Multistability and Chaos in a Semiconductor Microwave Device with Time–Delay Feedback

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We propose a tunable microwave device consisting of a Gunn diode with time–delay feedback, which will emit chaotic microwaves. The wavelength is controlled by two incident laser beams which trigger moving multiple Gunn domains. Predicted phenomena include the coexistence of stationary and chaotic states, complicated hysteresis domains, and persistent multistability. This device is potentially useful for applications such as secure microwave communications, memory devices and those involving photorefractive effects.

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Bistable dynamical systems with a time–delay feedback have been discovered in many disciplines. Most such investigations in physics focus on optical, electrical and electrooptic systems. The behavior of such systems depends crucially on the response time τ of the nonlinear system and the time delay T of the feedback loop. When T ≫ τ, in most cases bistability does not exist, but chaotic output may still be observed. When T ≳ τ, the region of most current research interest, complex dynamics and multistability are found. Recently, Goedgebuer et al. considered a tunable laser diode with feedback, which can generate chaos useful for secure communications. In their study, T ≫ τ and the contribution from the system’s response is neglected. In this paper, we propose a new electrooptic system which displays multistability and high-dimensional chaos when T ≳ τ. The dynamical system we consider is a microwave device (Gunn diode) stimulated by two incident laser beams. Both the Gunn diode and the laser beams are connected with electric circuit feedback. The system exhibits many interesting nonlinear features (see below) and is potentially useful for applications such as secure microwave communications, memory devices and those involving photorefractive effects.

The physical basis for the Gunn diode has been well known for decades. The main feature of this phenomenon is that the current is associated with the formation of a high-field domain, which creates a current oscillation. In recent years, many interesting nonlinear behaviors of the Gunn effect have been studied; some authors showed that optical waves can excite multiple Gunn-domain formation in deep-impurity-doped GaAs and semi-insulating GaAs. Due to the strong electric field in a Gunn domain, the refractive index of GaAs is changed via second-order optical nonlinearity, i.e., Pockel’s effect. Therefore, they predicted a new nonlinear optical effect in semiconductors. In this paper, we consider a moving multiple Gunn-domain in a shallow-impurity-doped GaAs generated by external optical waves with a feedback control. The interesting results in this system include the transition from monostability to multistability, complicated hysteresis loops, persistent bistability, and chaos, which have potential applications in secure microwave communications, memory cells and other areas.

It is worth pointing out that the variation of the refractive index in the sample considered by us can be strongly enhanced by time–delay feedback and is independent of the applied bias and number of free electrons. In the following, we give detailed descriptions of the physical mechanism for multiple Gunn-domains triggered by two incident laser beams in n-GaAs, the feedback loop to control the incident angle α of the laser beams, and the equation of the electric circuit.

Multiple Gunn—domains triggered by optical waves—

Two optical waves are incident on a biased shallow—impurity-doped GaAs as shown in Fig. 1. Taking the light energy just above the band gap of GaAs (1.42 eV), we can generate electron–hole pairs by optical excitation. The intensity I(x,t) of the mixing waves moving through the n-GaAs is given by I(x,t) = I0[1 + m cos(Kx + Ωt)], where Ω is the frequency difference of the two optical waves, K = 2π/λ the interference wave number, A the grating period, m the modulation depth of the interference grating, and I0 the average intensity. The generation-recombination processes include complete thermal ionization of donors, generation of electron–hole pairs by the optical waves at rate g, and recombination of electron–hole pairs with rate γ. The dynamical equations include the Gauss law, the continuity equations of electrons and holes, and the circuit equation.

Fig. 1. Schematic illustration of the proposed experimental apparatus.
\[ \frac{\partial E}{\partial x} = \epsilon \left[ (n - N_D^* - p) \right], \]
\[ \frac{\partial n}{\partial t} = g I(x, t) - \gamma n p - \frac{\partial}{\partial x} \left[ n v(E) - D_n \frac{\partial n}{\partial x} \right], \]
\[ \frac{\partial p}{\partial t} = g I(x, t) - \gamma n p + \frac{\partial}{\partial x} \left[ \mu_p E + D_p \frac{\partial p}{\partial x} \right], \]
\[ V = \int_0^L E(x, t) \, dx, \]

where \( n, p, N_D^*, L, V, \) and \( E \) are, respectively, the free electron density, the free hole density, the effective donor concentration, the sample length, the applied bias, and the electric field. \( D_n \) and \( D_p \) denote, respectively, the diffusion coefficients of electron and hole; \( \mu_p \) and \( v(E) \) are the hole mobility and the electron drift velocity, respectively. Due to the band structure of GaAs, we know that \( v(E) \) displays \( N \)-shaped negative differential mobility (NDM).

We use a Fourier series to solve eqs. (1)–(4) with complete basis functions \( \{ e^{i(kx+\Omega t)} \} \), where \( |k| \) is a positive integer, chosen based on the moving interference pattern of \( I(x, t) \). Therefore, the solutions of eqs. (1)–(4) can be described as follows,

\[ E(x, t) = E_0 + \sum_{\ell \neq 0} m_\ell E_\ell(t) e^{i(k_\ell x + \Omega t)}, \]
\[ n(x, t) = n_0 + \sum_{\ell \neq 0} m_\ell n_\ell(t) e^{i(k_\ell x + \Omega t)}, \]
\[ p(x, t) = p_0 + \sum_{\ell \neq 0} m_\ell p_\ell(t) e^{i(k_\ell x + \Omega t)}, \]
\[ v(E) = v_0 + \sum_{\ell \neq 0} \frac{\psi_{\ell 0}(t)}{j_\ell} \left[ \sum_{\ell \neq 0} m_\ell E_\ell(t) e^{i(k_\ell x + \Omega t)} \right], \]

where \( \bar{m} = m/2, v_0 = v(E_0) \) and \( \psi_{\ell 0}(t) = \frac{d^2 v(E)}{dt^2} \big|_{E=E_0} \). Note that the hole density \( p(x, t) \) in eq. (7) can be expressed in terms of the electric field \( E(x, t) \) and electron density \( n(x, t) \) via the relation in eq. (1). Furthermore, eqs. (5)–(8) converge when the modulation depth \( m \) is much smaller than 1. Then we can generate the solutions of eqs. (1)–(4) in order.

The zero-order solutions \( (E_0, n_0, p_0) \) of eqs. (1)–(4) are independent of space and time, thus it is straightforward to determine \( E_0 = V/L, n_0 = (N_D^* + \sqrt{N_D^* + 4gI_0/\gamma})/2, \) and \( p_0 = (-N_D^* + \sqrt{N_D^* + 4gI_0/\gamma})/2. \) For simplicity, from now on we consider the case when the free electrons are mostly from the donor impurities and the average intensity \( I_0 \) is very small so that \( n_0 \approx N_D^* \).

The first-order solutions \( (E_1, n_1, p_1) \) can be obtained by substituting the expansions of eqs. (5)–(8) into eqs. (1)–(4):

\[ \frac{dE_1}{dt} = -\left[ \frac{e}{\epsilon N_D^*(v_0^{(1)} + K^2 D_p + i\Omega - KE_0\mu_p)} \right] E_1, \]
\[ \frac{dn_1}{dt} = g I_0 - iKN_D^* \left( v_0^{(1)} - \frac{\epsilon}{\gamma} \right) E_1, \]
\[ \frac{dp_1}{dt} = g I_0 - iKN_D^* \left( v_0^{(1)} - \frac{\epsilon}{\gamma} \right) E_1, \]

where \( n_{1,\alpha} \) and \( n_{1,\beta} \) respectively. The indices \( \alpha \) and \( \beta \) denote steady-state and time-dependent solutions, respectively. The steady-state solutions are

\[ n_{1,\alpha} = \Gamma \times E_{1,\alpha}, \]
\[ E_{1,\alpha} = gI_0 \times \Psi^{-1}, \]

and

\[ \Psi = iKN_D^* \left( v_0^{(1)} - \frac{\epsilon}{\gamma} \right) \left[ \gamma N_D^* + K^2 D_n + i(\Omega + KI_0) \right] \Gamma. \]

The \( E_{1,\alpha} \) and \( n_{1,\alpha} \) have time dependence \( -e^{i\lambda t} \) where \( \lambda = -\gamma N_D^* - K^2 D_p + i(KE_0(\mu_p - \alpha)) \) or \( -\gamma N_D^* - K^2 D_n + i(\Omega + KI_0) \), which are obtained from eqs. (9)–(10). Hence the first-order solution for the electric field can be expressed as

\[ E_1 e^{i(K x + \Omega t)} = E_{1,0} e^{i(K x + \Omega t)} + c_1 e^{-\gamma N_D^* + K^2 D_n p/2} e^{iK x + \Omega t} + c_2 e^{-i\gamma N_D^* + K^2 D_n p/2} e^{iK x + \Omega t}, \]

where \( c_1 \) and \( c_2 \) are constants. The first term on the right hand side of eq. (13) shows the space-charge field induced by the optical-wave mixing, which has the same phase velocity as the moving interference pattern. The second term has phase velocity \( \mu_p E_0 \), but vanishes as \( t \to \infty \). The third term demonstrates the multiple Gunn-domain formation when \( -\gamma N_D^* - K^2 D_n > 0 \) (note that \( v_0^{(1)} < 0 \)), the domain velocity is \( v_0 \), and the distance between adjacent domains is \( \Lambda = K/2\pi \). Since \( n_0 \approx N_D^* \), the space-charge field can be neglected. Therefore, the underlying physics in eq. (13) is that the external laser beams will create a periodic domain train with wave number \( K \), but the domain train still sustains the drift velocity \( v_0 \) (i.e., bulk property), which is not influenced by the phase velocity \( \Omega/K \) of the interference pattern.\(^{11}\) These kinds of results are in complete agreement with the so-called “light-triggered Gunn-domain structure” by the numerical works of Subačius \textit{et al.}\(^{8}\) Therefore, the interference pattern will control the wavelength \( \lambda \) of the microwaves which is equal to \( \frac{1}{2} c \lambda \mu_p^{-1} \cos^{-1} \alpha \), where \( c \) is the speed of light and \( \lambda \) is the wavelength of the laser beams.

**Multistability and Chaos in the Proposed Experimental Setup**—Figure 1 shows the proposed experimental apparatus. The feedback loop consists of a pair of waveguides \( W \), a phase shifter whose fast and slow axes are at \( 45^\circ \) to a pair of crossed polarizers \( P \), a detector \( D_1 \), i.e., planar-doped barrier diode, with a time response \( \tau \sim 10 \) ns, a delay line with retardation time \( T \sim 1 \) ms, and a pair of acoustooptic scanners \( S \) to tune the incident angle \( \alpha \) of the laser beams. The microwave radiation power \( P_m \) from \( n \)-GaAs can be expressed as \( E_0^2 v_0^0 R^{-1} c^{-2} \lambda^2 \)\(^{12}\) where \( E_0 \) and \( R \) correspond to the rf field and resistance in the semiconductor, respectively. \( P_m \) is collected by a \( W \) and modulated by a phase shifter. The phase shifter induces the nonlinear power...
function $P_m \sin^2(\pi D/\lambda)$ detected by $D_e$ with a gain $\eta_1$, where $D$ is the length of the phase shifter. The purpose of $D_e$ is to convert the nonlinear power into an electric current $i(t)$. The current is then retarded by a delay line and enhanced by an amplifier with a gain $\eta_2$, then it drives $S$, tuning the incident angle of the laser beams. If the variation of $\alpha$ is linearly proportional to $i$ with a ratio $\alpha$, it is easy to get the relation between $\lambda(t)$ and $i(t)$: $\lambda(t) = \lambda_0 + A i(t)$, where $A \equiv a \lambda_0 \tan \alpha_0$ and $\lambda_0$ is the initial wavelength due to the initial incident angle $\alpha_0$. Therefore, the circuit equation for the experimental apparatus is

$$\tau \frac{d\lambda(t)}{dr} + \lambda(t) = \lambda_0 + A \eta_1 \eta_2 P_m(t - T) \sin^2 \left( \frac{\pi D}{\lambda(t - T)} \right).$$

(14)

Since $T \gg \tau$, the differential term in eq. (14) can be neglected. Then eq. (14) can be treated as a difference equation. For convenience of analysis, we make the difference equation dimensionless

$$\lambda_n = 1 + \beta \lambda_{n-1}^2 \sin^2 \left( \frac{\pi D}{\lambda_0} \frac{1}{\lambda_0} \right),$$

(15)

where $\lambda_n/\lambda_0 \rightarrow \lambda_n$, $\beta = a \lambda_0^2 \eta_1 \eta_2 \tan \alpha_0 E_{\text{rf}}^2 v_0^2 / R e^2$. Equation (15) indicates that $\lambda(t)$ is kept at a fixed value in each time interval between $(n - 1)T$ and $nT$. Therefore, the wavelength $\lambda(t)$ and output power $P_m(t)$ will show square-wave behavior. There are two important parameters in eq. (15), $D/\lambda_0$ and $\beta$. The experimental process for tuning these two parameters can be understood as follows. Selecting an arbitrary $\alpha_0$ to create initial wavelength $\lambda_0$, then $D/\lambda_0$ is determined by $\alpha_0$, since $D$ is fixed in the experiment. When $t$ is larger than $T$, the feedback loop begins to work. We only need to tune the gain of the amplifier in the experiment to effectively change the $\beta$ value in eq. (15). Therefore, $D/\lambda_0$ and $\beta$ in eq. (15) correspond to $\alpha_0$ and $\eta_2$ in the experiment, respectively.

In Fig. 2, we show various bifurcation diagrams of $\lambda_n$ vs $\beta$ at different values of $D/\lambda_0$. The transition from monostability to bistability is illustrated in Figs. 2(a)–2(c). The interesting phenomenon in the bistable regime is that persistent bistability can be observed when $D/\lambda_0$ reaches 1 [Fig. 2(c)]. This means that there is no discontinuous transition (i.e., nonhysteretic loop) between these two states when the $\beta$ value is slowly increased. The most important use of persistent bistability is that the variation of the refractive index in $n$-GaAs can be strongly enhanced. In the lower state, the $\lambda$ value is the same as the initial wavelength without feedback control. In the higher state, there is a longer distance between the adjacent high-field domains in $n$-GaAs. Thus, the maximum value of the electric field in the high-field domain in the higher state is larger than that in the lower state. Therefore, according to Pockel’s effect, there is a larger variation of refractive index in the higher state. This change in refractive index is due to the nonlinear feedback loop; both the free-electron density and the applied bias are kept constant here.

When we increase $D/\lambda_0$ to 1.4 [Fig. 2(d)], we find the period-doubling route to chaos occurs in the lower state and the higher state sustains a stationary response. The transition from the lower state to the higher state is due to the transient chaos when $\beta$ exceeds the critical value 0.796. Although our system is investigated in the $T \gg \tau$ regime, Figs. 2(b) and
In summary, we have studied moving multiple Gunn-domains in n-GaAs generated by external optical waves with a nonlinear feedback control. There are many interesting results and useful applications of this system including photorefractive effects enhanced by the feedback loop, secure microwave communication via chaos, Ikeda instability in microwave systems, and multistability used as a memory cell. To our knowledge, these applications of the Gunn effect are new, and present many interesting possibilities for future research.

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