Gauge symmetry breaking in Higher-Dimensional Gauge Theories and its Applications

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Key Point:

Two types of gauge symmetry breaking on multiply-connected spaces.

<table>
<thead>
<tr>
<th>Higgs mechanism</th>
<th>Higgs VEV</th>
<th>Higgs potential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hosotani mechanism</td>
<td>Wilson loop</td>
<td>Casimir energy</td>
</tr>
</tbody>
</table>
Plan of Talk

Introduction to The Hosotani Mechanism

1) Fermion Mass Generation
2) Symmetry breaking

Applications

1) Gauge-Higgs Unification
2) GUT breaking
3) New CP violating mechanism
Intro.

Hosotani Mechanism

On the non-simply connected space:

\[ F_{\mu\nu} = 0 \quad \Rightarrow \quad A_\mu = 0 \]

Wilson line phase

\[ W = P \exp i \oint_C g A_y \, dy \]
Two Pictures

\[ U(y) = e^{-i g \langle A_y \rangle y} \]

\[ U^{-1} \]

\[ \psi'(y + L) = e^{-i g \langle A_y \rangle L} \psi'(y) \]

\[ \langle A_y \rangle \equiv 0 \]

(singular) Gauge transf.

\[ \langle A_y \rangle \neq 0 \]

- Periodic b.c.
- Background \( \langle A_y \rangle \)
- A-B effect

\[ \psi'(y + L) = e^{-i g \langle A_y \rangle L} \psi'(y) \]

Wilson loop: \( \langle W \rangle = \exp \left( ig \langle A_y \rangle L \right) \)
Gauge Symmetry Breaking by Wilson loop

\[ G(T^a) \supset H(X^a) \]

\[ [\langle W \rangle, X^a] = 0, \quad \text{Unbroken} \]

\[ [\langle W \rangle, Y^a] \neq 0, \quad \text{Broken} \]

N.B. \[ \langle A_y \rangle \] is not “Higgs”!

\[ \langle A_y \rangle = \frac{1}{gR} \begin{pmatrix} 1/3 \\ 1/3 \\ -2/3 \end{pmatrix} \Rightarrow SU(2) \times U(1) \]

\[ \langle W \rangle = 1_3 \otimes e^{2/3 \pi i} \Rightarrow SU(3) \]
Dynamical Mass Generation

SU(3) with fundamental fermions

One-loop mass\(^2\) correction to \(A_y\)

- Finite temp.
  \(\Leftrightarrow\) 
  \(M^3 \times S^1\) with
  - boson: periodic
  - fermion: anti-periodic

- Spatial \(S^1\)
  - Boson, fermion: periodic (taken by hand)

\[
m^2 = \frac{1}{3} g^2 T^2 \left( Nc + \frac{1}{2} N_f \right)
\]

\(T \sim 1/R\)

\[
m^2 = \frac{1}{3} g^2 T^2 \left( Nc - N_f \right)
\]

Tachyonic!
(for trivial \(A_y\))
Effective Potential for “$\langle A_y \rangle$”

$$V_{h_{\text{cos}}}^{\text{fd}} = \frac{1}{L^D} \sum_{i=1}^{N_c} \sum_{n=1}^{\infty} \frac{\cos n \theta_i}{n^D}$$

$$\langle A_y \rangle = \frac{1}{g R} \text{diag}(\theta_1, \theta_2, \ldots, \theta_{N_c})$$

$$\langle W \rangle_{\text{fd}} = 1_{N_c} \cdot e^{\frac{N_c-1}{N_c} \pi i}$$

Fermion Mass Term:

$$\bar{\psi}(m - ig \langle A_y \rangle)\psi$$

New vacuum!

(but gauge symmetry is not broken)
Gauge Symmetry Breaking

c.f. Adjoint fermions (Gauge Symmetry Breaking!)

Effective potential

\[ V_{\text{hos}}^{ad} = \frac{C}{L^D} \sum_{i,j=1}^{N_c} \sum_{n=1}^{\infty} \frac{\cos n(\theta_i - \theta_j)}{n^D} \]

Wilson line

\[ \langle W \rangle_{\text{fd}} = \text{diag}(1, \omega, \omega^2, \cdots, \omega^{N_c}), \omega^{N_c} = 1 \]

gauge symmetry is maximally broken!

\[ SU(N_c) \to U(1)^{N_c-1} \]
Features

- Zero mode of the gauge field is "scalar" field
- "Scalar" mass is stable against the quantum correction, because of the gauge symmetry.
- "Scalar" field is in adjoint representation. Therefore, the breaking is suitable for GUT.
- "Scalar" field couplings is the new contribution to the fermion mass
applications
Gauge Higgs Unification

“Higgs” mass

Curvature of the Effective potential.
No quadratic divergence is seen in 1-loop.
(Hatanaka-Inami-Lim)

Symplest model:

SU(3) → SU(2) × U(1)
By orbifolding
We can get doublet higgs
Deconstruction and Little Higgs

- Dimensional Deconstruction: Discritizing Gauge Theory (quiver, moose, theory space)

- Little Higgs
  Gauge symmetry is replaced with some other symmetries
GUT breaking

Gauge field is in the adjoint representation. → Hosotani mechanism will be suitable for GUT breaking

but, How to control breaking pattern like $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$?
A suggestion: Matter representations

$$\langle A_y \rangle = \frac{1}{gR} \begin{pmatrix} \theta \\ -\theta \end{pmatrix}$$

$$V_{\text{eff}} L^D \propto -(D-2) \sum_{i,j=1}^{N_c} \sum_{n=1}^{\infty} \frac{\cos n(\theta_i - \theta_j)}{n^D_D}$$

$$+ 2 \left[ \frac{D}{2} \right] N_{ad} \sum_{i,j=1}^{N_c} \sum_{n=1}^{\infty} \frac{\cos n(\theta_i - \theta_j)}{n^D_D}$$

$$+ 2 \left[ \frac{D}{2} \right] N_{f} \sum_{i=1}^{N_c} \sum_{n=1}^{\infty} \frac{\cos n\theta_i}{n^D_D}$$
SU(3)

\[
\langle A_\mu \rangle = \frac{1}{g R} \begin{pmatrix}
\theta_1 \\
\theta_2 \\
-\theta_1 - \theta_2
\end{pmatrix}
\]

\[
SU(3) \rightarrow \left\{ \begin{array}{c}
SU(3) \\
SU(2) \times U(1) \\
U(1)^2
\end{array} \right\}
\]

Breaking pattern is determined by 
\((N_{ad}, N_f)\)
A model of CP violation from Extra Dimension

Talk Based on the work by
Intro.

► origin of CPV?
► CP violation from Ex-D
  ▪ Old days (Thirring, 1971)
  ▪ New approaches
    ► orbifold vs. CP (Darwin-Mohapatra)
    ► domain wall, multi-brane,
      (Sakamura ’00)(Dooling et.al,02)(Ichinose 02)(Branco et.al01)
    ► Using Hosotani mechanism
      (Cosme – Frere 02, Grzadkowski-Wudka, 03)
    ► ... And so many models...
CP violation from Hosotani mechanism

Cosme, et.al., PRD68:096001(03), hep-ph/0303037

- 5Dimensional Gauge theory ($\gamma_5 = i \gamma^4$)
- If space-time is non-simply connected, a non-vanishing Wilson-line phase is defined: $<W> = P \exp i \int g <A_y> dy$
- Gauge field VEV as a mass-term of fermions

$$\bar{\psi} \left( m - ig \left< A_y \right> \gamma_5 \right) \psi$$

- 😞 Size of extra dimension is related to the EW breaking scale (too large) ($\therefore g \left< A_y \right> \sim O(1) R^{-1}$)
**SU(3)$^3$ Model**

- $SU(3)_c \times SU(3)_l \times SU(3)_r$
- Space-time : $M^4 \times S^1/Z_2$

<table>
<thead>
<tr>
<th></th>
<th>$SU(3)_c \times SU(3)_l \times SU(3)_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_l$</td>
<td>$(3,3,1)$</td>
</tr>
<tr>
<td>$Q_r$</td>
<td>$(3,1,3)$</td>
</tr>
<tr>
<td>$A_c$</td>
<td>$(8,1,1)$</td>
</tr>
<tr>
<td>$A_r$</td>
<td>$(1,1,8)$</td>
</tr>
<tr>
<td>$A_l$</td>
<td>$(1,8,1)$</td>
</tr>
<tr>
<td>$\Phi_r$</td>
<td>$(1,1,8)$</td>
</tr>
<tr>
<td>$\Phi_l$</td>
<td>$(1,8,1)$</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>$(1,3,3)$</td>
</tr>
</tbody>
</table>
Orbifolding

\( \text{SU}(3)^3 \) broken to
\( \text{SU}(3)_c \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)_L \times \text{U}(1)_R \)

with:

\[ P_G = \text{diag}(-1,-1,+1)_L \times \text{diag}(+1,+1,-1)_R \]

\[ P_G = \text{diag}(-1,-1,+1) \]

\[ A_\mu^g(y)\lambda^g = +A_\mu^g(-y)P_G\lambda^gP_G^{-1} \]

\[ A_\nu^g(y)\lambda^g = -A_\nu^g(-y)P_G\lambda^gP_G^{-1} \]

\[ \psi(y) = \gamma_5 P_G \psi(-y) \]

\[ \phi^g(y)\lambda^g = -\phi^g(-y)P_G\lambda^gP_G^{-1} \]
### Zero modes

<table>
<thead>
<tr>
<th></th>
<th>SU(3)$_L$ x SU(3)$_R$</th>
<th>SU(2)$_L$ x SU(2)$_R$ x U(1)$_L$ x U(1)$_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_l$</td>
<td>$(3,1)$</td>
<td>$(2,1)_{(1,0)}$</td>
</tr>
<tr>
<td></td>
<td>$(u_L, d_L)$</td>
<td></td>
</tr>
<tr>
<td>$D_R$</td>
<td></td>
<td>$(1,1)_{(-2,0)}$</td>
</tr>
<tr>
<td>$Q_r$</td>
<td>$(1,3)$</td>
<td>$(1,2)_{(0,-1)}$</td>
</tr>
<tr>
<td></td>
<td>$(u^c_R, d^c_R)$</td>
<td></td>
</tr>
<tr>
<td>$D^c_L$</td>
<td></td>
<td>$(1,1)_{(0,2)}$</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>$(3,3)$</td>
<td>$(2,2)_{(-1,1)}$</td>
</tr>
<tr>
<td></td>
<td>$\sigma$</td>
<td></td>
</tr>
<tr>
<td>$\chi$</td>
<td></td>
<td>$(1,1)_{(2,-2)}$</td>
</tr>
<tr>
<td>$\Phi_R$</td>
<td>$(1,8)$</td>
<td>$(1,2)<em>{(0,3)} + (1,2)</em>{(0,-3)}$</td>
</tr>
<tr>
<td>$A_{r,M}$</td>
<td>$(1,8)$</td>
<td>$(1,3)<em>{(0,0)} + (1,1)</em>{(0,0)}$</td>
</tr>
<tr>
<td></td>
<td>$A_{r\mu}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$A_{r\nu}$</td>
<td>$(1,2)<em>{(0,3)} + (1,2)</em>{(0,-3)}$</td>
</tr>
</tbody>
</table>
Quark mass and CP phase

- $U(1)_R \times U(1)_L$ is broken to $U(1)_Y$ by $<\chi>$
- $SU(2)_R$ broken by $<\phi_R> = \nu_R \lambda_6$ and/or $g_R <A_{\nu_y}> = \nu_A \lambda_6$ ($<A_y>$ must be parallel to $<\phi_R>$)
- New CP phase by Hosotani term

\[
\begin{pmatrix}
\bar{d}_{rR} & \bar{D}_{lR} \\
\end{pmatrix}
\begin{pmatrix}
f_{\Sigma} \nu_\sigma & f_r \nu_r + i\nu_A \\
f_l \nu_l & M_{\langle \chi \rangle} \\
\end{pmatrix}
\begin{pmatrix}
d_{lL} \\
D_{rL} \\
\end{pmatrix}
\]

- $\nu_A$ scale is related to the size of 5th dimension and common part for all of generations.
- Up-type quarks get masses from $<\phi_L>$
Why parallel?

\[ \langle \phi_r \rangle = v_r \lambda_6, \]

\[ \text{tr} \ F_{\mu\nu} F^{\mu\nu} (\langle \phi_r \rangle, \langle A_{ry} \rangle) \supset \text{tr} \ [\phi_r, A_{ry}]^2 = 0 \]

\[ \Rightarrow \langle A_{ry} \rangle = \begin{pmatrix} -2a \\ a & b \\ b & a \end{pmatrix} \]

Orbifold projection forbids \( a \neq 0 \) \( \Rightarrow \langle A_{ry} \rangle \) is parallel to \( \lambda_{\bar{6}} \)
Comment on the relation between “Real CP violation” and this model

Masiero and Yanagida (hep-ph/9812225)

\[
\begin{pmatrix}
\bar{d}_L & \bar{D}_L \\
\end{pmatrix}
\begin{pmatrix}
0 & \phi_0 + \phi \\
\end{pmatrix}
\begin{pmatrix}
d_L \\
D_L \\
\end{pmatrix}
\]

Our model looks similar to the M.-Y. mechanism, but actually NOT.
Lepton sector

Lepton field “L” = (1_C, 3_L, 3_R)

orbifolding: B.C.

\[ P_G = \text{diag}(-1, -1, +1)_1 \times \text{diag}(+1, +1, -1)_r, \]
\[ \rightarrow \begin{pmatrix}
- & - & + \\
- & - & + \\
+ & + & - \\
\end{pmatrix} \]

Possible assignment:

\[
\begin{pmatrix}
N^0_L & E^+_{2L} & E^+_{2R} \\
E^-_{1L} & N^0_{1L} & N^0_{1R} \\
E^-_{1R} & N^0_{2R} & N^0_{2L}
\end{pmatrix}
\]
Mass terms for leptons

Possible mass terms:
$LL \Sigma, \text{Tr}(LL \phi_R), \text{Tr}(L \phi_L L)$

$L = \begin{pmatrix} N_L^0 & E_{2L}^+ & E_{2R}^+ \\ E_{1L}^- & N_{1L}^0 & N_{1R}^0 \\ E_{1R}^- & N_{2R}^0 & N_{2L}^0 \end{pmatrix}$

$\Sigma = \begin{pmatrix} \sigma \\ \chi \end{pmatrix} = \begin{pmatrix} S_{11}^0 & S_{12}^+ & 0 \\ S_{21}^- & S_{22}^0 & 0 \\ 0 & 0 & \chi \end{pmatrix}$

$\langle \phi_L \rangle = v_L \lambda_6, \quad \langle \phi_R \rangle = v_R \lambda_6,$

$\langle \phi_{L6} \rangle, \langle S_{11} \rangle, \langle S_{22} \rangle \ll \langle \phi_{R6} \rangle, \langle \chi \rangle,$
Mass Matrices

<table>
<thead>
<tr>
<th>charged</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{1L}^-$</td>
<td>$E_{1R}^-$</td>
<td>$E_{2L}^+$</td>
<td>$E_{2R}^+$</td>
</tr>
<tr>
<td>$E_{1L}^-$</td>
<td>$\phi_{L6}$</td>
<td>$\sum_{33}$</td>
<td></td>
</tr>
<tr>
<td>$E_{1R}^-$</td>
<td>$\phi_{L6}$</td>
<td></td>
<td>$S_{22}$</td>
</tr>
<tr>
<td>$E_{2L}^+$</td>
<td>$\sum_{33}$</td>
<td></td>
<td>$\phi_{R6}$</td>
</tr>
<tr>
<td>$E_{2R}^+$</td>
<td></td>
<td>$S_{22}$</td>
<td>$\phi_{R6}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>neutral</th>
<th>$N_{L}^0$</th>
<th>$N_{1L}$</th>
<th>$N_{2L}$</th>
<th>$N_{1R}$</th>
<th>$N_{2R}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{L}^0$</td>
<td>$\sum_{33}$</td>
<td></td>
<td>$S_{22}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_{1L}$</td>
<td>$\sum_{33}$</td>
<td></td>
<td>$S_{11}$</td>
<td>$\phi_{L6}$</td>
<td>$\phi_{R6}$</td>
</tr>
<tr>
<td>$N_{2L}$</td>
<td>$S_{22}$</td>
<td>$S_{11}$</td>
<td>$\phi_{R6}$</td>
<td>$\phi_{L6}$</td>
<td></td>
</tr>
<tr>
<td>$N_{1R}$</td>
<td>$\phi_{R6}$</td>
<td>$\phi_{L6}$</td>
<td>$S_{11}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_{2R}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \langle \phi_{L6} \rangle, \langle S_{11} \rangle, \langle S_{22} \rangle \ll \langle \phi_{R6} \rangle, \langle \chi \rangle , \]

- One pair of light Dirac fermions
- One ultra-light left-handed neutral fermion are obtained!!
Summary

Model of \((SU(3))^3\) on \(M^4 \times S^1/Z_2\),

- CP phase coming from \(<A_y>\) and it is common part of all generation.
- Size of XD is related to \(SU(2)_R\) breaking scale
- Up type quark mass from \(<\phi_L> \propto \lambda_6\)
- we can get light “electron” and super-light “neutrino” from Lepton multiplet L (through See-Saw mechanism)
- our model can chose correct helicity fields more naturally by imposing orbifold boundary conditions
- Left-Right assymmetry (why in “L” sector hosotani mech does not work?)?
Discussions

- **Flavor physics**
  - Can it yield CKM, MNS matrix?
  - Flavor symmetry ($S_3$)?
  - Solutions to Strong CP? Not yet, but it’s possible to extend flavor sector

- **SUSY?**