Supersymmetric radius stabilization
in Randall-Sundrum background

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Based on the work with Nobuhito Maru (RIKEN, Japan)
in preparation
1. Introduction

Brane World Scenario

String Theory : classical solution \quad (Polchinski '95)

D-brane: D+1 dim. object embedded in higher dim. space time

Open string $\rightarrow$ fermion, boson, gauge on D-brane
Closed string $\rightarrow$ graviton in the bulk
**Brane World Scenario as Phenomenological model**

We are confined on “3-brane”

Graviton lives in the bulk

**Beyond the standard model → Brane World Scenario**

- New property
  - “geometry”

**Typical Scenario:**

- **Large (flat) Extra Dimension** (Arkani-Hamed-Dimopoulos-Dvali, ’98)
- **Warped (small) Extra Dimension** (Randall-Sundrum, ’99)
(i) Large (flat) Extra Dimension Scenario (ADD scenario)

Alternative solution to hierarchy problem

without SUSY, TC, etc.

\[ M_W \sim 10^2 \text{GeV} \ll M_4 \sim 10^{19} \text{GeV} \]

If \[ M_W \sim M_{4+n} \Rightarrow \text{O.K.} \]

Low scale gravity model

\[
S_{4+n} = M_{4+n}^{2+n} \int d^4x d^n y \sqrt{-g_{4+n} R_{4+n}} \\
= M_4^2 \int d^4x \sqrt{-g_4 R_4}
\]

\[ M_4^2 = M_{4+n}^{2+n} V_n \]

\[ V_n : \text{volume of n extra-dim.} \]

\[ V_n = (2\pi r)^n \text{ (compactified on } T^n) \]

For \( M_{4+n} \sim 1 \text{ TeV} \)

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<th>( r )</th>
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\( \Leftarrow r < 218 \ \text{mm} \)

Hoyle et al., PRL 86 (2001) 1418
(ii) **Warped (small) Extra Dimension Scenario** (Randall-Sundrum scenario)

5-dim. Theory compactified on orbifold $s^1/Z_2$

\[ \Lambda < 0 \]

\[ \Lambda_{hid} \delta(y) \quad \Lambda_{vis} \delta(y - \pi) \]

Slice of AdS$_5$

\[ S_G = -\frac{1}{2} \int d^4x \int_0^\pi dy \sqrt{-g} \left( M_5^3 R_5 + \Lambda \right) \]

\[ S_{\text{vis}} = \int d^4x \sqrt{-g_{\text{vis}} \left( \mathcal{L}_{\text{vis}} - \Lambda_{\text{vis}} \right)} \]

Solution of Einstein Eqs.

\[ ds^2 = e^{-2\kappa r_c|y|} \eta_{\mu\nu} dx^\mu dx^\nu + r_c^2 dy^2 \]

**Flat 4D theory**

If

\[ \Lambda = -24 M_5^3 \kappa \]

\[ \Lambda_{hid} = -\Lambda_{vis} = 24 M_5^3 \kappa \]

Appropriate tuning of cosmological constants

in the bulk and on the branes
4-dim. Effective Planck scale

\[ V_5 = \int_0^\pi d y r_c e^{-2\kappa r_c |y|} = \frac{1}{2\kappa} \left( 1 - e^{-2\kappa r_c |y|} \right) \]
\[ \rightarrow M_4^2 = \frac{M_5^3}{2\kappa} \left( 1 - e^{-2\kappa r_c \pi} \right) \]
\[ \frac{M_5^3}{2\kappa} \quad \text{for } \kappa r_c \pi \gg 1 \]
\[ M_5^3 (\pi r_c) \quad \text{for } \kappa \to 0 \]

Alternative solution to hierarchy problem!

SM Higgs lives on the visible brane

\[ S_{Higgs} = \int d^4 x \sqrt{-g_{vis}} \left[ g_{vis}^{\mu\nu} (D_\mu H)^\dagger (D^\mu H) - \lambda (H^\dagger H - v_0^2)^2 \right] \]
\[ \sqrt{-g_{vis}} g_{vis}^{\mu\nu} = e^{-2\kappa r_c \pi} \eta^{\mu\nu} \implies H \rightarrow e^{\kappa r_c \pi} H \quad \text{rescale} \]
\[ S_{Higgs} = \int d^4 x \left[ \eta^{\mu\nu} (D_\mu H)^\dagger (D^\mu H) - \lambda (H^\dagger H - v^2)^2 \right] \]
\[ v = v_0 \times e^{-\kappa r_c \pi} \]

Even if \( v_0 \sim M_4 \), \( v \) can be the weak scale with \( \kappa r_c \sim 12 \)

Mild hierarchy
We are going to discuss ``Supersymmetric Brane World”

Why is SUSY needed?

ADD scenario, Randall-Sundrum scenario can provide alternative solutions to the gauge hierarchy problem **without SUSY**

**Alternative Motivation** to introduce extra-dimensions for SUSY models

**4D SUSY model** → solution to the gauge hierarchy problem

Phenomenological requirement

→ SUSY should be broken

SUSY breaking mediation → almost **flavor blind & CP invariant**

One of the important issues in SUSY models

→ Origin of SUSY breaking & its mediation
SUSY breaking mediation in supergravity

In supergravity, once the SUSY is broken, SUSY breaking effect can be mediated to the visible sector automatically through ``supergravity interactions’’  \( \leftrightarrow \) contact interactions

\[
\mathcal{L}_{\text{cont}} = \int d^4 \theta \ c_{ij} \ \frac{(Z^\dagger Z)(Q_i^\dagger Q_j)}{M_4^2}
\]

\( Z \) is hidden sector superfield :  \( F_Z \neq 0 \  \leftrightarrow \ \text{SUSY breaking} \)

\[
\int d^4 \theta \ c_{ij} \ \frac{(\bar{\theta}^2 F_Z^\dagger \theta^2 F_Z)Q_i^\dagger Q_j}{M_4^2} \rightarrow c_{ij} \ m_{3/2}^2 \bar{Q}_i \bar{Q}_j
\]

\( c_{ij} \neq \delta_{ij} \ \text{in general} \rightarrow \text{causes SUSY FCNC problem}!! \)
Sequestering Scenario \(\leftarrow\) alternative motivation of introduction of extra-dim.

Randall-Sundrum, ’99

SUSY breaking

hidden field \(Z\)

MSSM fields

5D brane world

5\textsuperscript{th} dim. is compactified on \(s^1/Z_2\)

hidden brane

visible brane

\(\delta(y)\)

\(\delta(y - \pi)\)

Naïve picture

\[
\mathcal{L}_{\text{cont}} = \int d^4 \theta \, c_{ij} \frac{(Z^\dagger Z)(Q_i^\dagger Q_j)}{M_4^2} \\
\leftarrow \mathcal{L}_{\text{cont}}^\mathbb{5} \propto \int dy \left[ (Z^\dagger Z) \delta(y) \right] \times \left[ (Q_i^\dagger Q_j) \delta(y - \pi) \right]
\]

is forbidden by Geometry !!

Proof:
by Luty & Sundrum
PRD 62, 035008 (’00)

More precise \(\rightarrow\) correction through bulk fields

\[
c_{ij} \sim \frac{1}{M_4^2 L^2} \ll 1 \quad \text{If} \quad M_4 \ll \frac{1}{L} \quad \text{volume suppression}
\]
SUSY breaking mediation?

No SUSY breaking mediation from contact terms

→ "Anomaly Mediation Scenario" has been discovered!

\[ M_{\lambda_i} = \frac{\alpha_i}{4\pi} b_i m_{3/2} \]

\[ \tilde{m}^2 = 2C_i b_i \frac{\alpha_i^2}{4\pi} m_{3/2} \]

Flavor blind!

--- Comments ------

Tachyonic slepton problem \( b_i < 0 \) for scalar leptons

Lots of ideas have been proposed to solve this problem

Pomarol-Rattazzi, Katz-Shdmi-Shirman,
Chacko-Luty-Maksymyk-Ponton,
Jack-Jones, Kitazawa-Maru-Okada,
Arkani-Hamed-Kaplan-Murayama-Nomura, etc
This is not the end of this story

Important issue in brane world scenario

→ **Radius Stabilization**

Brane world with SUSY

branes ← BPS states

configuration is static → No ``raidon’’ potential in SUSY limit

size of radius is undetermined

unstable under small fluctuations

SUSY is broken → **non-trivial radion potential comes out**

→ radius stabilization mechanism has

some important relations to SUSY breaking effect

→ New SUSY breaking effect in visible sector

through radius stabilization mechanism?
2. SUSY breaking and Radius stabilization

Basics: Supergravity Lagrangian in 4D

Superconformal framework

\[ \mathcal{L}_{SUGRA} = \int d^4 \theta \phi \phi^{\dagger} \left[ -3M_4^2 + K(Z_i^{\dagger}, Z_i) \right] \]
\[ + \left[ \int d^2 \theta \phi^3 W(Z_i) + h.c. \right] \]

compensating multiplet: \( \phi = 1 + \theta^2 F_\phi \)

This is a powerful tool to examine SUGRA models

(i) \( V_{SUGRA} \) can be obtained through global SUSY formula

(ii) SUSY breaking effect \( F_\phi = m_{3/2} \)

(iii) Suitable formalism for ``sequestering scenario’’

\[ \int d^4 \theta \phi^{\dagger} \phi(Q^{\dagger} Q) \rightarrow \int d^4 \theta Q^{\dagger} Q \]

No contact term \( \rightarrow \) No soft term in visible sector
SUSY brane world scenario

Setup of Model  Only 5D SUGRA multiplet in the bulk

![Diagram of SUSY brane world scenario](image)

5D N=1 SUSY $\leftrightarrow$ 4D N=2 SUSY

By orbifolding ($Z_2$ parity), N=2 $\rightarrow$ N=1 SUSY

5D SUGRA multiplet

Even: $e^a_{\mu}, \tilde{e}_5^a, B_{\mu}, \psi^+_{\mu}, \psi^-_{\mu}$

Odd: $e^5_\mu, e^5_{\mu}, \tilde{B}_\mu, \psi^-_{\mu}, \psi^+_{\mu}$

Massless mode \hspace{1cm} N=1 SUGRA

= 4D SUGRA multiplet

+ "Radion" chiral multiplet

\[ T = (g_{55} + iB_5) + \theta(\psi^-_5) + \theta^2 F_T \]

Marti & Pomarol,
PRD 64 (2001) 105025, etc
Simple model I (Flat extra-dim)

\[ \mathcal{L} = \int d^4 \theta \left[ -3 M_5^3 (T + T^\dagger) \phi^\dagger \phi \right] + \left[ \int d^2 \theta \phi^3 W_0 + h.c. \right] \]

From E.O.M for auxiliary fields

\[ F_\phi = 0, \quad F_T = \frac{W_0^\dagger}{M_5^3} \]

\( V = 0 \) Independent of \( T \) \( \rightarrow \) radius is undetermined!

If SUSY is manifest (\( W_0 = 0 \)) \( \rightarrow \) No corrections for \( V \)

If SUSY is broken (non-zero \( W_0 \)) \( \rightarrow \) mass shift for bulk gravitinos

\( \rightarrow V(T) \) through loop corrections

\[ V(T) \sim -3 \frac{\zeta(3)}{2 \pi^2 |T|^4} |F_T|^2 \]

\( \rightarrow \) Radius is destabilized!

Bulk massive hypermultiplet \( \rightarrow \) radius stabilized \( \langle |T| \rangle \sim 1/m \)

But serious problem: New FCNC soft term comes out
Simple model II (SUSY Randall-Sundrum model)

\[ \mathcal{L} = \int d^4 \theta \phi^\dagger \phi \left[ -3 \frac{M_5^3}{\kappa} \left( 1 - e^{-(T+T^\dagger)\kappa\pi} \right) \right] + \left[ \int d^2 \theta \phi^3 \left( W_0 - e^{-3T\kappa\pi W_\pi} \right) + h.c. \right] \]

From E.O.M for auxiliary fields

\[
\begin{align*}
F_\phi^\dagger &= \frac{\kappa}{M_5^3} W_0 \\
F_T^\dagger &= \frac{1}{\pi M_5^3} \left( W_0 - W_\pi e^{(T-2T^\dagger)\kappa\pi} \right)
\end{align*}
\]

\[ V = \frac{\kappa}{M_5^3} \left( |W_\pi|^2 e^{-2(T+T^\dagger)\kappa\pi} - |W_0|^2 \right) \]

If SUSY is manifest (all Ws=0)  \rightarrow  V=0  \rightarrow  radius is undetermined!

If SUSY is broken (non-zero W)  \rightarrow  Radius is destabilized!

\[ W_\pi \neq 0 \rightarrow T \rightarrow \infty \]
3. Simple model of Supersymmetric radius stabilization

Sequestering scenario + anomaly mediation $\rightarrow$ good for phenomenology

Important issue = radius stabilization

In simple model: SUSY $\rightarrow$ radius is undetermined

broken SUSY $\rightarrow$ radius is destabilized

New fields introduced to stabilize radius

$\rightarrow$ generate New soft terms in visible sector

larger than Anomaly Mediation contribution

In brane world SUSY model building,

we have to consider SUSY breaking, its mediation mechanism

$\&$ radius stabilization mechanism all together from the beginning

$\rightarrow$ it is very hard to construct realistic models
Need a model in which …

we can discuss radius stabilization as a different problem

from SUSY breaking and its mediations

**SUSY radius stabilization**

radius is stabilized at SUSY level with very heavy radion

If radion mass $\gg$ gravitino mass or even $\sqrt{F_Z}$

$\rightarrow$ SUSY breaking effects little affect the radion potential

$\rightarrow$ Radius is not destabilized by SUSY breaking effects
**Simple radius stabilization model**  
N. Maru & N. Okada, in preparation

**SUSY Randall-Sundrum model with a bulk hypermultiplet**

5-dim. Theory compactified on orbifold $s^1/Z_2$

\[
\mathcal{L}_G = -3M_5^3 \int d^4\theta (T + T^\dagger)e^{-(T + T^\dagger)\kappa |y|}\phi\phi
\]

\[
\mathcal{L}_H = \int d^4\theta \frac{T + T^\dagger}{2}(H_c^\dagger H_c + H^\dagger H)
+ \left[\int d^2\theta e^{-T\kappa |y|}H \left\{(-\partial_y + (c + \frac{1}{2})T\kappa \Theta(y))H_c + e^{-T\kappa |y|}W_b\right\} + h.c.\right]
\]

**Odd:**

\[
H_c = \Theta(y)f(x, |y|)
\]

**Even:**

\[
H = g(x, |y|)
\]

\[
W_b = J_0\delta(y) - J_\pi\delta(y - \pi)
\]

← Source on each branes
SUSY vacuum condition

\[ F_H = 0 \]
\[ \rightarrow \left[ -\partial_y + \left( c + \frac{1}{2} \right) T\kappa \Theta(y) \right] H_c + e^{-T\kappa|y|} (J_0 \delta(y) - J_\pi \delta(y - \pi)) = 0 \]

\[ \rightarrow \left[ -\partial_y + \left( c + \frac{1}{2} \right) T\kappa \right] f(y) = 0 \]

with boundary conditions

\[ f(0) = \frac{J_0}{2} \]
\[ f(\pi) = \frac{J_\pi}{2} e^{-T\kappa\pi} \]

SUSY vacuum condition \( \rightarrow \) \[ J_0 - J_\pi e^{-(c+\frac{3}{2})T\kappa\pi} = 0 \]

Radius is stabilized with appropriate sources on each branes and bulk mass \( c \)
Radion potential

\[ V(T) = \frac{1 - 2c}{e^{(1-c)(T+T^\dagger)\kappa \pi} - 1} |J_0 - J_\pi e^{-(c+\frac{3}{2})T\kappa \pi}|^2 \]

SUSY vacuum with finite r or infinite r

Radion mass (c=1/2)

\[ m_{radion}^2 \sim \frac{J_\pi^2}{M_4^2 r_0} e^{-2r_0\kappa \pi} \]

Example:

\[ \kappa \sim 0.1 \times M_5 \]
\[ J_\pi \sim (0.1 \times M_5)^{3/2} \]
\[ J_0 \sim 10^{-4} \times J_\pi \]

\[ m_{radion} \sim 10^{-5} \times M_4 \]
\[ \gg m_{3/2}, \sqrt{F_{hidden}} \]

No destabilization even with SUSY breaking!
**Phenomenological viability**

Bulk hypermultiplet H can connect **visible sector** to **hidden sector**

H mediated FCNC soft mass?

\[ H_0(x, y) = \frac{1}{N_0} e^{(1/2-c)r_0 \kappa |y|} \times h(x) \]

\[ \mathcal{L}_{int}^{h-H} \sim \int d^4\theta \frac{(H_0^\dagger H_0)(Z^\dagger Z)}{M_5^3} \delta(y) \]

\[ \mathcal{L}_{int}^{v-H} \sim \int d^4\theta \frac{(H_0^\dagger H_0)(Q_i^\dagger Q_j)}{M_5^3} \delta(y - \pi) \]

\[ \Delta \tilde{m}_{FCNC}^2 \sim \frac{1}{16\pi^2} \frac{|F_Z|^2}{M_4^2} \times f(c) \sim \frac{1}{16\pi^2} m_3^{2/3} \times f(c) \]

\[ H(y) \propto e^{(1/2-c)r_0 \kappa |y|} \rightarrow f(c) \sim \left( \frac{2c - 1}{1 - e^{(1-2c)r_0 \kappa \pi}} \right)^2 e^{(3-2c)r_0 \kappa \pi} \]

\[ \frac{\Delta \tilde{m}_{FCNC}^2}{\tilde{m}_{AMSB}^2} \leq 10^{-2} \quad \text{for} \quad c < -\frac{1}{2}, \quad \frac{3}{2} < c \]

So, viable!
4. Summary

We have proposed a simple SUSY model of radius stabilization in SUSY Randall-Sundrum model with a bulk hypermultiplet.

Radius is stabilized by SUSY vacuum conditions with appropriate sources on each branes and bulk mass.

Radion mass can be much larger than SUSY breaking scale.

→ SUSY breaking effect little affects on radion potential.

and radius stability is ensured even with SUSY breaking.

New unwanted soft breaking terms can be negligible.

Now we can forget about radius stabilization! Original sequestering scenario can work!