\eta)s}$$



$$\bar{P}^+ = \frac{p^+ + p'^+}{2}$$

$$\frac{1}{\left[uq + (x+\eta)\bar{P}\right]^2 - \boldsymbol{k}_\perp^2} \simeq \frac{1}{u(x+\eta)s - \boldsymbol{k}_\perp^2}$$

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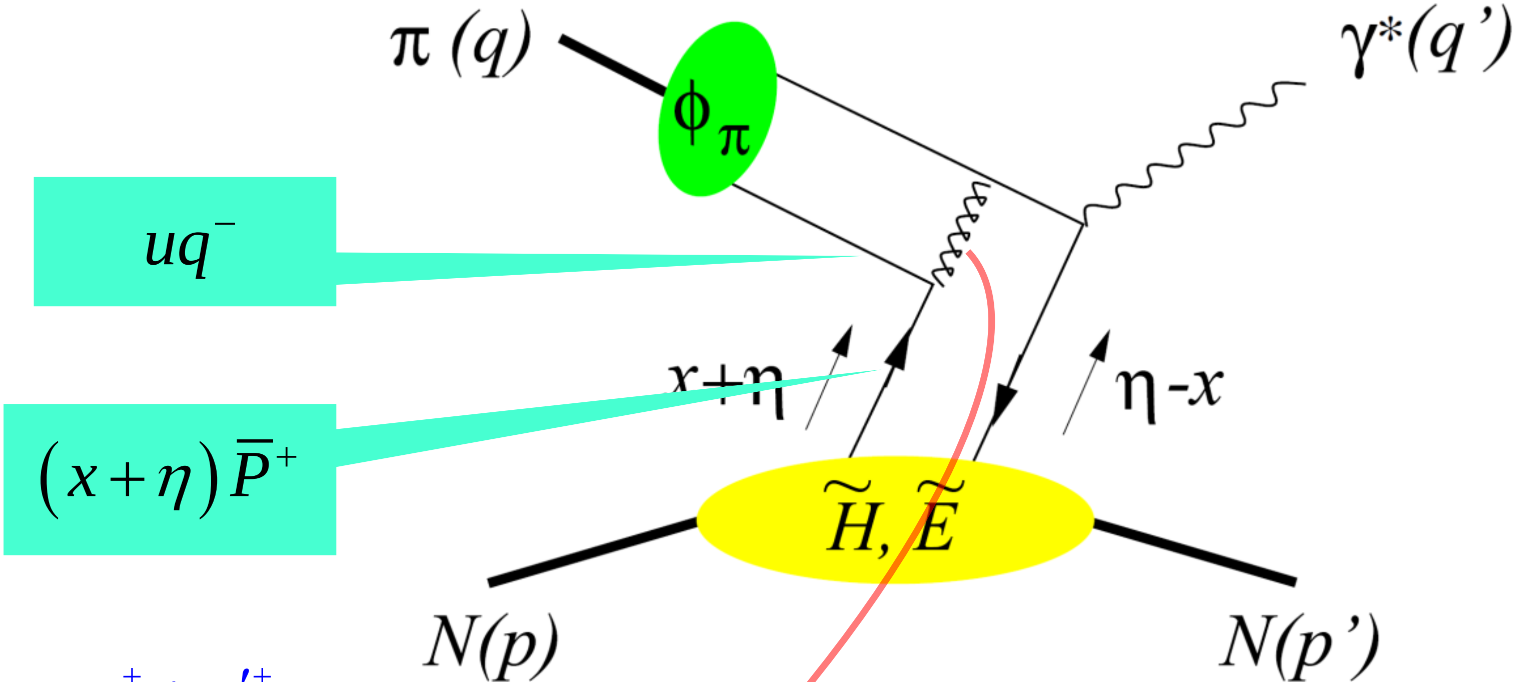

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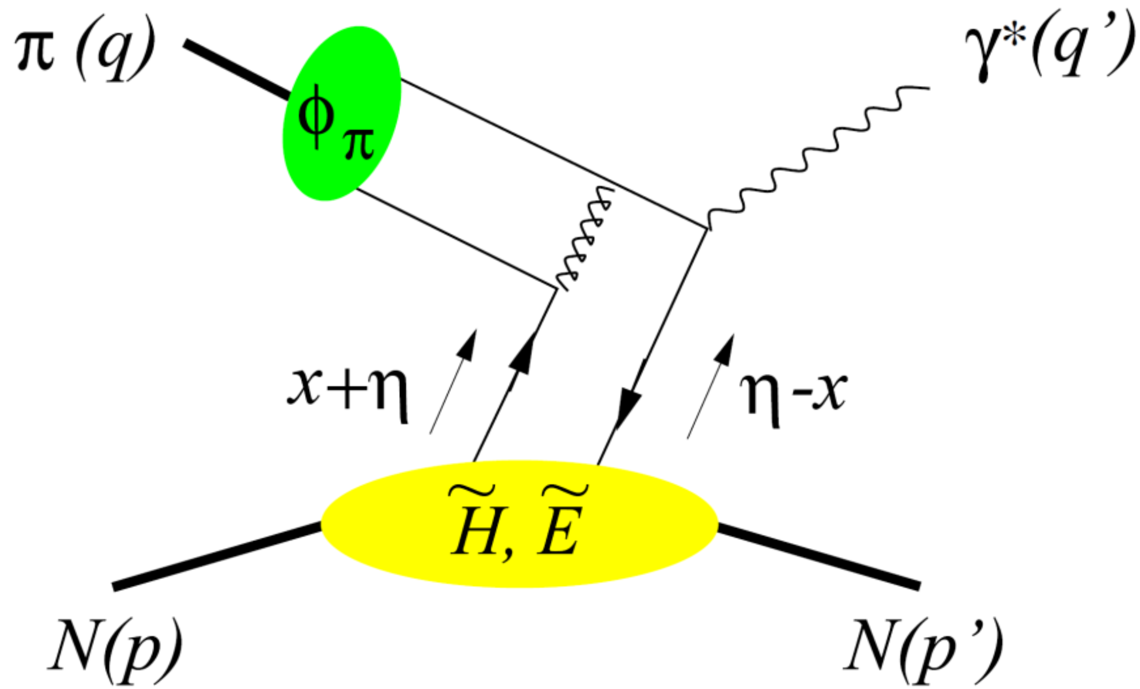
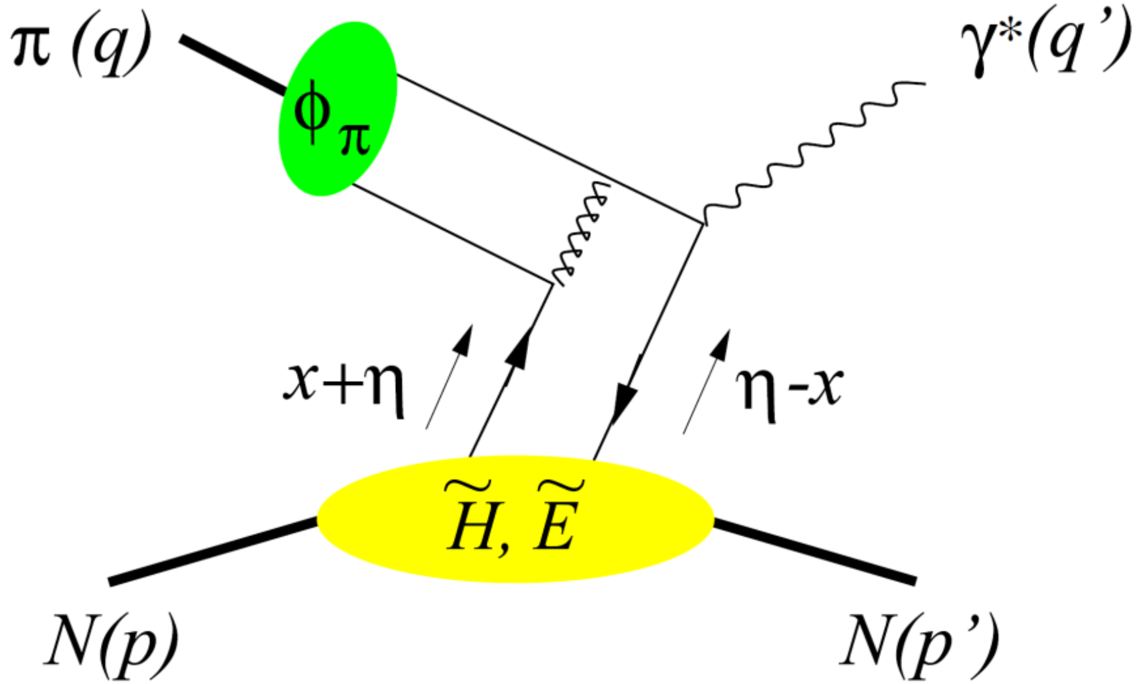


π (q)
γ*(q')
φ_π
x+η
η-x
H̃, Ẽ
N(p)
N(p')


π (q)
φπ
γ*(q')
x+η
η-x
H̃, Ẽ
N(p)
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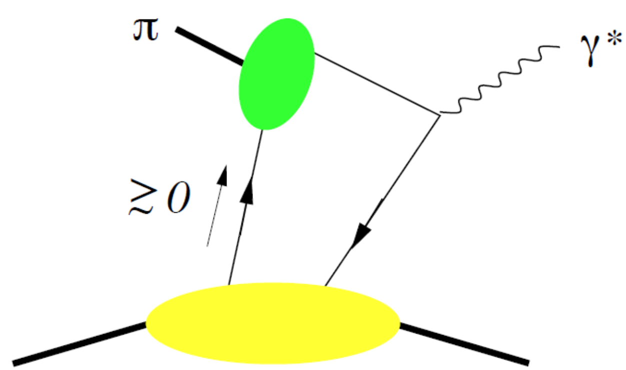
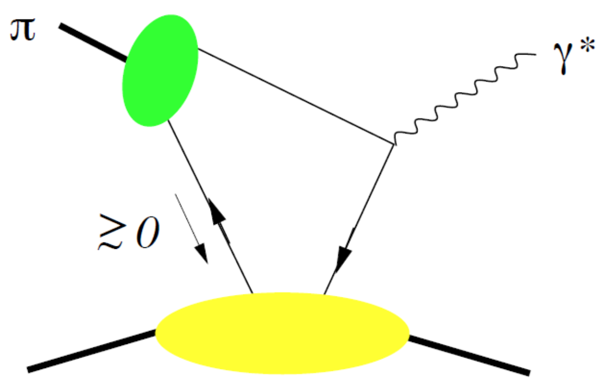
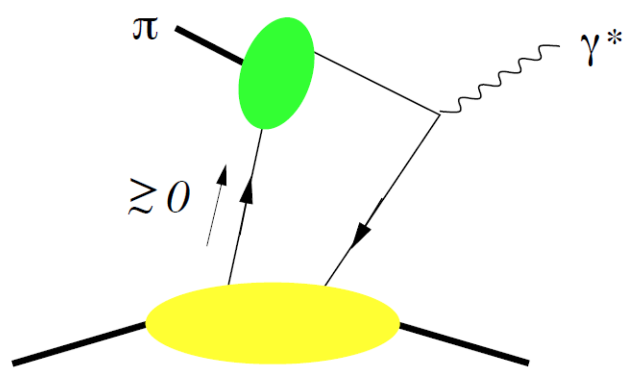
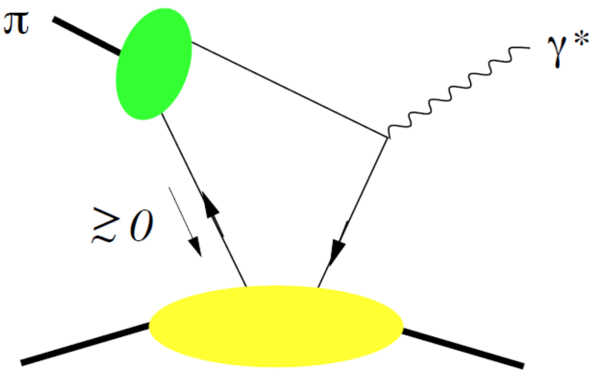

π
γ*
≳ 0

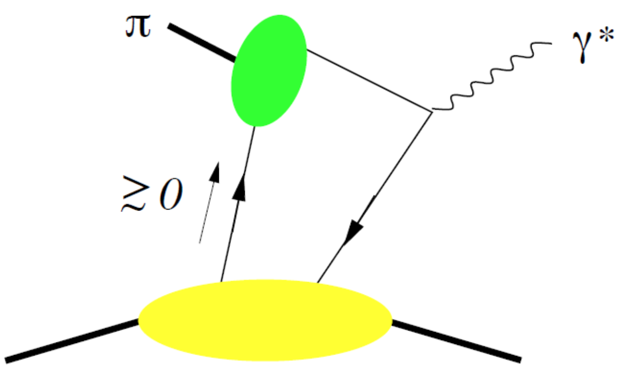
“nonfactorizable” mechanism





"nonfactorizable" mechanism





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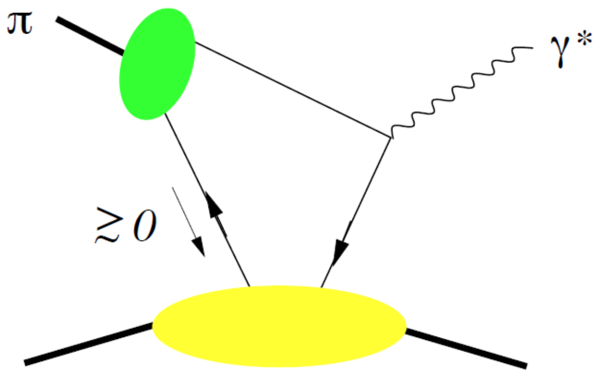
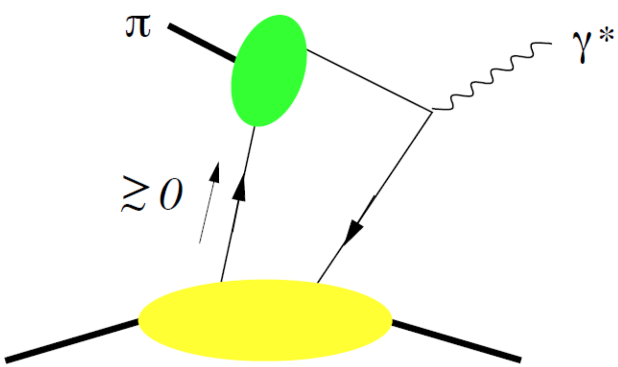


lower order in α_s



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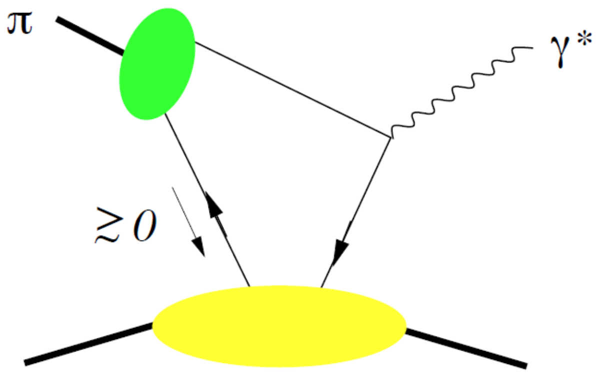
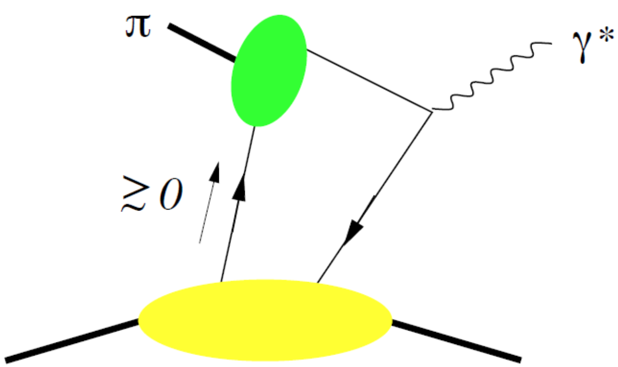


lower order in α_s

“Feynman mechanism”



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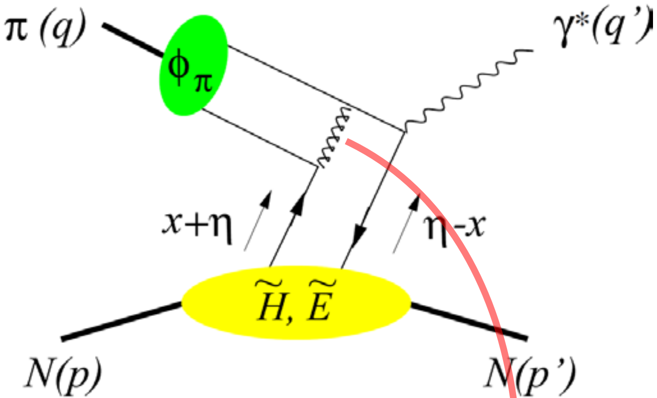
  nonfactorizable "Feynman mechanism" at lower order in α_s



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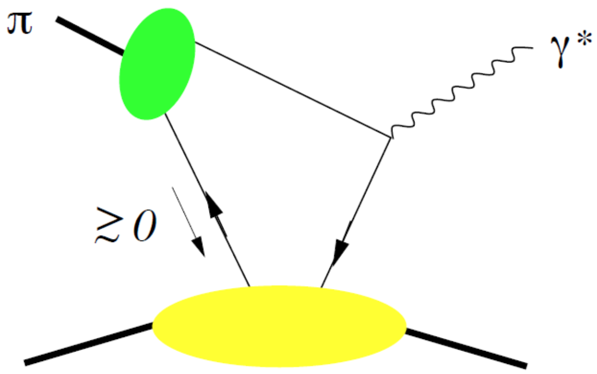
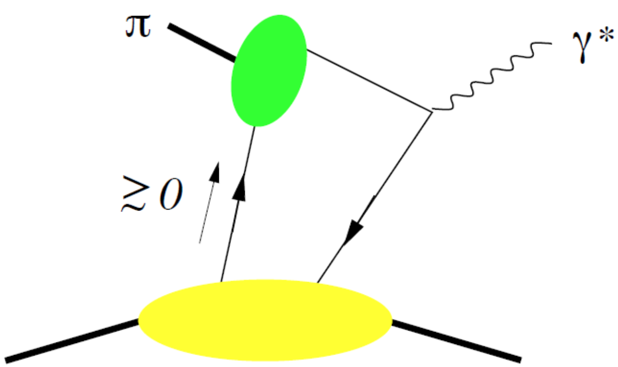
  nonfactorizable "Feynman mechanism" at lower order in α_s

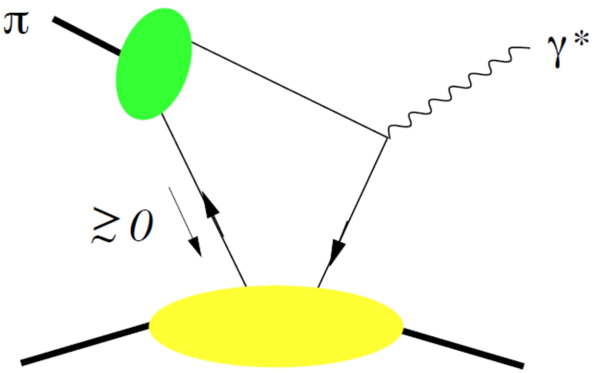
  relevent also for leading twist!

“nonfactorizable” mechanism

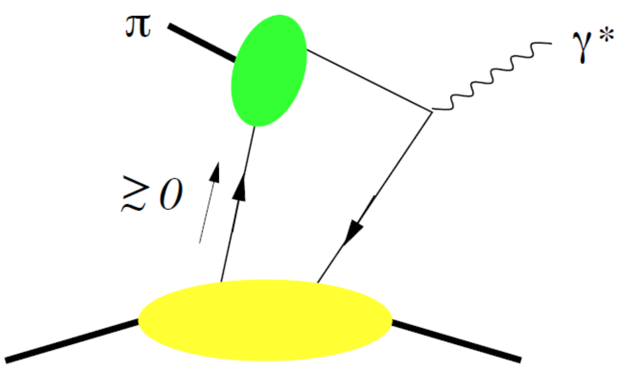




"nonfactorizable" mechanism



$$q^2 \neq m_\pi^2$$



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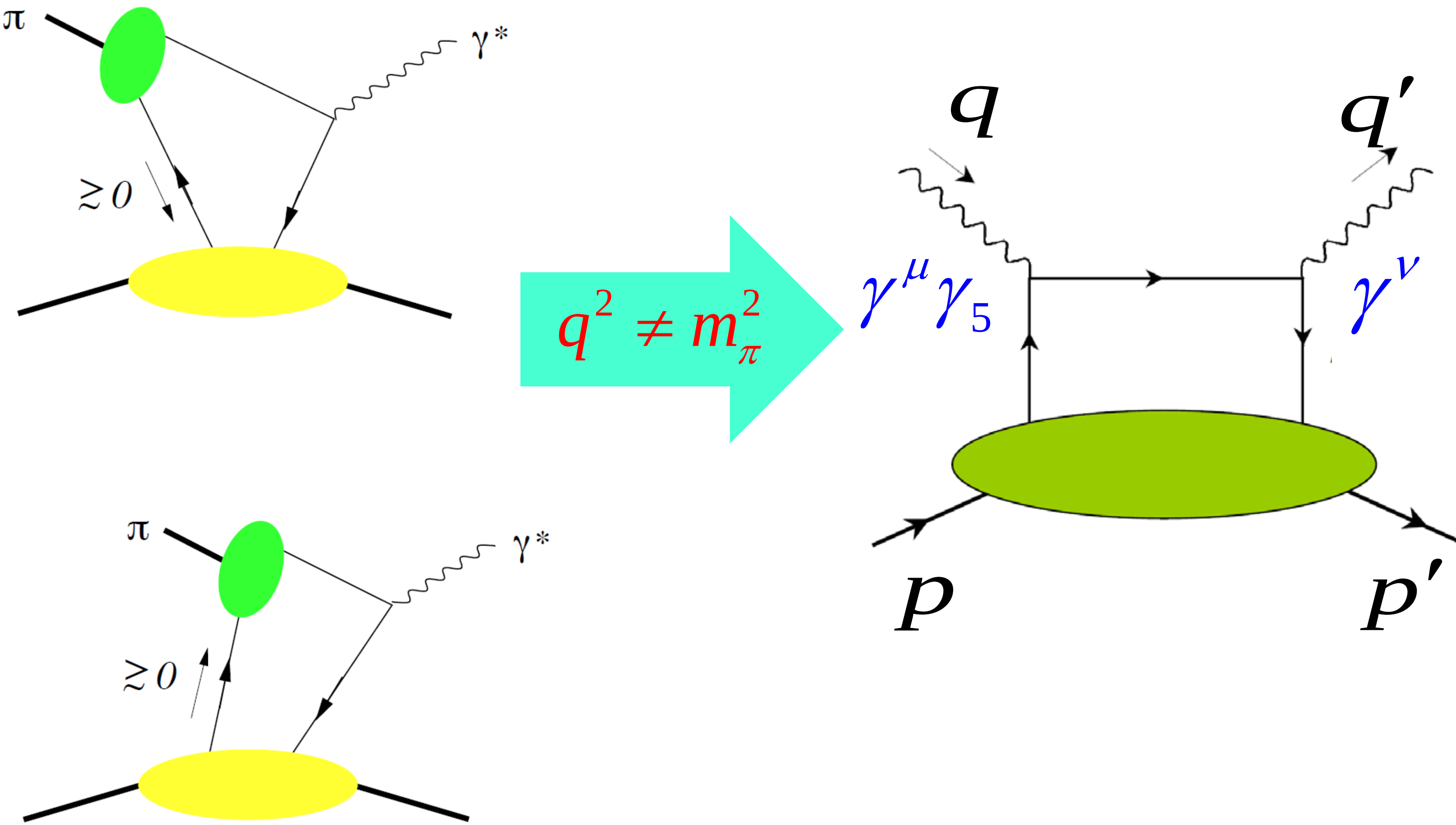

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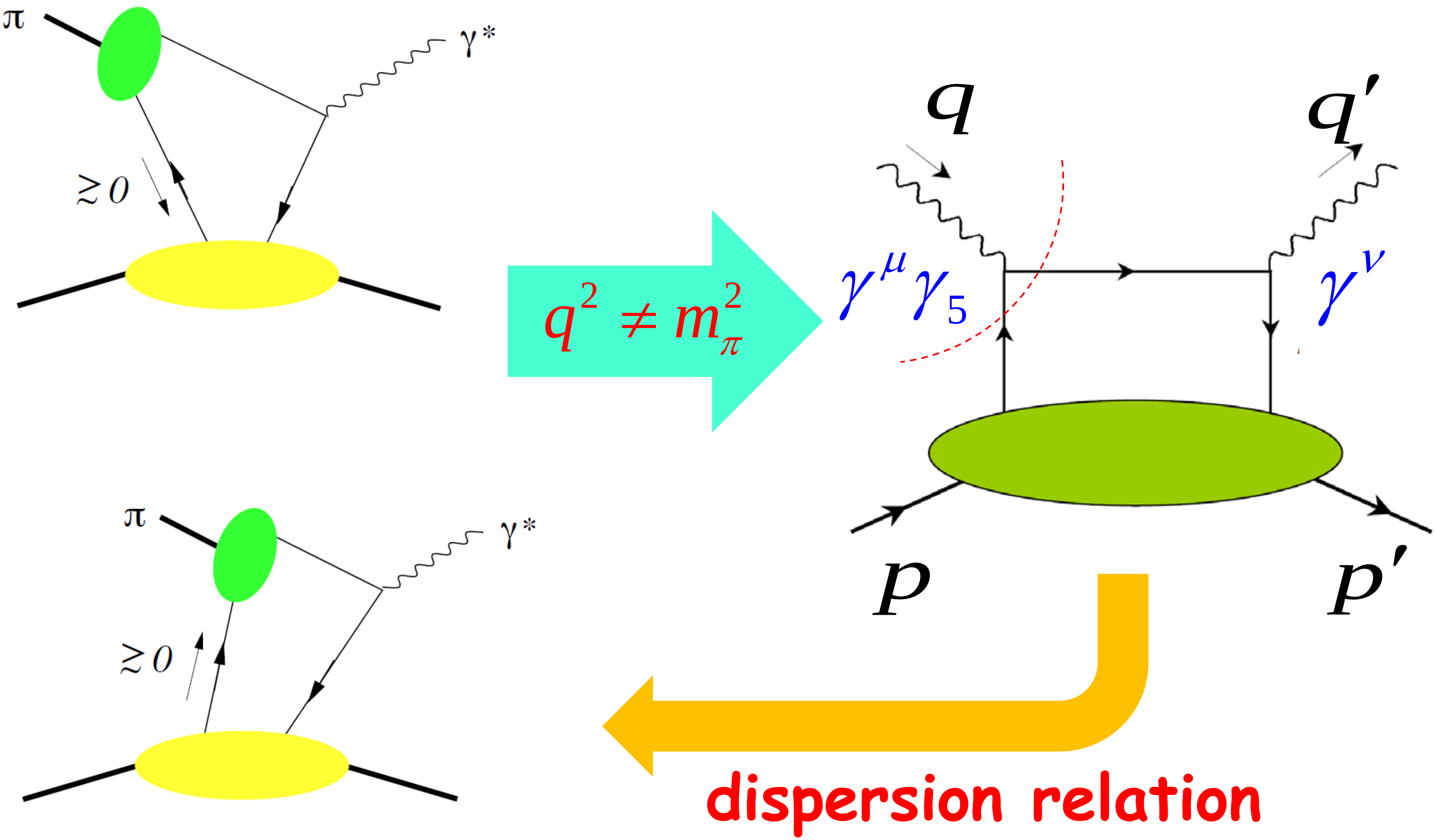

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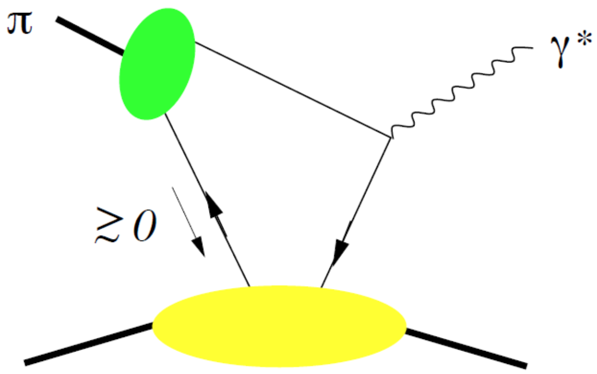
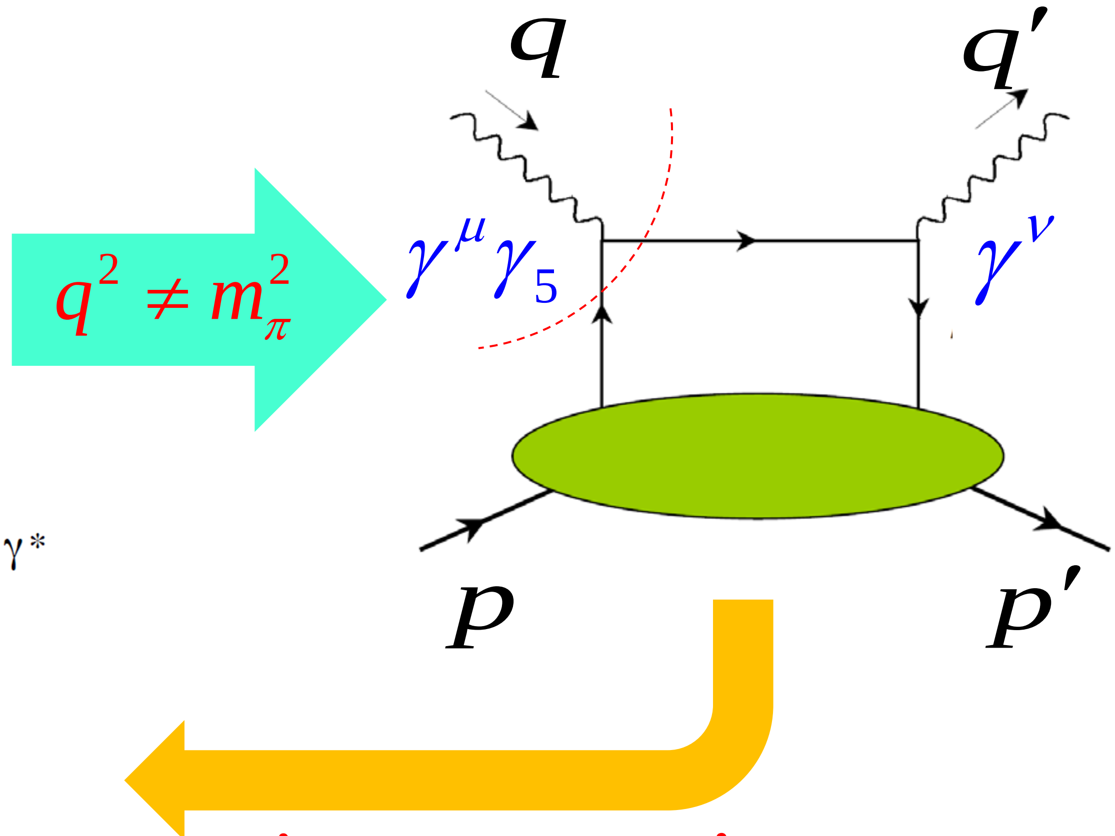
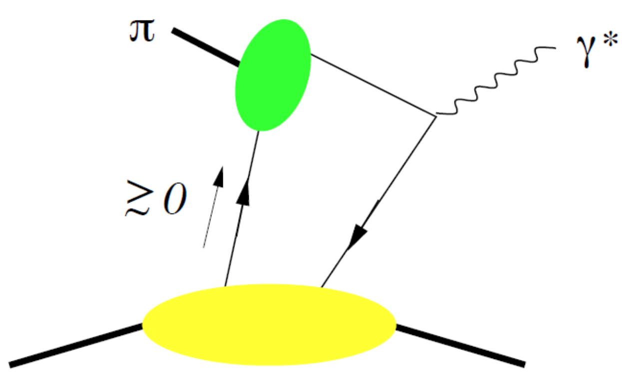
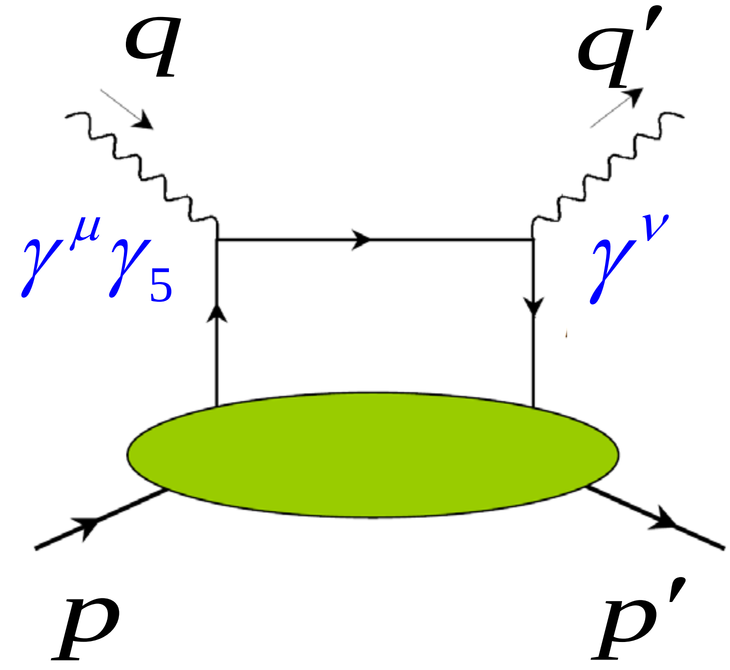






dispersion relation

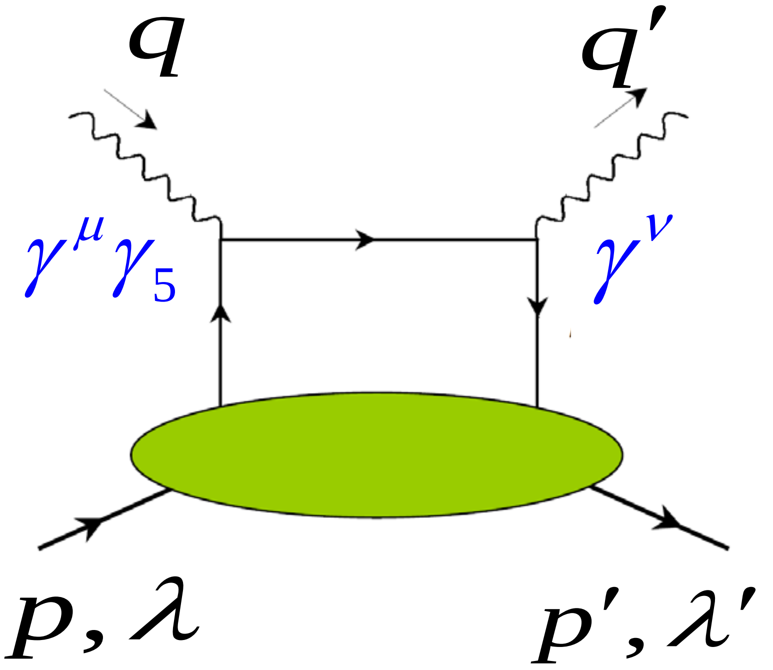
quark-hadron duality



q
q′
γ^μ γ_5
γ^ν
p
p′

$$\int d^4x e^{iq'\cdot x} \langle p'\lambda' | \mathrm{T}\, j^5_\mu(0)\, j^{\mathrm{em}}_\nu(x) | p\lambda \rangle$$

$$\equiv -iT_{\mu\nu}$$



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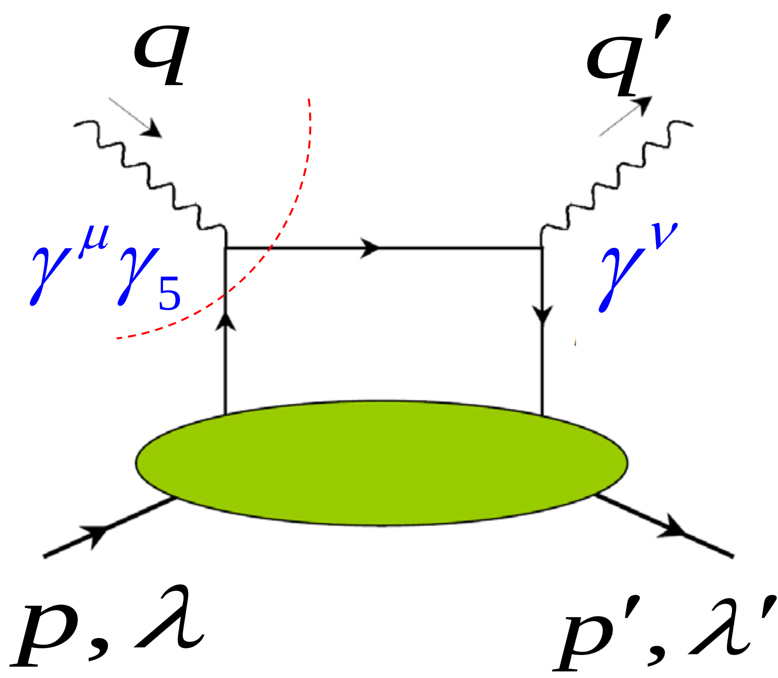

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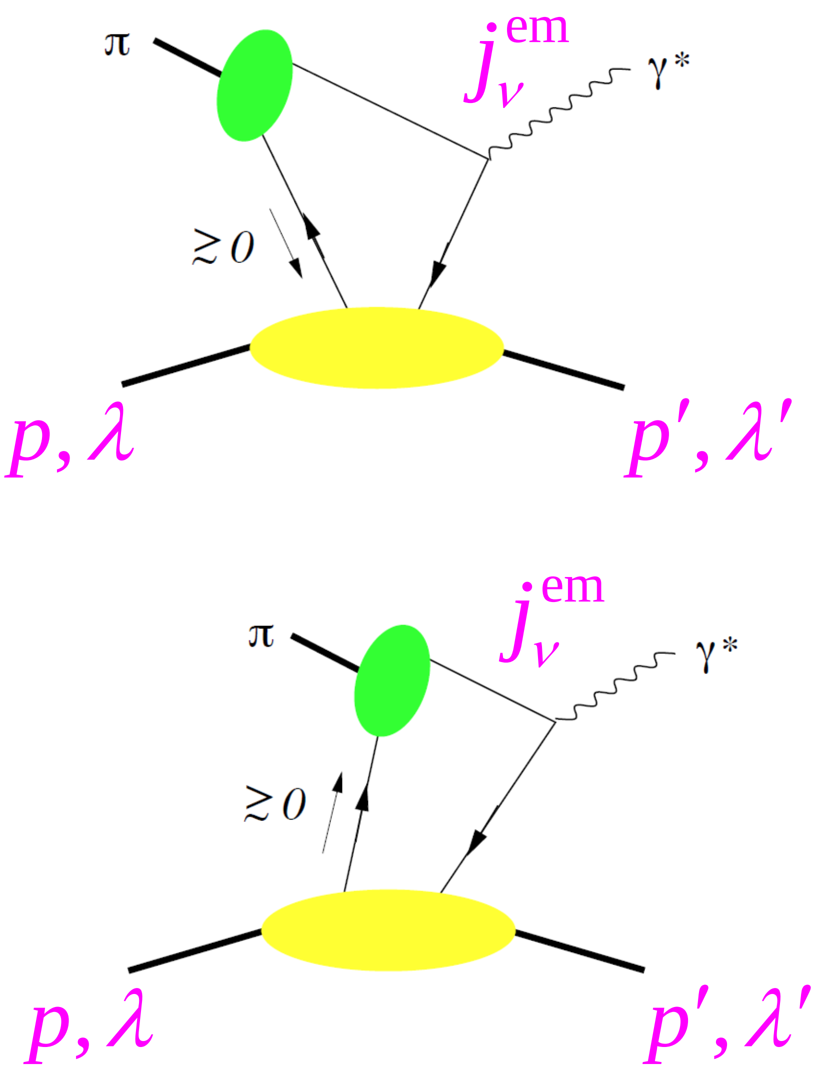
$$T_{\mu\nu} = iq_\mu f_\pi \frac{1}{q^2 - m_\pi^2} M_\nu^{\lambda',\lambda}(\pi^- p \to \gamma^* n) + \cdots$$

$$\langle 0 | j^5_\mu | \pi^-(k) \rangle = ik_\mu f_\pi$$

“nonfactorizable” mechanism



$$M_\nu^{\lambda',\lambda}(\pi^- p \to \gamma^* n)$$

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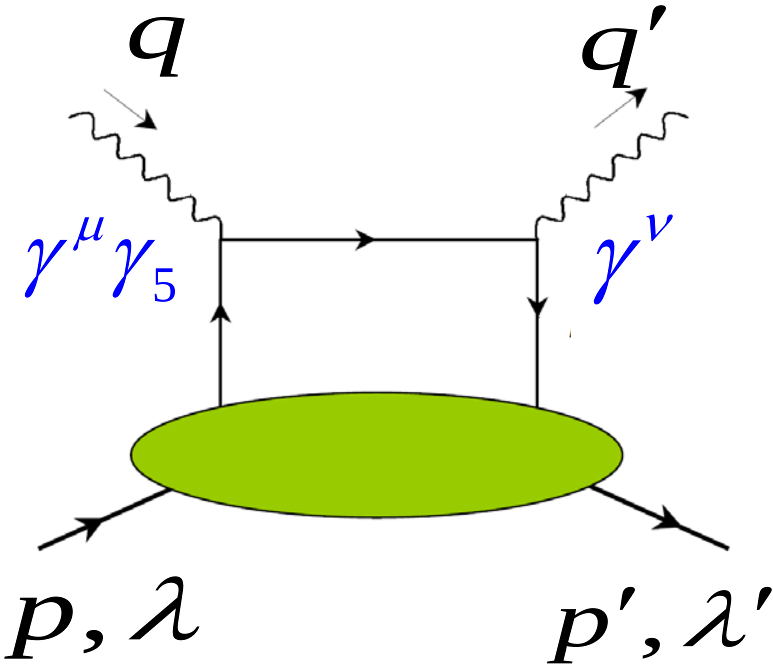

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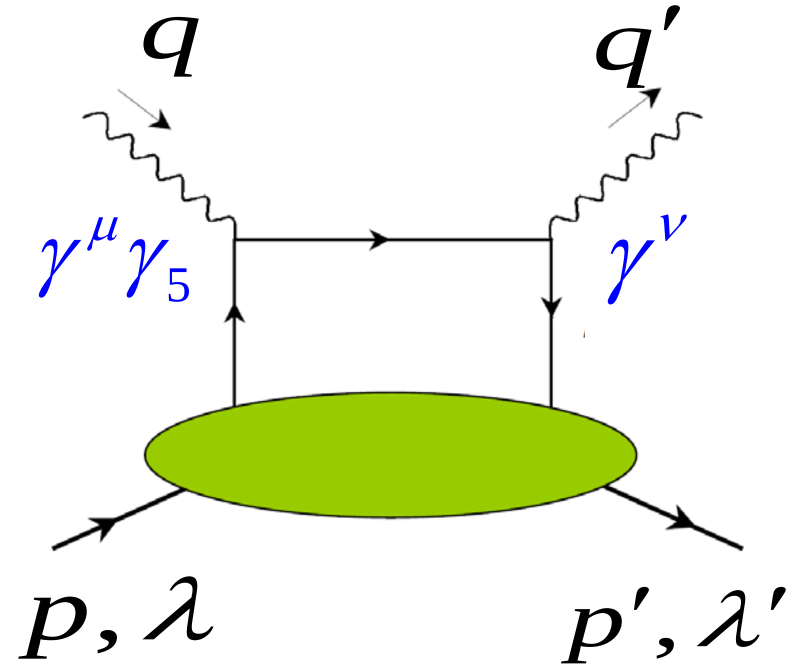

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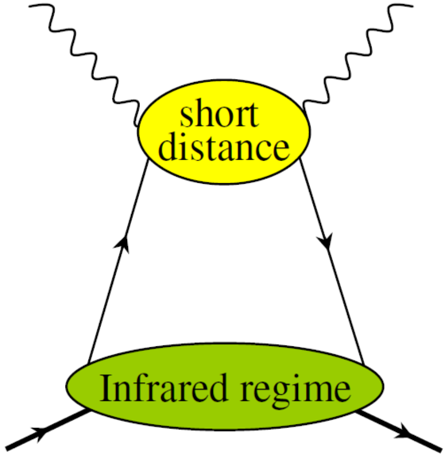
$$|q^2|, |q'^2| \gg \Lambda^2_{\mathrm{QCD}}$$



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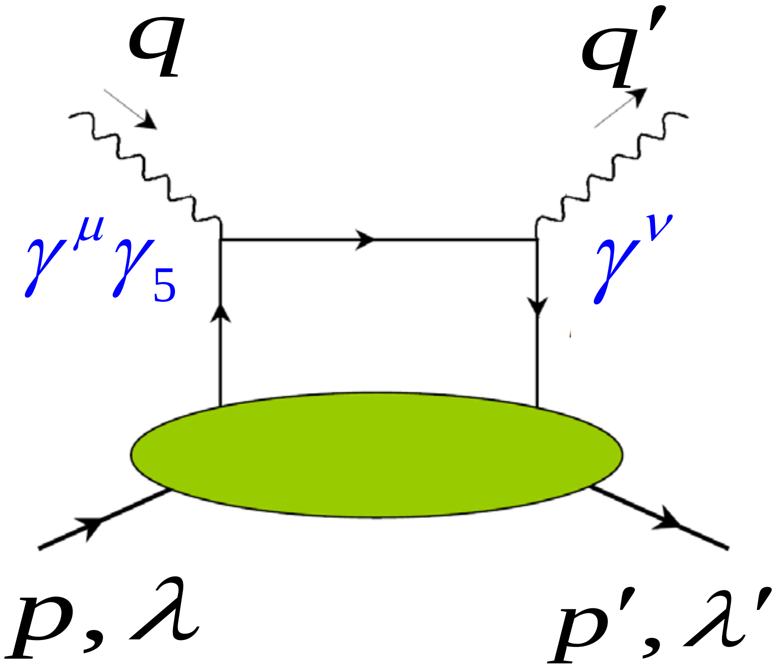

$$|q^2|, |q'^2| \gg \Lambda^2_{\mathrm{QCD}}$$

$$T_{\mu\nu} = -q_\mu g^-_\nu \int dx \frac{\frac{x+\eta}{2\eta}}{\frac{x-\eta}{2\eta}Q'^2 - \frac{x+\eta}{2\eta}q^2} \Big\{ \Big[e_u \tilde{H}^{du}(x,\eta,t) - e_d \tilde{H}^{du}(-x,\eta,t) \Big] \overline{u}(p'\lambda')\gamma^+\gamma_5 u(p\lambda)$$

$$+ \Big[e_u \tilde{E}^{du}(x,\eta,t) - e_d \tilde{E}^{du}(-x,\eta,t) \Big] \overline{u}(p'\lambda') \frac{\gamma_5 (p'-p)^+}{2M} u(p\lambda) \Big\} + \cdots$$

$$\int d^4x e^{iq'\cdot x} \langle p'\lambda' | \mathrm{T}\, j^5_\mu(0)\, j^{\mathrm{em}}_\nu(x) | p\lambda \rangle$$

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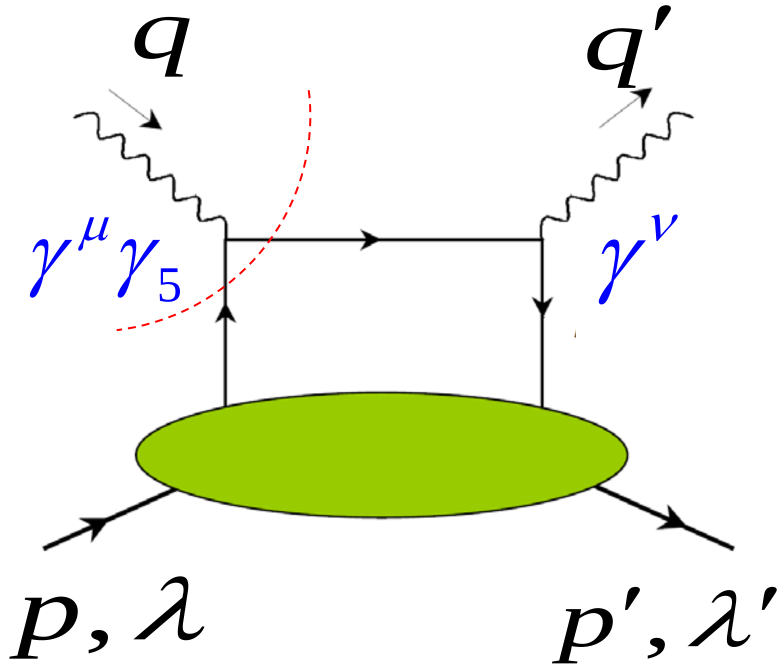

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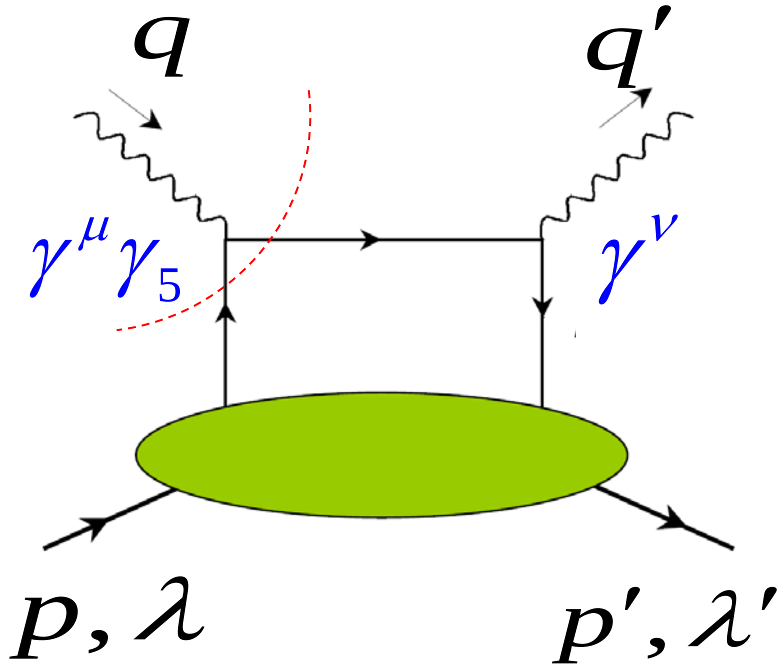

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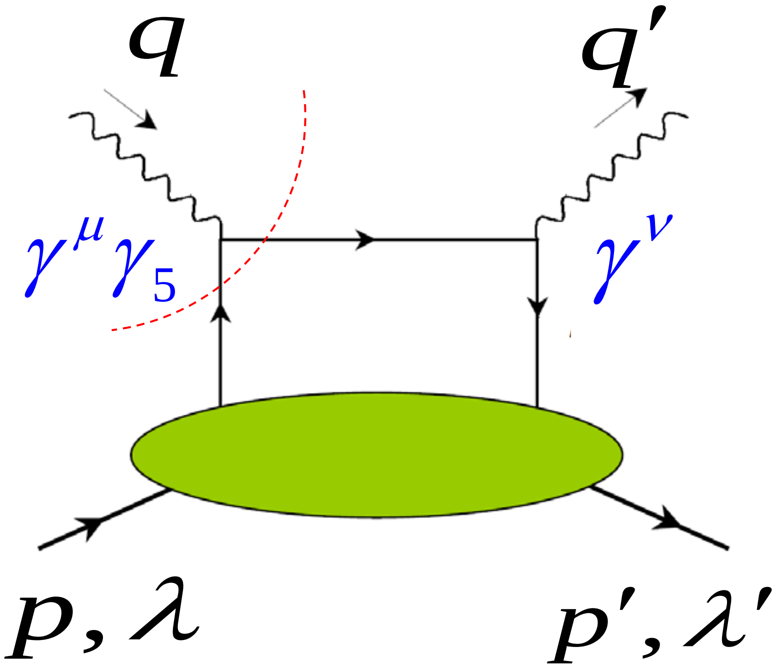

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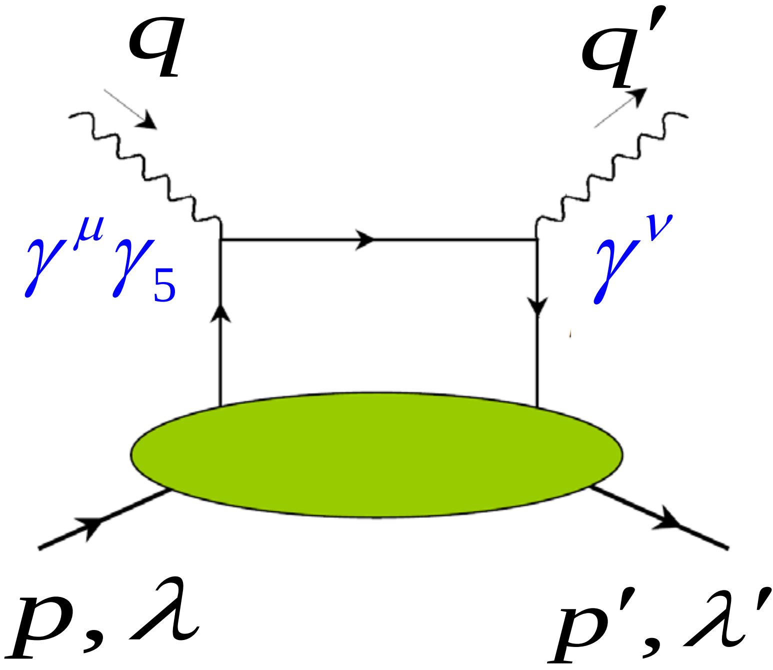
$$j^{\mathrm{em}}_\nu = e_u \bar{u}\gamma_\nu u + e_d \bar{d}\gamma_\nu d$$

$$T_{\mu\nu} = iq_\mu \left[f_\pi \frac{1}{q^2 - m_\pi^2} M_\nu^{\lambda',\lambda}(\pi^- p \to \gamma^* n) + \int_{q^2_{\mathrm{th}}}^\infty dm^2 \frac{\tilde{a}_\nu(m^2)}{q^2 - m^2} \right] + \cdots$$

$$\int d^4 x e^{iq'\cdot x}\langle p'\lambda' | \mathrm{T}\, j^5_\mu(0)\, j^{\mathrm{em}}_\nu(x) | p\lambda\rangle \equiv -iT_{\mu\nu}$$



$$|q^2|,|q'^2| \gg \Lambda^2_{\mathrm{QCD}}$$

$$T_{\mu\nu} = -q_\mu g^-_\nu \int dx \frac{\frac{x+\eta}{2\eta}}{\frac{x-\eta}{2\eta}Q'^2 - \frac{x+\eta}{2\eta}q^2}\Big\{\Big[e_u\tilde{H}^{du}(x,\eta,t) - e_d\tilde{H}^{du}(-x,\eta,t)\Big]\overline{u}(p'\lambda')\gamma^+\gamma_5 u(p\lambda)$$

$$+\Big[e_u\tilde{E}^{du}(x,\eta,t) - e_d\tilde{E}^{du}(-x,\eta,t)\Big]\overline{u}(p'\lambda')\frac{\gamma_5(p'-p)^+}{2M}u(p\lambda)\Big\} + \cdots$$

$$M_{\nu}^{\lambda',\lambda}(\pi^- p \to \gamma^* n)$$

$$= \frac{-i}{f_\pi} g_\nu^- \int_\eta^{x_0} dx\, e^{-\frac{x-\eta}{x+\eta}\frac{Q'^2}{M_B^2}} \left\{ \left[e_u \tilde{H}^{du}(x,\eta,t) - e_d \tilde{H}^{du}(-x,\eta,t) \right] \overline{u}(p'\lambda')\gamma^+\gamma_5 u(p\lambda) \right.$$

$$\left. + \left[e_u \tilde{E}^{du}(x,\eta,t) - e_d \tilde{E}^{du}(-x,\eta,t) \right] \overline{u}(p'\lambda') \frac{\gamma_5 (p'-p)^+}{2M} u(p\lambda) \right\}$$

$$\tilde{H}^{du}(x,\eta,t) = \tilde{H}^{u}(x,\eta,t) - \tilde{H}^{d}(x,\eta,t)$$

$$M_{\nu}^{\lambda',\lambda}(\pi^- p \to \gamma^* n)$$

$$= \frac{-i}{f_\pi} g_\nu^- \int_\eta^{x_0} dx\, e^{-\frac{x-\eta}{x+\eta}\frac{Q'^2}{M_B^2}} \Big\{ \Big[e_u \tilde{H}^{du}(x,\eta,t) - e_d \tilde{H}^{du}(-x,\eta,t) \Big] \overline{u}(p'\lambda')\gamma^+\gamma_5 u(p\lambda)$$

$$+ \Big[e_u \tilde{E}^{du}(x,\eta,t) - e_d \tilde{E}^{du}(-x,\eta,t) \Big] \overline{u}(p'\lambda') \frac{\gamma_5 (p'-p)^+}{2M} u(p\lambda) \Big\}$$

$$\tilde{H}^{du}(x,\eta,t) = \tilde{H}^{u}(x,\eta,t) - \tilde{H}^{d}(x,\eta,t)$$

$$x_0 = \eta \frac{Q'^2 + q_{\mathrm{th}}^2}{Q'^2 - q_{\mathrm{th}}^2}$$: **quark-hadron duality**

$$M_{\nu}^{\lambda',\lambda}(\pi^- p \to \gamma^* n)$$

$$= \frac{-i}{f_\pi} g_\nu^- \int_\eta^{x_0} dx\, e^{-\frac{x-\eta}{x+\eta}\frac{Q'^2}{M_B^2}} \Big\{ \Big[e_u \tilde{H}^{du}(x,\eta,t) - e_d \tilde{H}^{du}(-x,\eta,t) \Big] \overline{u}(p'\lambda')\gamma^+\gamma_5 u(p\lambda)$$

$$+ \Big[e_u \tilde{E}^{du}(x,\eta,t) - e_d \tilde{E}^{du}(-x,\eta,t) \Big] \overline{u}(p'\lambda') \frac{\gamma_5 (p'-p)^+}{2M} u(p\lambda) \Big\}$$

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Borel trnsf.: $$\hat{L}_{M_B}\left(\frac{1}{m^2 - q^2}\right) = \frac{1}{M_B^2} e^{-\frac{m^2}{M_B^2}}$$

$$M_{\nu}^{\lambda',\lambda}(\pi^- p \to \gamma^* n)$$

"Light-cone QCD SR (LCSR)"

$$= \frac{-i}{f_\pi} g_\nu^- \int_\eta^{x_0} dx\, e^{-\frac{x-\eta}{x+\eta}\frac{Q'^2}{M_B^2}} \left\{ \left[e_u \tilde{H}^{du}(x,\eta,t) - e_d \tilde{H}^{du}(-x,\eta,t) \right] \bar{u}(p'\lambda')\gamma^+\gamma_5 u(p\lambda) \right.$$

$$\left. + \left[e_u \tilde{E}^{du}(x,\eta,t) - e_d \tilde{E}^{du}(-x,\eta,t) \right] \bar{u}(p'\lambda') \frac{\gamma_5 (p'-p)^+}{2M} u(p\lambda) \right\}$$

$$\tilde{H}^{du}(x,\eta,t) = \tilde{H}^{u}(x,\eta,t) - \tilde{H}^{d}(x,\eta,t)$$

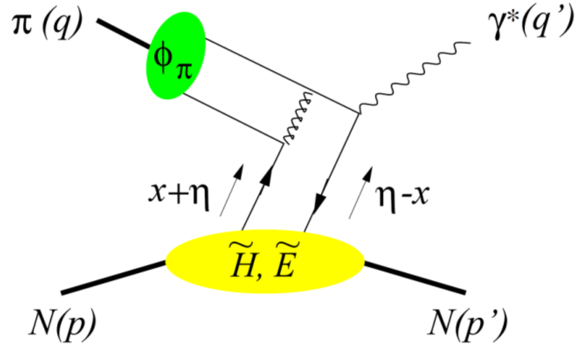
$$x_0 = \eta \frac{Q'^2 + q_{\mathrm{th}}^2}{Q'^2 - q_{\mathrm{th}}^2}$$: quark-hadron duality

Borel trnsf.: $\hat{L}_{M_B}\left(\frac{1}{m^2 - q^2}\right) = \frac{1}{M_B^2} e^{-\frac{m^2}{M_B^2}}$

long. photon

$$\frac{d\sigma}{dQ'^2\,dt\,d(\cos\theta)\,d\varphi} = \frac{\alpha_{em}}{256\,\pi^3}\frac{\tau^2}{Q'^6}\sum_{\lambda',\lambda}|M^{0\lambda',\lambda}|^2\sin^2\theta$$

$$\tau = \frac{Q'^2}{2p\cdot q} \qquad \eta = \frac{p^+ - p'^+}{p^+ + p'^+}$$



factorization

$$M^{0\lambda',\lambda}(\pi^- p \to \gamma^* n) = -ie\,\frac{4\pi}{3}\,\frac{f_\pi}{Q'}\,\frac{1}{(p+p')^+}\,\bar{u}(p',\lambda')\left[\gamma^+\gamma_5\,\tilde{\mathcal{H}}^{du}(\eta,t) + \gamma_5\frac{(p'-p)^+}{2M}\,\tilde{\mathcal{E}}^{du}(\eta,t)\right]u(p,\lambda)$$

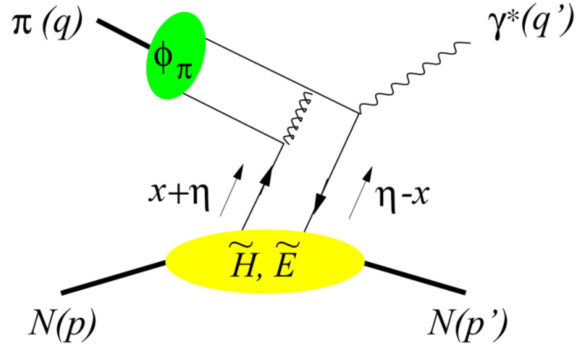
$$\tilde{\mathcal{H}}^{du}(\eta,t) = \frac{8\alpha_S}{3}\int_0^1 du\,\frac{\phi_\pi(u)}{4u(1-u)}\int_{-1}^1 dx\,\left[\frac{e_d}{-\eta-x-i\epsilon} - \frac{e_u}{-\eta+x-i\epsilon}\right]\left[\tilde{H}^d(x,\eta,t) - \tilde{H}^u(x,\eta,t)\right]$$

long. photon

$$\frac{d\sigma}{dQ'^2\,dt\,d(\cos\theta)\,d\varphi} = \frac{\alpha_{em}}{256\,\pi^3}\,\frac{\tau^2}{Q'^6}\,\sum_{\lambda',\lambda}|M^{0\lambda',\lambda}|^2\,\sin^2\theta$$

$$\tau = \frac{Q'^2}{2p\cdot q} \qquad \eta = \frac{p^+ - p'^+}{p^+ + p'^+}$$

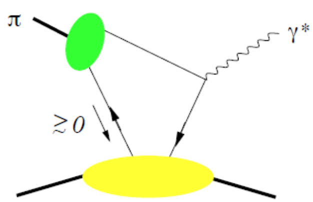
factorization



$$M^{0\lambda',\lambda}(\pi^- p \to \gamma^* n) = -ie\,\frac{4\pi}{3}\,\frac{f_\pi}{Q'}\,\frac{1}{(p+p')^+}\,\bar{u}(p',\lambda')\left[\gamma^+\gamma_5\,\tilde{\mathcal{H}}^{du}(\eta,t) + \gamma_5\,\frac{(p'-p)^+}{2M}\,\tilde{\mathcal{E}}^{du}(\eta,t)\right]u(p,\lambda)$$

$$\tilde{\mathcal{H}}^{du}(\eta,t) = \frac{8\alpha_S}{3}\int_0^1 du\,\frac{\phi_\pi(u)}{4u(1-u)}\int_{-1}^{1} dx\,\left[\frac{e_d}{-\eta-x-i\epsilon} - \frac{e_u}{-\eta+x-i\epsilon}\right]\left[\tilde{H}^d(x,\eta,t) - \tilde{H}^u(x,\eta,t)\right]$$

LCSR for nonfactorizable amp.



$$M^{0\lambda',\lambda}_{LCSR}(\pi^- p \to \gamma^* n)$$

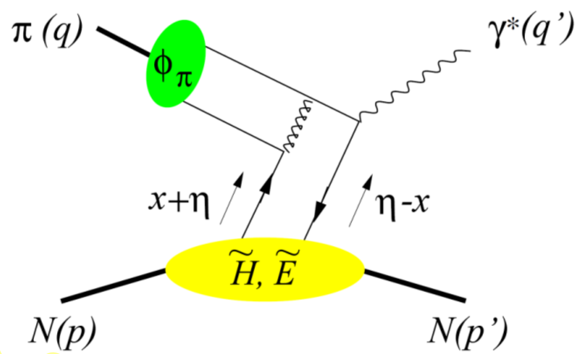
$$= -ie\,\frac{4q_{\rm th}^2\eta^2}{f_\pi Q'(1-\eta)^2}\,\frac{1}{(p+p')^+}\,\bar{u}(p',\lambda')\left[\gamma^+\gamma_5\tilde{H}^{du}_{LCSR}(\eta,t) + \gamma_5\,\frac{(p'-p)^+}{2M}\,\tilde{E}^{du}_{LCSR}(\eta,t)\right]u(p,\lambda)$$

$$\tilde{H}^{du}_{LCSR}(\eta,t) = \frac{Q'^2(1-\eta)^2}{8\eta^3 q_{\rm th}^2}\int_\eta^{x_0} dx\; e^{-\frac{x-\eta}{x+\eta}\frac{Q'^2}{M_B^2}}\left\{e_d\left[\tilde{H}^d(-x,\eta,t) - \tilde{H}^u(-x,\eta,t)\right] - e_u\left[x \to -x\right]\right\}$$

long. photon

$$\frac{d\sigma}{dQ'^2\, dt\, d(\cos\theta)\, d\varphi} = \frac{\alpha_{em}}{256\,\pi^3}\frac{\tau^2}{Q'^6}\sum_{\lambda',\lambda}|M^{0\lambda',\lambda}|^2\sin^2\theta$$

$$\tau = \frac{Q'^2}{2p\cdot q} \qquad \eta = \frac{p^+ - p'^+}{p^+ + p'^+}$$



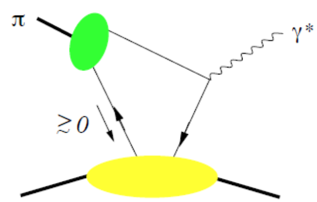
factorization

ERBL

$$M^{0\lambda',\lambda}(\pi^- p \to \gamma^* n) = -ie\frac{4\pi}{3}\frac{f_\pi}{Q'}\frac{1}{(p+p')^+}\bar{u}(p',\lambda')\left[\gamma^+\gamma_5\,\tilde{\mathcal{H}}^{du}(\eta,t) + \gamma_5\frac{(p'-p)^+}{2M}\,\tilde{\mathcal{E}}^{du}(\eta,t)\right]u(p,\lambda)$$

$$\tilde{\mathcal{H}}^{du}(\eta,t) = \frac{8\alpha_S}{3}\int_0^1 du\,\frac{\phi_\pi(u)}{4u(1-u)}\int_{-1}^{1}dx\left[\frac{e_d}{-\eta-x-i\epsilon} - \frac{e_u}{-\eta+x-i\epsilon}\right]\left[\tilde{H}^d(x,\eta,t) - \tilde{H}^u(x,\eta,t)\right]$$

LCSR for nonfactorizable amp.



$$M^{0\lambda',\lambda}_{LCSR}(\pi^- p \to \gamma^* n)$$

$$= -ie\frac{q_{\rm th}^2}{f_\pi Q'}\frac{1}{(p+p')^+}\bar{u}(p',\lambda')\left[\gamma^+\gamma_5\,\tilde{H}^{du}_{LCSR}(\eta,t) + \gamma_5\frac{(p'-p)^+}{2M}\tilde{E}^{du}_{LCSR}(\eta,t)\right]u(p,\lambda)$$

DGLAP

$$\tilde{H}^{du}_{LCSR}(\eta,t) = \frac{Q'^2}{2\eta q_{\rm th}^2}\int_\eta^{x_0}dx\, e^{-\frac{x-\eta}{x+\eta}\frac{Q'^2}{M_B}}\left\{e_d\left[\tilde{H}^d(-x,\eta,t) - \tilde{H}^u(-x,\eta,t)\right] - e_u\left[x \to -x\right]\right\}$$

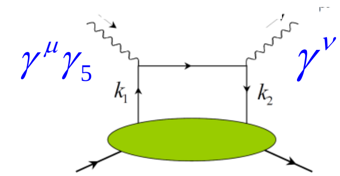
Summary & ToDo

$\pi^- p \to \gamma^* n \to \mu^+ \mu^- n$ at J-PARC　　GPDs

LO $\left(O(\alpha_s^2)\right)$ factorization formula is known, but it misses soft nonfactorizable mechanism (SNM)

LCSR at LO $\left(O(\alpha_s^0)\right)$ is derived for largely model-independent estimate for SNM

$\tilde{H},\ \tilde{E},\ q_{\mathrm{th}}^2 (\sim 0.7\ \mathrm{GeV}^2)$



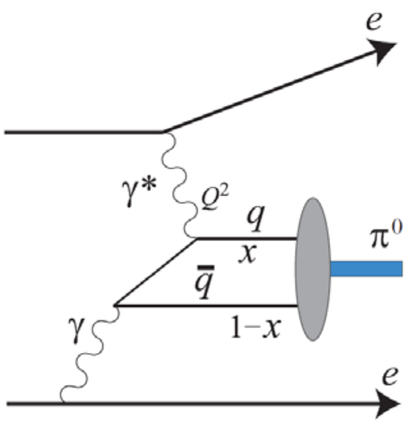
- numerical calculation at LO!
- NLO LCSR ⟷ quark $k_\perp$, pion pole contri.
- twist-3 LCSR ⟶ $M_{LCSR}^{\pm 1 \lambda', \lambda}(\pi^- p \to \gamma\ n)$

interplay of soft/hard QCD mechanism

Exclusive lepton pair production in πN scattering

$$\pi^- p \to \gamma^* n \to \mu^+ \mu^- n$$

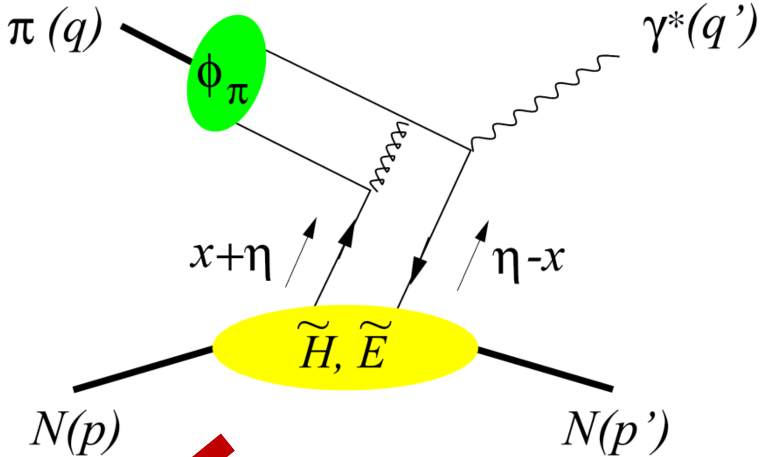
Berger, Diehl, Pire, PLB523(2001)265



@Belle, Babar

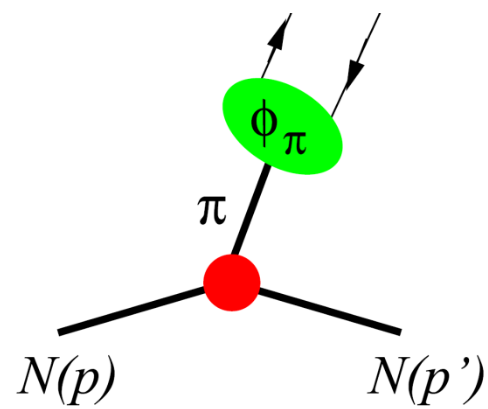
"exclusive limit of DY"

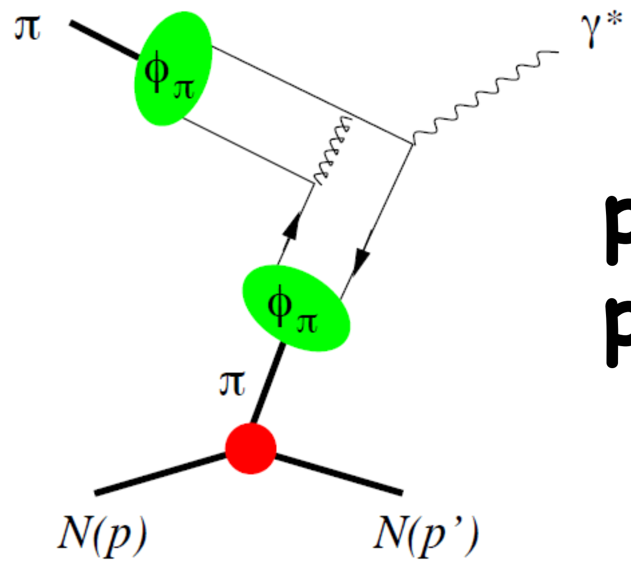
small $t = (q - q')^2$



$\Delta q(x)$

$t \to 0$





pion-pole contribution using pion form factor $F_\pi(Q'^2)$

Goloskokov, Kroll

$F_\pi(Q'^2)$: important soft nonfactorizable contr. was shown with LCSR

Braun, Khodjamirian, Maul