X-ray radiation from a relativistic charge in superconducting granule device by diffusional scattering of pseudophotons

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Abstract

A recent experiment was carried out to test a new type of X-ray detector. It consists of superconducting microspheres mixed randomly in dielectric material. X-ray radiation is generated from a relativistic charged particles passing through the detector. The number of X-ray photons detected depends on the particle energy, \( \gamma = E/mc^2 \), almost linearly in the energy range from 1 to 10 GeV. We explain this energy dependence by diffusional scattering of pseudophotons. This shows a closer agreement with the experimental result than the description from the conventional transition radiation theory. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Radiation of a moving charge in material has been of interest and studied with history. The existing processes of radiation from moving charged particle include transition radiation (TR) [1–3] and bremsstrahlung radiation (BR). These radiation energy loss along with the ionization effect constitute the major energy dissipation of a moving charge in the material. Besides the processes mentioned above, a new mechanism

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of radiation from a charge moving in an inhomogeneous material has been proposed by Gevorkian [4,5]. It describes the electromagnetic field accompanying the charged particle as a pseudophoton field formed by a collection of pseudophotons of various frequency. These pseudophotons can be scattered off to become real photons by an inhomogeneous dielectric medium when the charged particle passing through this medium. The scattering amplitude depends on the wave vector of the pseudophotons and the inhomogeneity. This theory has been used to describe one of the radiation contributions in a high-energy charged particle beam test on a superheated superconducting granule detector carried out by Yuan et al. [6,7]. Such a type of detector is in the category of cryogenic detector.

Cryogenic detector has become more and more popular during the past decade or so [8]. These include tunnel junction detector [9], bolometer [10], and superheated superconducting granule system [11,12]. These detectors operate at low temperature so that the sensitivity of energy detection is greatly improved over the measurement at room temperature. The working principle and the applications, however, are different for these detectors. The detector made of superheated superconducting microspheres, specifically, has been applied in the experiment of dark matter search [13,14], the detection of X-ray [15], α-particle [16,17], β-particle [18], etc. Much R&D in the laboratory has been devoted to the understanding of the basic properties of the superconducting grain transition in the area of condensed matter physics [16,17]. In general, a detector of this kind is made of type I superconducting microspheres mixed randomly in a dielectric material such as paraffin wax or varnish, etc., with diameter ranging from a few μm to the order of 50 μm. The geometric configuration of the system varies depending on the application.

One of the interesting applications of the superheated superconducting granule device is to serves as a transition radiation detector (TRD). Recently, Yuan et al. have carried out an experiment using high-energy charged particle beam, from 1 to 10 GeV, to test on such a type of detector. The detector consists of Sn grain mixed in paraffin wax to form cylindrical bar with pick up coil surrounding it to detect the pulsing signal from the superconducting grain transition. The purpose of this experiment is to utilize the dielectric interface of the Sn granule and the wax as a transition radiation (TR) radiator and the superconducting phase transition of the Sn grain itself as the detector of the emitted TR photon. Since the number of grain transitions is proportional to the number of photons absorbed, hence the photons emitted, it is, therefore, capable of showing the dependence of the radiation level on the energy of the incoming charged particle.

In the superconducting granule device mentioned above, the level of energy deposition on the Sn grain has different contributions from the various processes discussed. Since this system is a random dielectric material by nature, the newly proposed X-ray radiation mechanism by scattering of pseudophotons is capable of describing the experiment by Yuan et al. to a good agreement.
2. Superconducting granule detector

There exists a critical bulk transition line in the $H$-$T$ phase diagram of type I superconducting material, see Fig. 1. $H_c = H_c(T = 0)$ marks the critical field of the superconducting phase transition under magnetic field. Besides $H_c$, another two critical fields, the superheating field, $H_{sh}$, and the supercooling field, $H_{sc}$, also exist for the type I material. These fields are for the metastable states and are direct results by solving Ginzburg–Landau equation of the free energy for the superconductor including the surface super-current term. Depending on the size and density of the defects over the surface of the microsphere as well as the smoothness of the surface, the magnitudes of these two critical fields varies. This is because the defects and the sharp corner over the surface serves as the nucleation centers for the magnetic flux to penetrate into the superconductor. The ideal limit of these fields are predicted by theory, however, the experimentally determined value is always smaller. When applied as a cryogenic detector utilizing the property of the superheating state of the microsphere, the whole device is sent to this metastable state under an external magnetic field, $H_{app}$, with $H_c < H_{app} < H_{sh}$ and the temperature of the system, $T$, below the superconducting transition temperature, $T < T_c$. With an energy deposition on the grain, which exceeds the threshold energy $E_{th}$, the microsphere would proceed an irreversible phase transition to the normal state. The change of the magnetization, i.e., the break down of the Meissner state, will then induce a pulsing voltage on the surrounding pick up coil.

One can determine the threshold energy, $E_{th}$, for the phase transition by the following expression,

$$E_{th} = V_{grain} \int_T^{T+\Delta T} e(T) \,dT,$$  \hspace{1cm} (1)
where \( c(T) \) is the specific heat of the grain at temperature \( T \), \( V_{grain} \), the volume of the microsphere, and \( \Delta T \), the temperature increment needed to move the grain across the transition line.

The detector acts like threshold detector. A phase transition of the grain will occur at the absorption of photon with energy larger than the threshold energy or at the energy deposition from the moving charge by ionization under the condition that \( E_{ion} > E_{th} \). As a radiation detector, one can adjust \( E_{th} \), by varying the applied magnetic field, \( H_{app} \), so as not to observe the ionization effect. What is detected under such condition is then a result attribute to a single photon deposition.

3. Energy dissipation and absorption

The energy dissipation of an extremely relativistic charge moving in a medium like the detector system discussed above usually includes BR, TR, Cherenkov radiation, ionization, etc. These processes are well established. In the case that the particle velocity is smaller than the photon velocity in the medium, \( v < c/ \sqrt{\varepsilon} \), which is what we have in the detector system under discussion, no Cherenkov radiation is possible. The BR and TR are the radiation processes involving the emission and absorption of photons by the system with a wide energy range. The process of energy absorption, however, is not equal to that of energy emission. On the other hand, all of the energy loss due to the ionization effect is directly released to the host material. In order to understand the photons detected as measured by the superconducting grain transition, it is important to estimate the level of ionization deposition as the background effect and then calculate the number of radiated photons with the energy above the ionization level.

The number of TR photons can be calculated by the following expression [19],

\[
N_{TR} = \int_{E_{th}}^{E_m} \left( \frac{dN_{TR}}{dE} \right) dE,
\]

where \( E_m \) is the upper limit of the photon energy absorbable by the superconducting detector, \( E_{th} \), the threshold energy of the detector, and

\[
\frac{dN_{TR}}{dE} = \frac{\pi e^2}{E c} \left[ \left( \frac{\eta_i^2 + \eta_j^2}{\eta_i^2 - \eta_j^2} \right) \ln \left( \frac{\eta_j^2}{\eta_i^2} \right) - 2 \right],
\]

is the emission spectrum of TR per interface with \( \eta_i^2 = \zeta_i^2 + \gamma^{-2} \), in which \( \zeta_i^2 = \omega_{pi}^2/m_i \). \( \omega_{pi} \) is the plasma frequency of the host material \( i \). For Sn, the plasma frequency is approximately 50 eV, while for wax, 20 eV.

From the mass energy absorption coefficient [20], the absorption inelastic scattering length of a photon in the detector is easily estimated. For photon with energy of 200 KeV, the absorption inelastic scattering length is on the order of 1 cm, which is about the length of the detector. One can therefore set \( E_m = 200 \text{ KeV} \) in the integral of Eq. (2) to calculate the number of X-ray photons from various contributions.
In calculating the contribution from BR, the following formula is used:

\[ N_{BR} = N_s \int_{E_{th}}^{E_m} d\sigma(E), \] (4)

in which \( N_s \) is the number of scattering center in the particle trajectory and the interaction cross section is

\[ d\sigma(E) = 4\alpha^2 r_e^2 \frac{2(Z + 1)}{E} \left[ \left( 1 - \frac{2}{3} f + f^2 \right) \ln(183/Z^{1/3}) + X(Z) + \frac{f}{9} \right] dE. \] (5)

In the above expression, \( \alpha \) is the fine structure constant, \( Z \), the atomic number, \( r_e = 2.82 \times 10^{-13} \) cm, the classical electron radius, and \( f \), the fraction of particle energy remaining after an emission of BR photon of energy \( E \). The Coulomb correction function, \( X(Z) \), is used for the extremely relativistic case \([21,22]\).

The ionization level is estimated, for particle energy above 1 GeV, by applying the Bethe–Bloch equation for heavy charged particle and the modified one for the electron. For pion in Sn, \(-dE/dx = 12 \sim 14\) MeV/cm and electron, \(13 \sim 15\) MeV/cm. This translates to a maximum energy deposition of 50 KeV in a Sn grain of 35 \( \mu \)m in diameter. One can therefore filter out the ionization effect by setting the detection threshold, \( E_{th} \), to be above 50 KeV.

By considering the well-known processes mentioned above, the absorption of photons by the superconducting Sn grains in the detector due to radiation is obtained. The result shows that, from \( E_{th} = 50 \) KeV to \( E_m = 200 \) KeV, the number of BR photons is a constant and that of TR photons is \( \ln(\gamma) \) like. This behavior of energy dependence only explains part of the experimental fact which has a much stronger beam energy dependence \([6,7]\). It is therefore necessary to resort to a new mechanism of energy transfer from the relativistic charge to the Sn grain.

### 4. Pseudophoton field and random medium

The system of the superconducting microspheres mixed randomly in wax can be treated as a random media of inhomogeneous dielectric constant. Hence, part of the radiation from a moving charge in such a system can be described within the framework of radiation by the scattering of pseudophotons in a one-dimensional (1-D) random media proposed by Gevorkian \([4,5]\). In this treatment, the electromagnetic (EM) field originated from the moving charge is viewed as a pseudophoton field. The pseudophoton can be scattered off to become real photon if the charge is moving through a medium with inhomogeneous dielectric constant. The scattering probability depends on the photon momentum and the length scale of the inhomogeneity described by the correlation function of the dielectric constant. The scattering amplitude of the interaction between the pseudophoton and the medium is implicitly contained in the elastic mean-free path of the pseudophoton in the medium.
The number of pseudophotons associated with the moving charge is estimated to be

$$N_{ps}(\omega) \sim \int \frac{d\mathbf{q}}{(2\pi)^3} A^2(\mathbf{q}),$$

where $A(\mathbf{q})$ is the vector potential of the electromagnetic field surrounding the moving charge. The Maxwell equation for the vector potential is

$$\nabla^2 A + \frac{\varepsilon_0 c^2}{\mathbf{e}} A = j,$$

where the current resulting from the moving charge directed along the $z$-direction is

$$j(r, \omega) = -\frac{4\pi e}{c} \delta(x) \delta(y) e^{i\omega r/\mathbf{c}}.$$

In estimating the number of pseudophotons, one can replace the dielectric constant, $\varepsilon$, with the average one, $\varepsilon_a$, in Eq. (7), for the reason that the pseudophotons exist independent of the medium. Taking into account the symmetry condition for the vector potential which is directed along the $\mathbf{c}$-axis, one obtains the solution for the vector potential,

$$A(q) = -\frac{8\pi^2 e}{c} \frac{\delta(qz - \omega/\mathbf{c})}{k^2 - q^2},$$

where $k = \omega/\sqrt{\varepsilon_c}$. The number of pseudophotons at a given frequency $\omega$ is then reached by substituting Eq. (9) into Eq. (6), and replacing $\delta(0)$ by $L_c/2\pi$ in consideration of the finite size of the system,

$$N_{ps}(\omega) \sim \frac{\varepsilon^2}{c} \gamma_m^2 L_z,$$

where $\gamma_m = (1 - \beta^2\varepsilon)^{-1/2}$ is the Lorentz factor in the medium in which $v_p < c$ is always the case, and $L_z$ is the length of the system.

The pseudophoton will interact with the medium to emit real photon if the medium has an inhomogeneous dielectric constant. The interaction amplitude depends upon the varying dielectric constant and the wave vector of the pseudophoton. The result is implicitly contained in the elastic mean free path of the pseudophoton travelling in the medium. We consider here the system with 1-D randomness of dielectric constant interacting with the pseudophoton field.

One can construct a simple 1-D random system by randomly placing plates of dielectric constant $\varepsilon_p$ along the $z$-direction in a dielectric material of $\varepsilon_0$. Such a system is depicted in Fig. 2 which shows the trajectory and scattering of the pseudophotons as well.

The dielectric constant for such a system can be expressed as in the formula:

$$\varepsilon(z, \omega) = \varepsilon_0(\omega) + \sum_i \left[ \varepsilon_p(\omega) - \varepsilon_0(\omega) \right] \times \left[ \theta_{step}(z - z_i - a/2) - \theta_{step}(z - z_i + a/2) \right],$$

where $\theta_{step}$ is the step function and $a$, the thickness of the plates. One can rewrite the dielectric constant above as a sum of the average part, $\varepsilon_a$, and the varying part, $\varepsilon_r$,

$$\varepsilon(z, \omega) = \varepsilon_a + \varepsilon_r(z, \omega).$$
Fig. 2. The 1-D random system of dielectric material. The plates of thickness \(a\) are randomly distributed along the \(z\) direction. Trajectory of pseudophotons are plotted.

The average dielectric constant, \(\varepsilon_a = \langle \varepsilon(z, \omega) \rangle\), can be calculated via the expression below,

\[
\langle \varepsilon(z, \omega) \rangle = \int_0^{L_z} \prod_i \left( \frac{dz_i}{L_z} \right) \varepsilon(z, z_i, \omega) .
\]  

(13)

By putting Eq. (11) into Eq. (13), \(\varepsilon_a\) becomes

\[
\varepsilon_a = \varepsilon_0 + na(\varepsilon_p - \varepsilon_0),
\]  

(14)

where \(n = N/L_z\) in which \(N\) is the number of plates in the system.

One the other hand, the average over \(\varepsilon_r\) is zero, i.e., \(\langle \varepsilon_r \rangle = 0\). However, it is this varying part of the dielectric constant which is responsible for the interaction with the pseudophotons.

The correlation function in the coordinate space representation, for the 1-D random system is,

\[
B(z - z') = \frac{\omega^4}{c^4} \langle \varepsilon_r(z) \varepsilon_r(z') \rangle .
\]  

(15)

We use Eq. (11) to calculate the correlation function explicitly and obtain,

\[
B(q_z) = 4n(\varepsilon_p - \varepsilon_0) \frac{\omega^4 \sin^2(q_z a/2)}{q_z^2} .
\]  

(16)

By the technique of Green’s function one can connect the elastic mean free path to the correlation function by the following formula [4,5]:

\[
l(k) \sim \frac{4k^2}{B(0)} .
\]  

(17)

For the 1-D random system, this becomes

\[
l(k) \sim \frac{4c^2}{na^2(\varepsilon_p - \varepsilon_0)^2 \omega^2} .
\]  

(18)

Therefore, the pseudophotons moving with the charge particle is described by Eq. (10) and the scattering of these photons by the inhomogeneous medium can be described by the elastic mean free path shown in Eq. (17).
5. Single and diffusional scattering

In calculating the radiation intensity of a charged particle passing through the system of inhomogeneous dielectric constant, the radiation intensity can be divided into two parts, the contribution resulting from the single scattering of pseudophotons, \( I_S(\theta, \omega) \), and that from the diffusional scattering, \( I_D(\theta, \omega) \),

\[
I(\theta, \omega) = I_S(\theta, \omega) + I_D(\theta, \omega).
\]

In the weak scattering regime, \( \lambda \ll l \ll L \), with the condition that \( |\cos \theta| > |\hat{\lambda}|^{1/3} \), and in the limit that \( v \sqrt{\varepsilon} \to c \) (but \( v \sqrt{\varepsilon} < c \)), the explicit form of the radiation intensity has been obtained by calculation through Green’s function method from the Maxwell equation for the vector potential. Here, \( L \) is the characteristic size of the system. The radiation intensity due to single scattering is \([4,5]\)

\[
I_S(\theta, \omega) = \frac{e^2}{2c} \frac{\sin^2 \theta}{\gamma_m^2 + \sin^2 \theta} \frac{L_z B(|k_0 - k \cos \theta|)}{\epsilon k^2},
\]

where, \( k_0 = \omega/v \), and, \( \beta = v/c \). The contribution from the single scattering is proportional to the correlation function, \( B \), of the medium. One can view the correlation function as the form factor for the scattering of the pseudophotons. In the extremely relativistic region, \( \beta \to 1 \), the angle to have the maximum radiation intensity is, \( \theta = \gamma_m^{-1} \). Putting the correlation function, Eq. (16), into Eq. (20) and integrating over angle for the 1-D random system, one obtains that the radiated intensity is logarithmically dependent on the particle energy. This contribution is similar to that from the usual transition radiation.

For the radiation intensity attributed to the diffusional scattering, one has,

\[
I_D(\theta, \omega) = \frac{5e^2}{2c} \frac{L_z}{l(k)} \frac{L^2}{l^2(k)} \frac{\sin^2 \theta}{|\cos \theta|^{1/2}}.
\]

In the diffusional scattering of pseudophotons in the \( z \) direction, the following condition is satisfied, \( \lambda \ll l(\lambda) \ll L \), \( l_\omega(l, \lambda) \). \( l_\omega \) is the inelastic mean free path of the pseudophotons.

Consider the relative magnitude of these two contributions, we have, \( I_D/I_S \sim L_z^2/l^2 \), which is much greater than 1 in the weak scattering region. By integrating Eq. (21) over the angle \( \theta \), we have,

\[
I_D(\omega) \sim \frac{e^2}{c} \frac{L_z}{l(\omega)} \frac{L^2}{l^2(\omega)}.
\]

The above equation is clear on the nature of the diffusional scattering. Indeed, \( 1/l(\omega) \) is the probability of scattering, \( L_z^2/l^2 \) is the average number of scatterings and what is remaining in the equation is the number of pseudophotons created by the charge as described by Eq. (10).
6. Superconducting device

We apply the 1-D random system proposed above to the superconducting granule system in the experiment by Yuan et al. A sketch of the superconducting device is plotted in Fig. 3. In the figure, the pseudophoton trajectory is plotted together with that of the charged particle.

For the charge moving in the system depicted in the Fig. 3, it experiences the variation of the dielectric constant due to the presence of the superconducting granules. The pseudophotons will be scattered to become real photon by the variation of the dielectric constant. Since the pseudophoton is moving along the $z$ direction with the charged particle and in the X-ray region under consideration, the wavelength is much smaller than the grain diameter, $\lambda \ll a$, the pseudophoton experiences the randomness in the moving direction only, without much effect resulting from the transverse direction. This superconducting granule system is therefore of the same nature as the 1-D plate system described previously. The difference in the path length for the charged particle to pass through the plate system and the granule system is by a geometric factor only.

In the emission of X-ray, we apply the plasma formula for the dielectric constant

\[
\varepsilon_p \approx 1 - \frac{\omega_p^2}{\omega^2},
\]

\[
\varepsilon_0 \approx 1 - \frac{\omega_{p0}^2}{\omega^2},
\]

(23)

where $\omega_{p0}$ is the plasma frequency of the plate or, in the case of granule system, of the Sn and $\omega_{p0}$ is that of the homogeneous medium. The average dielectric constant of the system then becomes

\[
\varepsilon_d(\omega) \approx 1 - \frac{(1 - na)\omega_{p0}^2 + (na)\omega_p^2}{\omega^2}.
\]

(24)

By applying the explicit form of the dielectric function above, the Lorentz factor in the medium becomes

\[
\gamma_m = \gamma \left(1 + \frac{\omega_p^2}{\omega_e^2}\right)^{-1/2},
\]

(25)

where one defines the characteristic frequency,

\[
\omega_e = \gamma \sqrt{(na)\omega_p^2 + (1 - na)\omega_{p0}^2}.
\]

(26)
The characteristic frequency $\omega_c$ is a result of the grain concentration, $n$, grain diameter, $a$, the particle energy, $\gamma$, as well as the dielectric properties of the material, $\omega_p$ and $\omega_{p0}$. For the Sn grain system of 13% volumetric filling factor and the grain size of 35\,\mu m in diameter, one obtains $n \sim 130$ cm$^{-1}$ and $na \sim 0.46$. One can use $\omega_p \sim 50$ eV as the plasma frequency of the Sn grain and $\omega_{p0} \sim 20$ eV, for the wax. The characteristic frequency becomes

$$\omega_c \sim (37 \text{ eV})\gamma,$$

which ranges from 75 to 750 KeV corresponding to the beam energy from 1 to 10 GeV.

Another characteristic frequency of the 1-D system can be defined by substituting the plasma formula, Eq. (23), into the condition of Born approximation, $ka\sqrt{\varepsilon_p/\varepsilon_0} - 1 \ll 1$, for pseudophoton scattering from a grain

$$\omega \geq \frac{a(\omega_p^2 - \omega_{p0}^2)}{2c} \equiv \omega_b.\tag{28}$$

$\omega_b$ depends on the dielectric constant of Sn and wax as well as the diameter of the Sn grain only. The elastic mean-free path, Eq. (18), then takes the form

$$l(\omega) \approx \frac{1}{n} \left( \frac{\omega}{\omega_b} \right)^2.\tag{29}$$

For the superconducting granule system under discussion, $\omega_b \sim 120$ KeV.

The characteristic frequency, $\omega_c$, is important in the consideration of the radiation intensity from the charge particle as a function of the particle energy. On the other hand, $\omega_b$ marks the lower-energy limit above which the diffusional scattering becomes important.

### 7. Emission and absorption of photons

The emission of photons can be viewed as the result of elastic scattering of pseudophotons, while the disappearance of the pseudophoton in the medium before real photon emission can be described as the absorption of the pseudophoton by the medium and characterized by the inelastic scattering length of the pseudophoton. The real photons emitted from the pseudophoton field experience two consequences, being absorbed by the superconducting granule system or emerging out of the system. The absorption of the photon by the superconducting granule is also characterized by the inelastic scattering length.

The detection threshold of the X-ray photon energy in the experiment by Yuan et al. is set to be above 100 KeV, which is in the diffusional scattering region. This scattering contribution is described by Eq. (22). The photon or the pseudophoton absorption by the random medium is similar to the case of wave propagating through the medium and get absorbed. In the weak absorption limit, the elastic mean free path is much smaller than the elastic one, $l \ll l_{\omega_b}(\omega)$ and the quantity, $L_{\text{eff}} \equiv \sqrt{T \cdot l_{\text{in}}}$, instead of $L$, becomes
the effective size of the system, provided that $L_{\text{eff}} \leq L$ [23]. Under such condition, the
diffusional radiation intensity of Eq. (22) then becomes

$$I_D \sim \frac{e^2}{c} \frac{\gamma^2}{1 + \frac{\omega^2}{\omega_1^2}} \frac{L_2 \cdot I_m(\omega)}{I(\omega)}.$$  \hspace{1cm} (30)

In the above equation, the ratio $l_m/l$ is the mean number of scatterings of photon or
pseudophoton before absorption. The inelastic mean free path in the X-ray region is
determined by the absorption mainly via photoelectric effect

$$l_m^{-1}(\omega) = (1 - na)N_0 \sigma_{ph}^0(\omega) + (na)N_p \sigma_{ph}^P(\omega),$$ \hspace{1cm} (31)

where $N_0$ and $N_p$ are the atom numbers in unit volume of the homogeneous medium
and of the plate, respectively. $\sigma_{ph}^0(\omega)$ and $\sigma_{ph}^P(\omega)$ are the corresponding photoelectric
cross sections.

In different frequency regime, the frequency dependence of the radiation intensity
is derived as follows. For $\omega_b < \omega < \omega_L$, where $\omega_L$ is the frequency at which
$\sqrt{I(\omega_L) \cdot I_m(\omega_L)} = L$. The effective size of the system is determined by the absorption
of pseudophoton. In the frequency region discussed above, the photoabsorption cross
section is hydrogen-like, $\sigma_{ph}(\omega) \propto \omega^{-3.5}$. From Eqs. (30) and (31), we obtain the fre-
cquency dependence of the radiation intensity, $I(\omega)$, and the number of photon $N(\omega)$,
emitted from the pseudophoton field.

$$I(\omega) \sim \frac{e^2}{c} \frac{\gamma^2}{1 + \frac{\omega^2}{\omega_1^2}} \frac{\omega^{-1/2}}{1 + \alpha^2 \omega^2/\omega^2},$$ \hspace{1cm} (32)

and

$$N(\omega) \sim \frac{\gamma^2}{2} \frac{\omega^{-3/2}}{1 + \alpha^2 \omega^2/\omega^2}.$$ \hspace{1cm} (33)

For $\omega > \omega_L$, the diffusion trajectory is cut on the system characteristic size. We have
the formula for $I(\omega)$ and $N(\omega)$ in this frequency range as

$$I(\omega) \sim \frac{e^2}{c} \frac{\gamma^2}{1 + \alpha^2 \omega^2/\omega^2} \frac{\omega^{-6}}{1 + \alpha^2 \omega^2/\omega^2},$$ \hspace{1cm} (34)

and

$$N(\omega) \sim \frac{\gamma^2}{2} \frac{\omega^{-7}}{1 + \alpha^2 \omega^2/\omega^2}.$$ \hspace{1cm} (35)

where $\alpha = e^2/hc$ is the fine structure constant.

From the above equations, by integrating over frequencies and taking into account
that $\omega_\gamma \propto \gamma$, one obtains the explicit beam energy dependence of the total radiation
intensity, $I_T^{\text{em}}(\gamma)$, and the total number of emitted photons, $N_T^{\text{em}}(\gamma)$, coming out of the
pseudophoton field in the following two frequency ranges:

For $\omega_\gamma < \omega_L$,

$$I_T^{\text{em}}(\gamma) \sim \gamma^{2.5},$$

$$N_T^{\text{em}}(\gamma) \sim \gamma^{1.5}.$$ \hspace{1cm} (36)
For \( \omega_c > \omega_c \), the radiation reaches the saturation region,

\[
I^\text{em}(\gamma) \sim \text{Constant},
\]

\[
N^\text{em}(\gamma) \sim \text{Constant}.
\]  

(37)

Since the frequency, \( \omega_c \), is a signature which marks that the effective size of the system under diffusional propagation of pseudophoton equals the system size, the radiation intensity of the charged particle is saturated once the pseudophoton field is cut by the system size spatially. The saturation region is therefore determined by the system size \( L^2 \) and the particle energy, \( \gamma \). From \( l(\omega_c) \cdot l_{\text{in}}(\omega_c) = L^2 \), together with \( l_{\text{m}}(\omega_c) \sim \omega_c^{1.5} \) and \( l(\omega_c) \sim \omega_c^2 \), we have \( \omega_c \approx L^{4/11} \).

In formulae (36) and (37), we have found the energy dependence of the emitted radiation intensity and the number of emitted photons. On the other hand, in the experiment [6,7], the number of absorbed photons is detected. These two are related to each other if we consider the photon emitted from the pseudophoton field as well as the photon absorbed by the system at the same time. Since the effective linear size of the system, \( L_{\text{eff}} \), is smaller than the length of the system, \( L_z \), there exist many such subsystems in the medium along the path of moving charge in the \( z \)-direction. After real photons are formed with a spatial extent of \( L_{\text{eff}} \), they propagate through the rest of the system with part of them absorbed by the system. The number of real photons absorbed is related to the total number of real photons in the medium, \( N_{ab}(\omega) \leq N^\text{em}(\omega) \), while the rest of them emerges out of the system. The absorption probability of a diffusing photon in a system of effective size \( L_{\text{eff}} \) is \( L_{\text{eff}}/l_{\text{in}} = \sqrt{l/l_{\text{in}}} \). So, in the weak absorption limit, one can evaluate the total number of absorbed photons as

\[
N_{ab} \leq \int_{\omega_c}^{\omega} d\omega \left[ \frac{l(\omega)}{l_{\text{in}}(\omega)} \right]^{1/2} N^\text{em}(\omega).
\]  

(38)

By taking into account the frequency dependence of \( \omega_c, l(\omega), l_{\text{in}}(\omega), \) and \( N^\text{em}(\omega) \), we obtain from Eq. (38), \( N_{ab} \sim \gamma^x \), with \( 1.5 \geq x \geq \frac{3}{4} \). The upper bound of the exponent is obvious from Eq. (36). So, the number of grain transitions measured in the experiment [6,7] has a \( \gamma \) dependence to the power approximately equal to one.

Now, let us make some numerical estimations. For a detector system as described in the previous section, with 13% of volumetric filling factor of Sn grains 35 \( \mu \)m in diameter, distributed randomly in wax, one obtains the elastic mean-free path from Eq. (29) as

\[
l(\omega) \sim 77 \text{ \( \mu \)m} \left( \frac{\omega}{120 \text{ KeV}} \right)^2.
\]  

(39)

For the photon energy of 200 and 300 KeV, the elastic mean-free paths are roughly 0.02 and 0.05 cm, respectively, according to the above formula. The inelastic mean-free path, \( l_{\text{in}} \), can be obtained from the mass energy absorption coefficient. One has 1.46 and 3.44 cm at the corresponding photon energy discussed above. In this energy range under consideration, the weak absorption limit, \( l \ll l_{\text{in}} \), is valid. In addition, the corresponding effective size of the system are calculated to be 0.175 and 0.406 cm for photons of 200
and 300 KeV. For the detector size used in the experiment, 0.3 cm, the characteristic frequency, $\omega_L$, is then within this energy range.

So, indeed, one has a system of 1-D random medium. The number of the radiated X-ray photons is in the region described by the diffusional scattering of pseudophotons,

8. Conclusion

The superconducting granule device in the experiment by Yuan et al. is applied as a energy detecting device of the extremely relativistic charged particle. The energy dissipation of the charged particle in passing through the device by the processes of transition radition, bremsstahlung radiation, and ionization has been investigated. It is shown that they do not explain the strong energy dependence of the number of emitted photons observed in the experiment. The diffusional scattering mechanism of radiation gives a closer agreement on the experimental result.

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References