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Contents:
• Introduction (Structure function, GPDs, Effective models…)
• NJL model, Faddeev eq.
• Radyushkin’s ansatz
• Summary and conclusions
Introduction

Hadron properties and hadron interactions
• very interesting and challenging
• lightest baryon, nucleon is actively studied
• static properties, electro-magnetic form factors, structure functions

QCD
• fundamental theory of hadrons
• useful and practical at high energy
• non-perturbative character at low energy

lattice QCD simulation is developing
low energy effective model is necessary
**Inclusive** lepton-nucleon deep inelastic scattering (DIS)

Under Bjorken limit \( Q^2 \) & \( \nu \to \infty \), \( x \) : finite

structure functions are scaled to \( F_2(x) = 2xF_1(x) = x \sum_f e_f^2 (q_f(x) + \bar{q}_f(x)) \)  Callan Gross relation

\( q_f(x), \bar{q}_f(x) \) : parton distributions
$F_2^{p,n}(x)$ nucleon structure

EXP. : HERA, RHIC, EMC
Theory : NJL model, Bag model, Chiral quark-soliton model …

EMC effects for pol. and unpol. structure functions,

$F_2^n(x)/F_2^p(x)$ …

I. Cloet, W. Bentz et al., in progress.
HM, W. Bentz, A.W. Thomas et al., NPA735, 482 (’04); NPA703, 785 (’02).

qualitative features are reasonably reproduced
Exclusive hard electro-production

- Deeply virtual compton scattering (DVCS)
  \[ \gamma^* p \rightarrow \gamma p \]
- Hard electroproduction of vector and pseudo-scalar mesons
  \[ \gamma^* p \rightarrow M p \]

Generalized parton distributions (GPDs)

\[ \sigma, \frac{d\sigma}{d\Delta^2} \bigg|_{\Delta^2=0} \]

Experimentally: measurements of GPDs are expected

(HERMES, CLAS, ZEUS etc.)

- Pseudo-scalar meson
  \[ H^q, E^q \]
- Vector meson, \( \gamma \)
  \[ H^q, E^q \]

Not appear in DIS
There are several other theoretical works of nucleon GPDs:

Bag model: X. Ji, W. Melnitchouk, and X. Song, PRD56, 5511 (‘97)

Chiral-quark soliton model: V.Yu. Petrov, et al., PRD57, 4325 (‘98)

Constituent quark model: S.Scopetta, and V. Vento, EPJA16, 527 (‘03);
S. Boffi, B. Pasquini and M. Traini, NPB649, 243 (‘03); NPB680, 147 (‘04)

Light front model: B.C. Tiburzi and G.A. Miller, PRC64, 065204 (‘01);
PRD65, 074009 (‘02)

Radyushkin’s ansatz: A.V. Radyushkin, PLB449, 81 (‘99)

simply and intuitively construct GPDs by combining forward PDs, form factors, double distributions
Our method

Constituent quark model

Nambu-Jona-Lasinio (NJL) model based on the Faddeev approach

Relativistic field theory

- Dynamical breaking of Chiral Symmetry
- Covariant and Field theoretic
- q-q correlations
- non-renormalizable theory → cutoff scheme (Pauli-Villars cutoff)
- no confinement

quark-diquark model
Definition of GPDs

- DVCS

\[
\begin{align*}
K^+ &= x\bar{P}^+, \quad \Delta^+ = -2\xi\bar{P}^+ \\
\bar{P} &= \frac{p + p'}{2}, \quad \Delta = q' - q, \quad a^+ \equiv (a^0 \pm a^3)/\sqrt{2} \\
(0 < \xi < 1, \quad -1 < x < 1)
\end{align*}
\]
\[
\begin{align*}
\bar{P}^+ \int dy^- e^{ixy^-} \langle p', \lambda' | \psi_q (-y/2) \psi_q (y/2) | p, \lambda \rangle \bigg|_{y^+ = y_\perp = 0} & \quad \text{: Leading twist} \\
= \frac{1}{4} \left\{ (\gamma^-)_{\alpha\beta} \bar{u}(p', \lambda') \left[ H^q (x, \xi, \Delta^2) \gamma^+ + E^q (x, \xi, \Delta^2) i \sigma^+ \sigma^+ \frac{\Delta_\kappa}{2m_N} \right] u(p, \lambda) \\
+ (\gamma^5 \gamma^-)_{\alpha\beta} \bar{u}(p', \lambda') \left[ \bar{H}^q (x, \xi, \Delta^2) \gamma^+ \gamma^5 + \bar{E}^q (x, \xi, \Delta^2) \gamma^5 \frac{\Delta^+}{2m_N} \right] u(p, \lambda) \right\}
\end{align*}
\]

Put \(\gamma_{\beta\alpha}^+\)

\[
\begin{align*}
\bar{P}^+ \int dy^- e^{ixy^-} \langle p', \lambda' | \bar{\psi}_q (-y/2) \gamma^+ \psi_q (y/2) | p, \lambda \rangle \bigg|_{y^+ = y_\perp = 0} & \\
= \bar{u}(p', \lambda') \left[ H^q (x, \xi, \Delta^2) \gamma^+ + E^q (x, \xi, \Delta^2) i \sigma^+ \sigma^+ \frac{\Delta_\kappa}{2m_N} \right] u(p, \lambda)
\end{align*}
\]
Relations between GPDs → quark distributions, form factors

\[ H^q(x, \xi = 0, \Delta^2 = 0) = q(x) \]  \[
\tilde{H}^q(x, \xi = 0, \Delta^2 = 0) = \Delta q(x) \]

\( \int_{-1}^{1} dx H^q(x, \xi, \Delta^2) = F_1^q(\Delta^2), \quad \int_{-1}^{1} dx E^q(x, \xi, \Delta^2) = F_2^q(\Delta^2) \)

\( \int_{-1}^{1} dx \tilde{H}^q(x, \xi, \Delta^2) = g_A^q(\Delta^2), \quad \int_{-1}^{1} dx \tilde{E}^q(x, \xi, \Delta^2) = g_p^q(\Delta^2) \)

- Peak of free quark distributions \( xq(x) \) in nucleon is located at \( x=1/3 \).
  - Deviation from 1/3 is due to the q-q correlations.
- How about GPDs? Partonic interpretation is quite difficult!

Outgoing and incoming quarks have different momentum fraction

\[ \frac{k'^+}{p'^+} = \frac{x - \xi}{1 - \xi}, \quad \frac{k^+}{p^+} = \frac{x + \xi}{1 + \xi} < 1 \]

\( x < -\xi \) \quad \( -\xi < x < \xi \)

\( \xi < x \)

Behavior of GPDs?
NJL model

\[ \mathcal{L}_{I,NJL} = \times \]

transform to Fierz symm. form

\[ \mathcal{L}_{I,\pi} = G_\pi \text{ pionic channel } (0^-, I=1) \]
\[ \mathcal{L}_{I,s} = G_s \text{ scalar diquark channel } (0^+, I=0) \]
\[ \mathcal{L}_{I,a} = G_a \text{ axial vector diquark channel } (1^+, I=1) \]

\[ G_a = 0 \]
\[ G_\pi G_s \text{ reflect } \mathcal{L}_{I,NJL} \]
$m_Q$: current quark mass

$\Lambda$: cutoff

$G_\pi$: pionic channel coupling constant

$M_Q = 400$ MeV (Gap eq.)

$M_N = 940$ MeV (Faddeev eq.)

$\Lambda = \text{combine and determine}$

$G_\pi = 93$ MeV

$M_\pi = 140$ MeV (BS eq.)
Faddeev equation

Nucleon \rightarrow \text{Faddeev eq. in ladder approx.} \rightarrow \text{quark-diquark bound state}

\begin{align*}
G & = Z \Pi_{QD} \Gamma \\
\text{static approximation}
\end{align*}
Cutoff scheme

Pauli-Villars regularization:
• conserves gauge invariance
• applicable to any integral e.g., Euclidean, 3-momentum, LC …

\[
\frac{1}{k_1 - M} \cdots \frac{1}{k_n - M} \rightarrow \sum_{i=0}^{m} c_i \left\{ \frac{k_1 + M}{k_1^2 - M^2 - \Lambda_i^2} \cdots \frac{k_n + M}{k_n^2 - M^2 - \Lambda_i^2} \right\}
\]

where \( c_0 = 1 \) and \( \Lambda_0 = 0 \), \( n \) is # of particles in loop integral.

Conditions for convergence of loop integral should satisfy

\[
\sum_{i=0}^{m} c_i = \sum_{i=0}^{m} c_i \Lambda_i^2 = 0 \quad \rightarrow \quad m_{\text{min}} = 2
\]

In the limit \( \Lambda_1 \rightarrow \Lambda_2 \)

\[
\sum_{i=0}^{2} c_i f(\Lambda_i^2) = f(0) - f(\Lambda^2) + \Lambda^2 \frac{\partial}{\partial \Lambda^2} f(\Lambda^2)
\]
Results of Parameters

current quark mass: \( m_Q = 9 \text{ MeV} \)
cutoff: \( \Lambda_{PV} = 740 \text{ MeV} \)
pionic coupling const.: \( G_\pi = 10.4 \text{ GeV}^{-2} \)
diquark coupling const.: \( G_s / G_\pi = 0.65 \)
diquark mass: \( M_D = 590 \text{ MeV} < 2M_Q = 800 \text{ MeV} \)
GPDs in the NJL model

Definition of GPDs: \[
\bar{P}^+ \int \frac{dy^- e^{ix_p^+y^-}}{2\pi} \left< p'| \psi_q(-y/2)\gamma^+ \psi_q(y/2) | p \right> \bigg|_{y^+=y^-=0}
\]
\[
= u(p') \left[ H^q(x,\xi,\Delta^2)\gamma^+ + E^q(x,\xi,\Delta^2) \frac{i\sigma^{+\kappa}\Delta_\kappa}{2m_N} \right] u(p)
\]
where \( \bar{P} = \frac{p+p'}{2} \), \( x \) is quark LC momentum fraction \( \bar{K}^+ = x\bar{P}^+ \), and \( \xi \) is skewness \( \Delta^+ = -2\xi\bar{P}^+ \)

Diagrammatically depicted as

In actual calculation we calculate in the forward direction \( \bar{\Delta}_\perp = 0_\perp \) for simplicity

Then \( \Delta^2 = \Delta_{\text{min}}^2 (\xi) = \frac{-4M_N^2\xi^2}{1-\xi^2} \)
GPDs are related to quark distributions as
\[ H^q(x, \xi = 0, \Delta^2 = 0) = q(x), \]
Satisfy following sum rules for PDs and GPDs
\[
\int_{-1}^{1} dx q(x) = \int_{0}^{1} dx (q(x) - \bar{q}(x)) = \# \text{ of valence quarks in nucleon} \\
\int_{0}^{1} dx x \sum_{f} q_{f}(x) = 1 \quad \text{Momentum sum rule}
\]
\[
\int_{-1}^{1} dx H^q(x, \xi, \Delta^2) = F_1^q(\Delta^2), \quad \int_{-1}^{1} dx E^q(x, \xi, \Delta^2) = F_2^q(\Delta^2)
\]
above sum rules are automatically satisfied in our model
The following summation of ring diagrams are non-zero at finite $\Delta^2$ improve $\Delta^2$ of EM Form Factors in our model

\[ q - \Delta + \Delta + \Delta + \cdots = q \]

“vector meson dominant”
Results

• nucleon form factors $F_{1,2}^{p,n}(\Delta^2)$, NJL input GPDs, Radyushkin’s ansatz with NJL input PDs for GPDs $H^{u,d}(x,\xi,\Delta^2), E^{u,d}(x,\xi,\Delta^2)$
• At low energy scale, no $Q^2$-evolution
\( F_1^p(\Delta^2) \) vs. \(-\Delta^2 [\text{GeV}^2]\)

- NJL model
- +vertex corrections
- dipole fits to EXP.
$F_1^n(\Delta^2)$

NJL model

$+$vertex corrections

dipole fits to EXP.

$-\Delta^2 \ [\text{GeV}^2]$
\[ F_2^p(\Delta^2) \] (a)

- \( \Delta^2 \) [GeV^2]

**NJL model**

**dipole fits to EXP.**

+ vertex corrections

\[ -\Delta^2 \] [GeV^2]
\( F_2^n(\Delta^2) \)

\( -\Delta^2 \text{ [GeV}^2] \)

NJL model + vertex corrections

dipole fits to EXP.
Radyushkin’s Ansatz of GPDs

- intuitive and simple, only need PDs and nucleon form factors
- non-zero contribution btw. \(-1<x<1\)
- satisfies all sum rules

- \(\xi\)-dependent ansatz for \(H^g, \tilde{H}^g\) (Radyushkin et al., PLB 449, 81 (1999))

\[
\begin{align*}
\xi \rightarrow x &= \beta + \alpha \xi \\
\end{align*}
\]
\[ \Delta \to 0 \]

- \( \beta P^+ \) in s-channel
- quark distribution \( q(\beta) \) in \( \beta \)

\[ P \to 0 \]

- \( \frac{1}{2} (1 + \alpha) \Delta^+ \)
- \( \frac{1}{2} (1 - \alpha) \Delta^+ \)
- meson distribution amplitude in \( \alpha \)
- e.g. \( \frac{3}{4} (1 - \alpha)(1 + \alpha) \)

\[ H^q(x, \xi, \Delta^2) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(x - \beta - \alpha \xi) F^q(\beta, \alpha, \Delta^2) \]

\[ F^q(\beta, \alpha, \Delta^2) = h(\alpha, \beta) q(\beta) \frac{F_1^q(\Delta^2)}{F_1^q(0)} \]

- profile function;
- e.g. \( \frac{3}{4} \frac{1}{1 - \beta - \alpha^2} \)

- input PDs,
- we use NJL input PDs

- dipole fits to the EXP.
\[ H_{uv}(x, \xi, \Delta^2) \]
\[ \Delta^2 = \Delta_{\text{min}}^2(\xi) = \frac{-4M_N^2 \xi^2}{1 - \xi^2} \]

Full NJL calc.
Radyushkin’s ansatz
+ NJL input forward PDs

\[ \xi = 0 \quad \Delta^2 = 0 \, \text{GeV}^2 \]
\[ \xi = 0.1 \quad \Delta^2 = -0.04 \, \text{GeV}^2 \]
\[ \xi = 0.3 \quad \Delta^2 = -0.35 \, \text{GeV}^2 \]
$H^d(x, \xi, \Delta^2)$

$\Delta^2 = \Delta^2_{\min}(\xi) = \frac{-4M_N^2 \xi^2}{1-\xi^2}$

--- Full NJL calc.
- - - Radyushkin’s ansatz
+NJL input forward PDs

$\Delta^2 = -0.04 \text{ GeV}^2$

$\Delta^2 = 0 \text{ GeV}^2$

$\Delta^2 = -0.35 \text{ GeV}^2$
\[ E_{uv}(x, \xi, \Delta^2) = \Delta^2 = \Delta_{\text{min}}^2(\xi) = \frac{-4M_N^2 \xi^2}{1 - \xi^2} \]
\[ E_{d_y}^{d_v}(x, \xi, \Delta^2) \quad \Delta^2 = \Delta_{\min}^2(\xi) = -\frac{4M_N^2\xi^2}{1-\xi^2} \]
Summary and conclusions

• We calculated GPDs of the nucleon in the NJL model based on the Faddeev approach in the forward direction $\tilde{\Delta}_\perp = \tilde{0}_\perp$
• Sum rules btw. Form factors and GPDs are automatically satisfied.
• We consider only valence quarks, then ‘non-zero’ region of GPDs are $-\xi < x < 1$.
  \[ NJL \text{ model is field theoretic.} \]
  \[ \text{consistent with Radyushkin’s ansatz} \]

Compared with Radyushkin’s ansatz:
• With increasing $\xi$, GPDs calculated in NJL model show a strong variation, and a peak is shifted toward large $x$.
• Dependence on $\Delta^2$ is weaker.

$E^q(x,\xi,\Delta^2)$

• Their magnitudes are too small.
• Behavior of $E^u(x,\xi = \Delta^2 = 0)$ is not very similar to $u(x)$, whereas that of $E^d(x,\xi = \Delta^2 = 0)$ is very similar to $d(x)$

\[ \text{strong diquark correlations} \]
Outlook

- pion cloud and “d-term” effects
  \[ \xi < x < \xi \text{ will be modified} \]
- \( Q^2 \) evolution in \( -\xi < x < \xi \)
- medium effects, pol. GPDs, higher twist \( etc. \)