# Art of Spin Decomposition

*--- A practical perspective*

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**Oct 5th @ Spin 2015**

# Recall of the Controversies: gauge invariance

Jaffe-Manohar [NPB337:509 (1990)]

$$\boxed{\vec{J}_{\text{total}}} = \int d^3x \psi^+ \frac{1}{2}\vec{\Sigma}\psi + \int d^3x \vec{x} \times \psi^+ \frac{1}{i}\vec{\nabla}\psi + \int d^3x \vec{E} \times \vec{A} + \int d^3x \vec{x} \times E^i \vec{\nabla} A^i$$

Ji [PRL78:610 (1997)], Chen-Wang [CTP27:212 (1997)]

$$\boxed{\vec{J}_{\text{total}}} = \int d^3x \psi^+ \frac{1}{2}\vec{\Sigma}\psi + \int d^3x \vec{x} \times \psi^+ \frac{1}{i}\vec{D}\psi + \int d^3x \vec{x} \times \left(\vec{E} \times \vec{B}\right)$$

Chen-Lü-Sun-Wang-Goldman [PRL100:232002 (2008); 103:062001 (2009)]

$$\boxed{\vec{J}_{\text{total}}} = \int d^3x \psi^+ \frac{1}{2}\vec{\Sigma}\psi + \int d^3x \vec{x} \times \psi^+ \frac{1}{i}\vec{D}_{\text{pure}}\psi + \int d^3x \vec{E} \times \vec{A}_{\text{phys}} + \int d^3x \vec{x} \times E^i \vec{\mathrm{D}}_{\text{pure}} A^i_{\text{phys}}$$

Wakamatsu [PRD81:114010(2010); 83:014012 (2011); 84:037501 (2011)]

$$\boxed{\vec{J}_{\text{total}}} = \int d^3x \psi^+ \frac{1}{2}\vec{\Sigma}\psi + \int d^3x \vec{x} \times \psi^+ \frac{1}{i}\vec{D}\psi + \int d^3x \vec{E} \times \vec{A}_{\text{phys}} + \int d^3x \vec{x} \times (E^i \vec{\mathrm{D}}_{\text{pure}} A^i_{\text{phys}} + \vec{A}^a_{\text{phys}} \rho^a)$$

# A practical perspective:

- Construction of Spin Eigenstate
- Simplification of Spin Structure
- Experimental differentiation of Spin from OAM
- Enlarged inspection: The angular momentum tensor and flux density

# Hint from a forgotten practice: Why photon is ignored for atomic spin?

$$i\partial_t \psi_e = H_e \psi_e = E_e \psi_e,$$

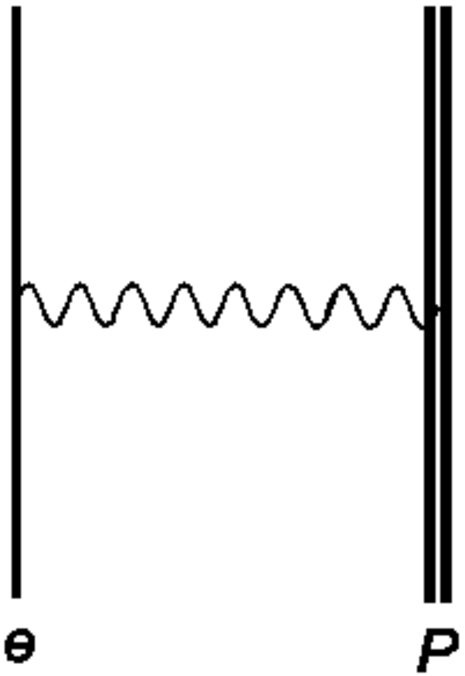
$$\vec{J}_e^2 \psi_e = j(j+1)\psi_e, J_e^z \psi_e = m\psi_e,$$

$$H_e = \vec{\alpha} \cdot \frac{1}{i}\vec{D}_e + M_e\beta + q_e A^0,$$

$$\vec{J}_e = \frac{1}{2}\vec{\Sigma} + \vec{x} \times \frac{1}{i}\vec{\partial}$$

**Do these solutions make sense?!**

# The atom as a whole




$$H_{\text{atom}} = \int d^3x\,\psi_e^\dagger\Big(\vec{\alpha}\cdot\frac{1}{i}\vec{D}_e + M_e\beta\Big)\psi_e$$
$$+ \int d^3x\,\psi_p^\dagger\Big(\vec{\alpha}\cdot\frac{1}{i}\vec{D}_p + M_p\beta\Big)\psi_p$$
$$+ \int d^3x\,\frac{1}{2}(\vec{E}^2 + \vec{B}^2),$$

$$\vec{J}_{\text{atom}} = \int d^3x\,\psi_e^\dagger\Big(\frac{1}{2}\vec{\Sigma} + \vec{x}\times\frac{1}{i}\vec{\partial}\Big)\psi_e$$
$$+ \int d^3x\,\psi_p^\dagger\Big(\frac{1}{2}\vec{\Sigma} + \vec{x}\times\frac{1}{i}\vec{\partial}\Big)\psi_p$$
$$+ \int d^3x(\vec{E}\times\vec{A} + E_i\vec{x}\times\vec{\partial}A_i)$$
$$\equiv \vec{J}_e + \vec{J}_p + \vec{J}_\gamma.$$

# Close look at the photon contribution

$$
\begin{aligned}
\mathscr{L}^A &= \frac{1}{2}(\partial_\mu A_\nu \partial^\nu A^\mu - \partial_\mu A_\nu \partial^\mu A^\nu) + eA_\mu j^\mu, \\
S_{ij}^A &= \int d^3x[\dot{A}_j A_i - \dot{A}_i A_j \\
&+ \partial_j A^0 A_i - \partial_i A^0 A_j], \\
L_{ij}^A &= \int d^3x[\dot{A}_k (x_j \partial_i - x_i \partial_j) A_k \\
&+ \partial_k A^0 (x_j \partial_i - x_i \partial_j) A_k].
\end{aligned}
$$

**The static terms!**

# Justification of neglecting photon field

$$\int d^3x[\partial_j A^0 A_i - \partial_i A^0 A_j + \partial_k A^0 (x_j \partial_i - x_i \partial_j) A_k]$$
$$= \int d^3x A^0 (x_j \partial_i - x_i \partial_j)(\partial_k A_k). \quad (16)$$

Thus, in Coulomb gauge, $\vec{\partial} \cdot \vec{A} = 0$, the static terms in $J^A_{ij}$ vanish and $J^A_{ij}$ simplifies to

$$^C J^A_{ij} = \int d^3x[\dot{A}_j A_i - \dot{A}_i A_j + \dot{A}_k (x_j \partial_i - x_i \partial_j) A_k]^C. \quad (17)$$

# A critical gap to be closed

The stationary condition only means that the gauge-invariant physical observables (like the electric current $j^\mu$ or electromagnetic field $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$) are time-independent, while the gauge-potential $A^\mu$ may contain spurious (nonphysical) time-dependence [12]. This gap is closed by noting that in Coulomb gauge $A^\mu$ can be expressed in terms of $F^{\mu\nu}$ [13]:

$$^C A^\mu = \frac{1}{\vec{\partial}^2} \partial_i F^{i\mu}. \tag{18}$$

Hence, in Coulomb gauge, $A^\mu$ is time-independent if $F^{\mu\nu}$ is, and Eq. (17) dictates that $^C J^A_{ij}$ vanishes for a stationary system.

# The same story with Hamiltonian

$$\begin{aligned} H_{\text{atom}} &= \int d^3x(\psi_e^\dagger i\partial_t\psi_e + \psi_p^\dagger i\partial_t\psi_p - E^i\partial_t A^i - \mathcal{L}) \\ &= \int d^3x\psi_e^\dagger\left(\vec{\alpha}\cdot\frac{1}{i}\vec{D}_e + M_e\beta + q_eA^0\right)\psi_e \\ &\quad + \int d^3x\psi_p^\dagger\left(\vec{\alpha}\cdot\frac{1}{i}\vec{D}_p + M_p\beta + q_pA^0\right)\psi_p \\ &\quad - \int d^3x\left[E^i\partial_t A^i + \frac{1}{2}(\vec{E}^2 - \vec{B}^2)\right], \end{aligned} \tag{8}$$

where the Lagrangian is

$$\begin{aligned} \mathcal{L} &= \bar{\psi}_e(i\gamma_\mu D_e^\mu - M_e)\psi_e + \bar{\psi}_p(i\gamma_\mu D_p^\mu - M_e)\psi_p \\ &\quad - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}. \end{aligned} \tag{9}$$

# The fortune of using Coulomb gauge

$$
\begin{aligned}
H_{\text{atom}} = & \int d^3x \psi_e^\dagger \left( \vec{\alpha} \cdot \frac{1}{i} \vec{D}_e + M_e \beta + q_e A^0 \right) \psi_e \\
& + \int d^3x \psi_p^\dagger \left( \vec{\alpha} \cdot \frac{1}{i} \vec{\partial} + M_p \beta \right) \psi_p \\
& + \int d^3x \frac{1}{2} \left[ \vec{E}_\perp^2 - \vec{A}_\perp \cdot \partial_t^2 \vec{A}_\perp + (\vec{j}_e - \vec{j}_p) \cdot \frac{1}{\vec{\partial}^2} \partial_t^2 \vec{A}_\perp \right] \\
& - \vec{j}_p \cdot \vec{\partial} \frac{1}{\vec{\partial}^2} (\vec{\partial} \cdot \vec{A}) + j_e^0 \partial_t \frac{1}{\vec{\partial}^2} (\vec{\partial} \cdot \vec{A}). \qquad (10)
\end{aligned}
$$

# Gauge-invariant revision – Angular Momentum

$$\vec{J}_{\mathrm{atom}} = \int d^3x \psi_e^\dagger \left[\frac{1}{2}\vec{\Sigma} + \vec{x} \times \frac{1}{i}(\vec{\partial} - iq_e A_\parallel)\right]\psi_e$$
$$+ \int d^3x \psi_p^\dagger \left[\frac{1}{2}\vec{\Sigma} + \vec{x} \times \frac{1}{i}(\vec{\partial} - iq_p \vec{A}_\parallel)\right]\psi_p$$
$$+ \int d^3x [\vec{E}_\perp \times \vec{A}_\perp + E_\perp^i \vec{x} \times \vec{\partial} A_\perp^i]$$
$$\equiv \mathbf{J}_e + \mathbf{J}_p + \mathbf{J}_\gamma.$$

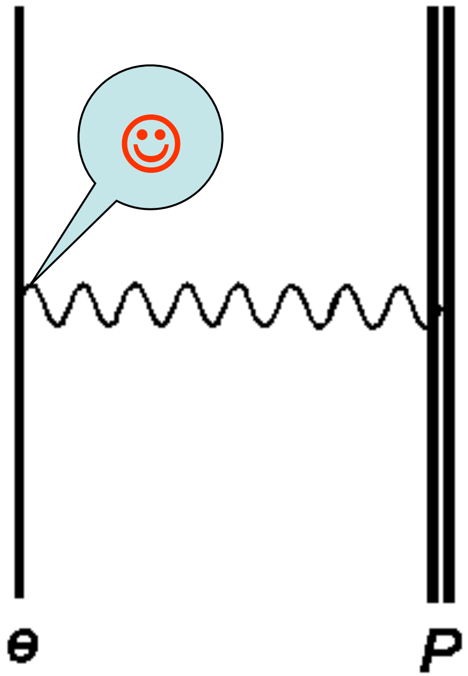
# Gauge-invariant revision
# -Momentum and Hamiltonian

$$\vec{P} = \int d^3x \left( \psi_e^\dagger \frac{1}{i} \vec{\bar{D}}_e \psi_e + \psi_p^\dagger \frac{1}{i} \vec{\bar{D}}_p \psi_p + E_\perp^i \vec{\partial} A_\perp^i \right), \qquad (16)$$

and the Hamiltonian is

$$\begin{aligned} H_{\text{atom}} &= \int d^3x ( \psi_e^\dagger i \bar{D}_e^0 \psi_e + \psi_p^\dagger i \vec{\bar{D}}_p^0 \psi_p - E_\perp^i \partial_t A_\perp^i - \mathcal{L}) \\ &= \int d^3x \psi_e^\dagger \left( \vec{\alpha} \cdot \frac{1}{i} \vec{D}_e + M_e \beta + q_e \hat{A}^0 \right) \psi_e \\ &\quad + \int d^3x \psi_p^\dagger \left( \vec{\alpha} \cdot \frac{1}{i} \vec{D}_p + M_p \beta + q_p \hat{A}^0 \right) \psi_p \\ &\quad + \int d^3x \frac{1}{2} (\vec{E}_\perp^2 + \vec{B}^2 - \vec{E}_\parallel^2). \end{aligned} \qquad (17)$$

# The covariant scheme



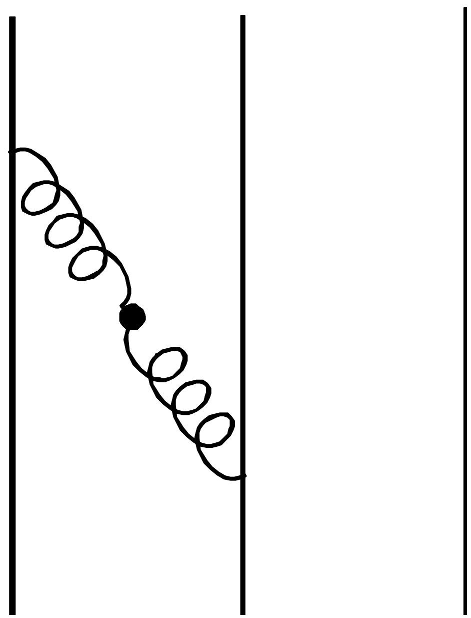

$$\vec{J}_{\rm atom} = \int d^3x \psi_e^\dagger \left( \frac{1}{2}\vec{\Sigma} + \vec{x} \times \frac{1}{i}\vec{D}_e \right) \psi_e + \int d^3x \psi_p^\dagger \left( \frac{1}{2}\vec{\Sigma} + \vec{x} \times \frac{1}{i}\vec{D}_p \right) \psi_p + \int d^3x \vec{x} \times (\vec{E} \times \vec{B}) \equiv \vec{J}'_e + \vec{J}'_p + \vec{J}'_\gamma.$$

$$\vec{J}'_\gamma = \vec{J}_\gamma + \int d^3x \vec{x} \times \rho \vec{A},$$

**spurious photon angular momentum**

# Gluon angular momentum in the nucleon:

## Tree-level



$$\vec{J}'_g = \int d^3x\, \vec{r} \times (\vec{E} \times \vec{B})$$

$$> 0$$

$$\vec{J}_g = \int d^3x\, \vec{E} \times \vec{A}_\perp + \int d^3x \vec{r} \times (E^i \vec{\nabla} A^i{}_\perp)$$

$$= 0$$

**One-gluon exchange has the same property as one-photon exchange**

# Physical part of the non-Abelian gluon field

$$\hat{A}_\mu = A_\mu - \bar{A}_\mu$$

$$\bar{F}_{\mu\nu} \equiv \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu + ig[\bar{A}_\mu, \bar{A}_\nu] = 0$$

$$\bar{\mathcal{D}}_i \hat{A}_i \equiv \partial_i \hat{A}_i + ig[\bar{A}_i, \hat{A}_i] = 0$$

$$\hat{A}_\mu = \frac{1}{\vec{\partial}^2}\partial_i F_{i\mu} + ig\frac{1}{\vec{\partial}^2}\left\{\left[\frac{1}{\vec{\partial}^2}\partial_k F_{ki}, \partial_i \frac{1}{\vec{\partial}^2}\partial_k F_{k\mu} - \partial_i A_\mu\right] - \partial_i\left[A_i, \frac{1}{\vec{\partial}^2}\partial_k F_{k\mu}\right] + \partial_\mu\left[\frac{1}{\vec{\partial}^2}\partial_k F_{ki}, A_i\right]\right\} + \mathcal{O}(g^2)$$

# Manipulating the gluon spin

$$\vec{J}_{\text{QED}} = \int d^3x\, \psi^\dagger \frac{1}{2}\vec{\Sigma}\psi + \int d^3x\, \vec{x} \times \psi^\dagger \frac{1}{i}\vec{\hat{D}}\psi + \int d^3x\, \vec{E} \times \vec{\hat{A}} + \int d^3x\, \vec{x} \times E_i \vec{\partial} \hat{A}_i \equiv \vec{S}_e + \vec{L}_e + \vec{S}_\gamma + \vec{L}_\gamma$$

$$\vec{J}_{\text{QCD}} = \int d^3x\, \psi^\dagger \frac{1}{2}\vec{\Sigma}\psi + \int d^3x\, \vec{x} \times \psi^\dagger \frac{1}{i}\vec{\hat{D}}\psi + \int d^3x\, \vec{E} \times \vec{\hat{A}} + \int d^3x\, \vec{x} \times E_i \vec{\hat{\mathcal{D}}} \hat{A}_i \equiv \vec{S}_q + \vec{L}_q + \vec{S}_g + \vec{L}_g$$

$$\vec{E} = -\partial_t \vec{A} - \vec{\partial} A^0 = -\partial_t \vec{\hat{A}} - \vec{\partial} \hat{A}^0 \equiv \vec{E}^{\text{dy}} + \vec{E}^{\text{st}}$$

$$\vec{E} = -\partial_t \vec{A} - \vec{\nabla} A^0 + ig[\vec{A}, A^0] = -\bar{\mathcal{D}}_t \vec{\hat{A}} - \vec{\bar{\mathcal{D}}} \hat{A}^0 + ig[\vec{\hat{A}}, \hat{A}^0] \equiv \vec{E}^{\text{dy}} + \vec{E}^{\text{st}} + \vec{E}^{\text{nl}}$$

$$\vec{S}_g^{\text{st}} \equiv \int d^3x\, \vec{E}^{\text{st}} \times \vec{\hat{A}} = \int d^3x\, (-\vec{\bar{\mathcal{D}}} \hat{A}^0) \times \vec{\hat{A}}$$

$$\vec{S}_g^{\text{dy}} \equiv \int d^3x\, \vec{E}^{\text{dy}} \times \vec{\hat{A}} = \int d^3x\, (-\bar{\mathcal{D}}_t \vec{\hat{A}}) \times \vec{\hat{A}}$$

$$\vec{S}_g^{\text{nl}} \equiv \int d^3x\, \vec{E}^{\text{nl}} \times \vec{\hat{A}} = \int d^3x\, ig[\vec{\hat{A}}, \hat{A}^0] \times \vec{\hat{A}}$$

# Beyond the static approximation

$$\Delta g \equiv \langle p\sigma| \int d^3x(\vec{E}\times\vec{A})^3|p\sigma\rangle_{A^+=0} = \sigma\cdot 2\frac{\alpha_s}{\pi}\ln\frac{Q^2}{m^2}$$

$$\alpha_s(Q^2) = \frac{g^2(Q^2)}{4\pi}$$

$$\begin{aligned} S_g &\equiv \langle p\sigma| \int d^3x(\vec{E}\times\vec{A}_\perp)^3|p\sigma\rangle \\ &= \langle p\sigma| \int d^3x(\vec{E}\times\vec{A})^3|p\sigma\rangle_{\vec{\nabla}\cdot\vec{A}=0} = \frac{5}{9}\Delta g, \end{aligned}$$

$$= \frac{12\pi}{(33-2n_f)\ln(Q^2/\Lambda^2)}.$$

$$S_g^{\rm dy} = \frac{1}{5}S_g = \frac{1}{9}\Delta g = \sigma\cdot\frac{2}{9}\frac{\alpha_s}{\pi}\ln\frac{Q^2}{m^2}, \qquad (13)$$

which is largely negligible. E.g., the specific renormalization by choosing the ultraviolet cutoff $Q^2$ to be the same as the scale for $\alpha_s(Q^2)$ gives $S_g^{\rm dy} \simeq 0.1\sigma$.

# A key issue in spin decomposition

$$\vec{J}_{\rm QCD} = \int d^3x\, \psi^\dagger \frac{1}{2}\vec{\Sigma}\psi + \int d^3x\, \vec{x} \times \psi^\dagger \frac{1}{i}\vec{\hat{D}}\psi$$

$$+ \int d^3x\, \vec{E} \times \vec{\hat{A}} + \int d^3x\, \vec{x} \times E_i \vec{\hat{\mathcal{D}}} \hat{A}_i$$

$$\equiv \vec{S}_q + \vec{L}_q + \vec{S}_g + \vec{L}_g$$

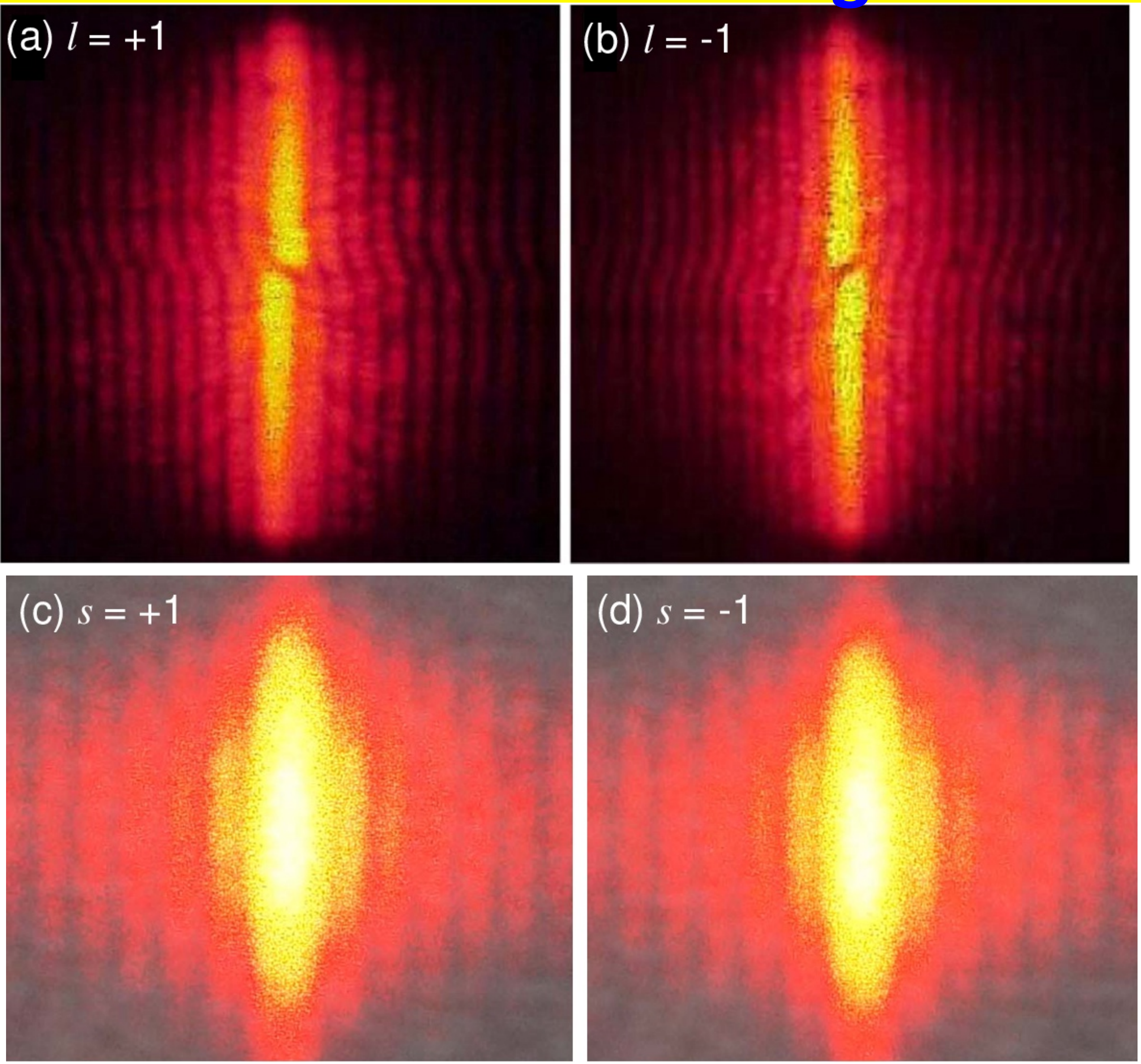
$$\vec{J}_{\rm QCD} = \int d^3x\, \psi_q^\dagger \frac{1}{2}\vec{\Sigma}\psi_q + \int d^3x\, \vec{x} \times \psi_q^\dagger \frac{1}{i}\vec{D}\psi_q$$

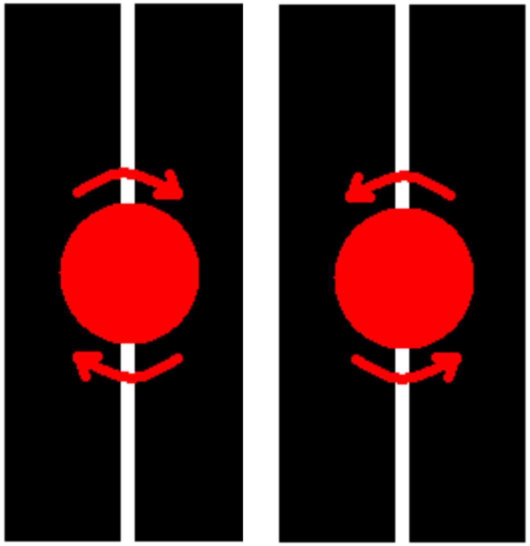
$$+ \int d^3x\, \vec{x} \times (\vec{E} \times \vec{B})$$

$$\equiv \vec{S}_q + \vec{L}'_q + \vec{J}'_g,$$

**Should we separate spin from OAM?**

# The Experimental answer:
# two kinds of angular momenta



**Spin is a kind of angular momentum without circulation of momentum**

# Enlarged inspection:

# the angular momentum tensor

Canonical: $T_C^{\mu\nu}(x) = \frac{\partial L(\phi_i, \partial_\mu \phi_i)}{\partial(\partial_\mu \phi_i)} \partial^\nu \phi_i - g^{\mu\nu} L$

Symmetric: $T_{symm}^{\mu\nu}(x) = \frac{1}{\sqrt{-g}} \frac{\delta I_M}{\delta g_{\mu\nu}(x)}$

$$M_C^{\mu\nu\lambda} = x^\mu T_C^{\lambda\nu} - x^\nu T_C^{\lambda\mu} + \frac{\partial L}{\partial(\partial_\lambda \phi_a)} \Sigma_{ab}^{\mu\nu} \phi_b$$

$$M_?^{\mu\nu\lambda} = x^\mu T_{symm}^{\lambda\nu} - x^\nu T_{symm}^{\lambda\mu}$$

**Surprising: none is satisfactory!**

# A new perspective: Constrains on $T^{ab}$ from quantum measurement

**If a quantum wave is in mutual eigenstates of more than one observables, then the associated currents must be proportional to each other**

$$E.g.: \vec{j}_E \propto \vec{j}_{p_i} \propto \vec{j}_{s_i} \propto \vec{j}_q$$

$$\hat{H}\psi = E\psi, \hat{P}_i\psi = p_i\psi, \hat{S}_i\psi = s_i\psi, \hat{Q}\psi = q\psi$$

# The Symmetric $T^{ab}$ stands no chance!

$$j^\mu = e\bar{\psi}\gamma^\mu\psi$$

$$T^{\mu\nu}_{symm} = \frac{i}{4}\bar{\psi}(\gamma^\mu\partial^\nu + \gamma^\nu\partial^\mu)\psi + h.c. \quad T^{\mu\nu}_{cano} = \frac{i}{2}\bar{\psi}\gamma^\mu\partial^\nu\psi + h.c.$$

$$P^i = T^{0i}_{cano} = \frac{i}{2}\bar{\psi}\gamma^0\partial^i\psi + h.c. \quad K^i = T^{i0}_{cano} = \frac{i}{2}\bar{\psi}\gamma^i\partial^0\psi + h.c. \approx \frac{\mathcal{E}}{e}j^i$$

# But the canonical $T^{ab}$ is not fully OK

$$T_C^{\mu\nu}(x) = \frac{\partial L(\phi_a, \partial_\mu\phi_a)}{\partial(\partial_\mu\phi_a)}\partial^\nu\phi_a - g^{\mu\nu}L$$

$$T_C^{i0} = \frac{\partial L(\phi_a, \partial_\mu\phi_a)}{\partial(\partial_i\phi_a)}\partial^0\phi_a, \quad T_C^{ii}(x) = \frac{\partial L(\phi_a, \partial_\mu\phi_a)}{\partial(\partial_i\phi_a)}\partial^i\phi_a + L$$

# An improved $T^{ab}$: free field

$$T^{\mu\nu}_{revised} = \frac{\partial L(\phi_a, \partial_\mu \phi_a)}{\partial(\partial_\mu \phi_a)} \overleftrightarrow{\partial}^\nu \phi_a, \quad \overleftrightarrow{\partial}^\nu \equiv \frac{1}{2}\left(\overrightarrow{\partial}^\nu - \overleftarrow{\partial}^\nu\right)$$

$$T^{\mu\nu}_{C} = \frac{\partial L(\phi_a, \partial_\mu \phi_a)}{\partial(\partial_\mu \phi_a)} \partial^\nu \phi_a - g^{\mu\nu} L$$

**E.g. photon**

$$T^{\mu\nu}_{revised} = -F^{\mu\rho} \overleftrightarrow{\partial}^\nu A_\rho$$

$$T^{\mu\nu}_{C} = -F^{\mu\rho} \partial^\nu A_\rho + \frac{1}{4} g^{\mu\nu} F^2$$

$$T^{\mu\nu}_{symm} = -F^{\mu\rho} F^{\nu}{}_{\rho} + \frac{1}{4} g^{\mu\nu} F^2$$

# Proof of validness (general free fields)

$L(\phi_a, \partial_\mu \phi_a)$ is quadratic in and $\partial_\mu \phi$ ($\phi$, $\phi^*$ independent)

$$= \sum_a \frac{1}{2}\left( \frac{\partial L}{\partial \phi_a}\phi_a + \frac{\partial L}{\partial(\partial_\rho \phi_a)}\partial_\rho \phi_a \right) = \sum_a \frac{1}{2}\partial_\rho\left( \frac{\partial L}{\partial(\partial_\rho \phi_a)}\phi_a \right)$$

$$T_C^{\mu\nu} = \frac{\partial L}{\partial(\partial_\mu \phi_a)}\partial^\nu \phi_a - g^{\mu\nu} L, \quad T_{revised}^{\mu\nu} = T_C^{\mu\nu} + \partial_\rho B^{[\rho\mu]\nu}$$

$$B^{[\rho\mu]\nu} = \sum_a \frac{1}{2}\partial_\rho\left( g^{\mu\nu}\frac{\partial L}{\partial(\partial_\rho \phi_a)}\phi_a - g^{\rho\nu}\frac{\partial L}{\partial(\partial_\mu \phi_a)}\phi_a \right)$$

# The interacting fields: scalar case

$$T^{\mu\nu}(x) = \frac{1}{\sqrt{-g}} \frac{\delta I_M}{\delta g_{\mu\nu}(x)}$$

$$I_\phi = \int d^4x \sqrt{-g} \left( \frac{-1}{2} D_\mu \phi D^\mu \phi + \frac{1}{8} R \phi^2 \right)$$

$$\rightarrow T^{\mu\nu}_{new} = \frac{\partial L(\phi_a, \partial_\mu \phi_a)}{\partial(\partial_\mu \phi_a)} \overleftrightarrow{\partial}^\nu \phi_a$$

**This gives a gravitational theory different from Einstein's GR**

# Trouble with the Angular momentum tensor

$$M^{\mu\nu\lambda} = x^{\mu} T^{\lambda\nu}_{symm} - x^{\nu} T^{\lambda\mu}_{symm}$$

$$M^{\mu\nu\lambda}_{C} = x^{\mu} T^{\lambda\nu}_{C} - x^{\nu} T^{\lambda\mu}_{C} + \frac{\partial L}{\partial(\partial_{\lambda}\phi_{a})} \Sigma^{\mu\nu}_{ab} \phi_{b}$$

$$M^{\mu\nu\lambda}_{revised?} = x^{\mu} T^{\lambda\nu}_{revised} - x^{\nu} T^{\lambda\mu}_{revised} + \frac{\partial L}{\partial(\partial_{\lambda}\phi_{a})} \Sigma^{\mu\nu}_{ab} \phi_{b}$$

$$+\frac{1}{2} g^{\lambda\nu} \frac{\partial L}{\partial(\partial_{\mu}\phi_{a})} \phi_{a} - \frac{1}{2} g^{\lambda\mu} \frac{\partial L}{\partial(\partial_{\nu}\phi_{a})} \phi_{a}$$

**Our trick applies to longitudinal spin flux only, but not to transverse flux of angular momentum!**

# Conclusion:

So far there exists no satisfactory expression of angular momentum tensor, even for a free field!

And thus no satisfactory way of spin decomposition.

**Thank you!**