Data Analysis in High Energy Physics

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Outline

- Some Observables of Interest
- Elementary Observables
- More Complex Observables
- Analysis tools.
Some Observables of Interest

- Total Interaction/Reaction Cross Section
- Differential Cross Section
- Particle mass or width
- Branching Ratio
• Cross Section defines the strength of a particular interaction between two particles.
Cross Section
Interaction Matrix Element

\[ a + b \rightarrow c + d \]

Flux \( \phi = n_a v_i \)

\( v_i \) : velocity of the incident beam relative to the target

Number of reactions pre target particle \( W \)

\[ W = \phi \sigma = \frac{2\pi}{2} |M_{if}|^2 \rho_f \]

Interaction Matrix Element \( M_{if} = \int \Psi_i^* U \Psi_i dV \)

Energy density of final state \( \rho_f = dN / dE \)

\[ dN = \frac{V}{(2\pi\hbar)^3} p^2 dp d\Omega \]

\[ \frac{d\sigma}{d\Omega} \equiv \frac{W}{v_i} = \frac{1}{4\pi^2 \hbar^4} |M_{if}|^2 \frac{p_f^2}{v_i v_f} \]
Scattering Cross Section

- **Differential Cross Section**
  \[
  \frac{d\sigma}{d\Omega}(E,\Omega) = \frac{1}{F} \frac{dN_s}{d\Omega}
  \]

- **Total Cross Section**
  \[
  \sigma(E) = \int d\Omega \frac{d\sigma}{d\Omega}(E,\Omega)
  \]

- **Average number of scattered into dΩ**
  \[
  N_s(E,\Omega) = FAN\delta x \frac{d\sigma}{d\Omega}(E,\Omega)
  \]
Most particles studied in particle physics or high energy nuclear physics are unstable and decay within a finite lifetime.

Particles decay randomly (stochastically) in time. The time of their decay cannot be predicted. Only the probability of the decay can be determined.

The probability of decay (in a certain time interval) depends on the life-time of the particle. In traditional nuclear physics, the concept of half-life is commonly used.
Radioactive Decay of Nuclei with Half Life = 1 Day

Undecayed Nuclei

Days

Half-Life
• The number of particle (nuclei) left after a certain time “t” can be expressed as follows:

\[ N(t) = N_0 e^{-\frac{t}{\tau}} \]

\[ \frac{dN(t)}{dt} = -\frac{1}{\tau} N(t) \]

• where “\(\tau\)” is the mean life time of the particle

• “\(\tau\)” can be related to the half-life “\(t_{1/2}\)” via the simple relation:

\[ t_{1/2} = -\ln(\frac{1}{2})\tau = 0.693 \tau \]
### Examples - particles

<table>
<thead>
<tr>
<th>Type</th>
<th>Mass (MeV/c²)</th>
<th>( \tau ) or ( \Gamma )</th>
<th>( c\tau )</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton (p)</td>
<td>938.2723</td>
<td>&gt;1.6x10^{25} y</td>
<td>Very long…</td>
<td>Baryon</td>
</tr>
<tr>
<td>Neutron (n)</td>
<td>939.5656</td>
<td>887.0 s</td>
<td>2.659x10^{8} km</td>
<td>Baryon</td>
</tr>
<tr>
<td>N(1440)</td>
<td>1440</td>
<td>350 MeV</td>
<td>Very short!</td>
<td>Baryon resonance</td>
</tr>
<tr>
<td>( \Delta(1232) )</td>
<td>1232</td>
<td>120 MeV</td>
<td>Very short!!</td>
<td>Baryon resonance</td>
</tr>
<tr>
<td>( \Lambda )</td>
<td>1115.68</td>
<td>2.632x10^{-10} s</td>
<td>7.89 cm</td>
<td>Strange Baryon resonance</td>
</tr>
<tr>
<td>Pion (( \pi^+ ))</td>
<td>139.56995</td>
<td>2.603x10^{-8} s</td>
<td>7.804 m</td>
<td>Meson</td>
</tr>
<tr>
<td>Rho - ( \rho(770) )</td>
<td>769.9</td>
<td>151.2 MeV</td>
<td>Very short</td>
<td>Meson</td>
</tr>
<tr>
<td>Kaon (K^+)</td>
<td>493.677</td>
<td>1.2371 x 10^{-8} s</td>
<td>3.709 m</td>
<td>Strange meson</td>
</tr>
<tr>
<td>( D^+ )</td>
<td>1869.4</td>
<td>1.057x10^{-12} s</td>
<td>317 ( \mu )m</td>
<td>Charmed meson</td>
</tr>
</tbody>
</table>
## Examples - Nuclei

Radioactive Decay Reactions Used to data rocks

<table>
<thead>
<tr>
<th>Parent Nucleus</th>
<th>Daughter Nucleus</th>
<th>Half-Life (billion year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Samarium ($^{147}$Sa)</td>
<td>Neodymium ($^{143}$Nd)</td>
<td>106</td>
</tr>
<tr>
<td>Rubidium ($^{87}$Ru)</td>
<td>Strontium ($^{87}$Sr)</td>
<td>48.8</td>
</tr>
<tr>
<td>Thorium ($^{232}$Th)</td>
<td>Lead ($^{208}$Pb)</td>
<td>14.0</td>
</tr>
<tr>
<td>Uranium ($^{238}$U)</td>
<td>Lead ($^{206}$Pb)</td>
<td>4.47</td>
</tr>
<tr>
<td>Potassium ($^{40}$K)</td>
<td>Argon ($^{40}$Ar)</td>
<td>1.31</td>
</tr>
</tbody>
</table>
• By virtue of the fact that a particle decays, its mass or energy \((E=mc^2)\), cannot be determined with infinite precision, it has a certain *width* noted \(\Gamma\).

• The width of an unstable particle is related to its life time by the simple relation

\[
\Gamma = \frac{h}{\tau}
\]

• \(h\) is the Planck constant.
• In general, particles can decay in many ways (modes).
• Each of the decay modes have a certain relative probability, called branching fraction or branching ratio.
• Example \((K^0_s)\) Neutral Kaon (Short)
  – Mean life time = \((0.8926\pm0.0012)\times10^{-10}\) s
  – \(c\tau = 2.676\) cm
  – Decay modes and fractions

<table>
<thead>
<tr>
<th>mode</th>
<th>(\Gamma_i/\Gamma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pi^+\pi^-)</td>
<td>((68.61 \pm 0.28)) %</td>
</tr>
<tr>
<td>(\pi^0\pi^0)</td>
<td>((31.39 \pm 0.28)) %</td>
</tr>
<tr>
<td>(\pi^+\pi^-\gamma)</td>
<td>((1.78 \pm 0.05)\times10^{-3})</td>
</tr>
</tbody>
</table>
Elementary Observables

- Momentum
- Time-of-Flight
- Energy Loss
- Particle Identification
- Invariant Mass Reconstruction
Momentum Measurements

- Definition
  - Newtonian Mechanics
    \[ p = mv \]
  - Special Relativity
    \[ p = \gamma mv \]
    \[ \gamma = \frac{1}{\sqrt{1-(v/c)^2}} \]

- But how does one measure “p”?
• Use a spectrometer with a constant magnetic field B.
• Charged particles passing through this field with a velocity “v” are deflected by the Lorentz force.

\[ F_M = q \vec{v} \times \vec{B} \]

• Because the Lorentz force is perpendicular to both the B field and the velocity, it acts as centripetal force “F_c”.

\[ F_c = \frac{mv^2}{R} \]

• One finds

\[ p = \gamma mv = qBR \]
Momentum Measurements Technique

- Knowledge of B (magnetic field) and R (bending radius) needed to get “p”
- B is determined by the construction/operation of the spectrometer.
- “R” must be measured for each particle.
Bending Magnet in Spectrometer

\[ \frac{p_{xz}}{q/e} = -\frac{e/c}{\sin \theta_2 - \sin \theta_1} \int B_y \, dz \]
Momentum Measurement

B = 0.5 T

Collision Vertex

Radius: R

Trajectory is a helix in 3D; a circle in the transverse plane

$\pi^+$

$p = \gamma m v = qBR$
TPC Inside the Solenoid Magnet at SPring-8
Pad Plane of TPC

- Inner radius: < 1.25 cm.
- Outer radius: < 30 cm.
- Maximum drift distance: about 70 cm.
- ~1000 pads and ~100 wires for readout,
- $\sigma_{xy} \sim 350\text{mm}$ and $\sigma_z \sim 500\text{mm}$,
- $B = 1.5 \sim 2.5\text{T.}$
32-channel SPring-8 FADC cards

- Use TEXONO FADC and IHEP BES version as the starting point.
- 40 MHz clock, maximum 1024 sampling bins for one strobe.
- 10-bit FADC: ADC input 0-2 V range.
- Shift register inside FPGA: max length = 100 time bin.
- On-board threshold suppression performed by FPGA.
- Buffer FIFO: 16 bits x 4096 depth dual port memory, large enough to hold 5 events before issuing IRQ.
- CPLD: controlling VME actions.
- Adjustable zero-suppression level channel by channel and number of events per IRQ.
- VME 9U: 32 channels/module; 8 detachable cards/module; 4 channel/card.
Determination of Particle Trajectory
Operation of Time-Projection Chamber
Time-of-Flight (TOF) Measurements

- Typically use scintillation detectors to provide a “start” and “stop” time over a fixed distance.
- Electric Signal Produced by scintillation detector
  
  ![Diagram showing scintillation detector and time measurement](image)

- Use electronic Discriminator
- Use time-to-digital-converters (TDC) to measure the time difference $= \text{stop} - \text{start}$.

- Given the known distance, and the measured time, one gets the velocity of the particle

  $$ v = \frac{\Delta d}{\Delta t} = \frac{\Delta d}{c_1(t_{\text{stop}} - t_{\text{start}} + c_2)} $$
More Complex Observables

- Particle Identification
- Invariant Mass Reconstruction
- Identification of decay vertices
• Particle Identification or PID amounts to the determination of the mass of particles.
  – The purpose is not to measure unknown mass of particles but to measure the mass of unidentified particles to determine their species e.g. electron, pion, kaon, proton, etc.

• In general, this is accomplished by using to complementary measurements e.g. time-of-flight and momentum, energy-loss and momentum, etc
LEPS Detector System

- Dipole Magnet (0.7 T)
- Start counter
- Aerogel Cherenkov Detector (n=1.03)
- Silicon Vertex Detector
- MWDC 1
- MWDC 2
- MWDC 3
- TOF wall

γ
Target, Upstream Spectrometer, Dipole Magnet
Dipole Magnet and Drift Chambers
Time-of-Flight Wall
Particle Identification by TOF

Reconstructed mass

K/π separation (positive charge)

\[ m = \frac{\gamma v}{p}, \quad v = \frac{l}{TOF} \]
• The energy loss of charged particles passing through a gas is a known function of their momentum. (Bethe-Bloch Formula)

\[-\frac{dE}{dx} = 2\pi N_\alpha r_e^2 m_e c^2 \rho \frac{Z z^2}{A \beta^2} \ln(...)\]
Particle Identification by dE/dx

**dE/dx PID range:**
- ~ 0.7 GeV/c for K/π
- ~ 1.0 GeV/c for K/p
How Do We Identify Resonances?

**Resonance**: Broad states with finite widths and lifetimes, which can be formed by collision between the particles into which they decay.

\[ M_{inv} = \sqrt{(E_1 + E_2)^2 - (\vec{P}_1 + \vec{P}_2)^2} \]

\[ p + Be \rightarrow e^+ + e^- + X \]
Invariant Mass Reconstruction

- In special relativity, the energy and momenta of particles are related as follows:

\[ E^2 = p^2 c^2 + m^2 c^4 \]

- This relation holds for one or many particles. In particular for 2 particles, it can be used to determine the mass of the parent particle that decayed into two daughter particles.
Invariant Mass Reconstruction (cont’d)

- Invariant Mass
  \[ m_p^2 c^4 = E_P^2 - p_P^2 c^2 \]

- Invariant Mass of two particles
  \[ m_p^2 c^4 = (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2 c^2 \]

- After simple algebra
  \[ m_p^2 c^4 = m_1^2 c^4 + m_2^2 c^4 + 2(E_1 E_2 - p_1 p_2 c^2 \cos \theta) \]
Example – Lambda Reconstruction

Good pairs have the right invariant mass and accumulate in a peak at the Lambda mass.

Bad pairs produce a more or less continuous background below and around the peak.
STAR STRANGENESS!

(K0s, \Lambda, \bar{\Omega}^+, \Omega^-)

\sigma = 4 \text{ MeV/c}^2

\sigma = 17 \text{ MeV}

\Lambda

\bar{\Omega}^+, \Omega^-

K^*, K^+

STAR Preliminary

STAR Preliminary

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Finding V0s

Primary vertex

dcav0

dcap

dca

dcan

proton

pion
In case you thought it was easy...

After
Strange Baryon Ratios

Reconstruct:
\[\sim 0.84 \Lambda/\text{ev}, \quad \sim 0.61 \overline{\Lambda}/\text{ev}\]

\[\text{Ratio} = 0.73 \pm 0.03 \text{ (stat)}\]

Reconstruct:
\[\sim 0.006 \Xi^-/\text{ev}, \quad \sim 0.005 \Xi^+/\text{ev}\]

\[\text{Ratio} = 0.82 \pm 0.08 \text{ (stat)}\]
Resonances

\[ \phi \rightarrow K^+ K^- \]

\[ K^0 \rightarrow K^+ \pi^- \]

\[ \bar{K}^0 \rightarrow K^- \pi^+ \]
Offline Analysis

RAW DATA

Detector

Electronics

DAQ

Physics event

LEPSana

Ntuple

Analysis Code

Physics Results

Calibration Parameters
Introduction of PAW

Basic Principles

- PAW is an INTERACTIVE SYSTEM
- PAW provides a set of COMMANDS acting on specific objects
- The commands structure of PAW is a TREE structure
- The general structure of the tree is:
  \[
  \text{OBJECT/\textit{ACTION}}
  \]
  Example:
  \[
  \text{NTUPLE/PLOT}
  \]
  \[
  \text{HISTOGRAM/PROJECT}
  \]
  \[
  \text{VECTOR/DRAW}
  \]
- PAW commands can be grouped into MACROS
- PAW++ provide a Motif based User Interface to PAW
- PAW and PAW++ have the SAME basic functionality
Introduction of PAW

PAW

HIGZ
The Graphics Package
basic graphics and
graphics editor for
pictures in data base

SIGMA
Arrays Manipulation

KUIP
User Interface
Command Processor
Menu Dialogue
Motif Interface

MINUIT
Minimization Package

HPLOT
The Plotting Package

HBOOK
Histogramming
N-Tuples
Statistical Analysis

ZEBA
Data Structure Manager
Input/Output Server
Data Base Manager

COMIS
FORTRAN Interpreter

ZEBA FILES

ZEBA MEMORY
Introduction of PAW

Data Analysis - Data Presentation

Data

- Vectors
  - 1D
  - 2D
  - 3D
- Histograms
  - 1D
  - 2D
  - 3D
- Ntuples
  - RWN
  - CWN

Analysis

- Fitting
- Smoothing
- Array Manipulation
- FORTRAN Interpreter
- Cuts
- Projections

Presentation

- 1D, 2D, and 3D Plots

Repositories

- ASCII
- RZ
PAW:
http://wwwas.web.cern.ch/wwwasd/paw/

PAW is conceived as an instrument to assist physicists in the analysis and presentation of their data. It provides interactive graphical presentation and statistical or mathematical analysis, working on objects familiar to physicists like histograms, event files (Ntuples), vectors, etc. PAW is based on several components of the CERN Program Library. Like CERN Program Library PAW usage and/or redistribution is granted under the terms of the GNU General Public License. For any questions about PAW contact Paw.Support@Cern.Ch or send a bug report. You may also find useful informations in the cern.heplib news group.

PAW Release Notes
(Last update: September 16th 2002)
Release notes describing all changes (features added or bug fixed) in each PAW release.

Known bugs
(Last update: October 27th 2004)
A list of known bugs with workarounds. You can send a bug report to the PAW support person.

PAW Frequently Asked Questions
(last update: June 7th 2004)
Frequently asked questions are listed here as well as answers to them. A list with some usage statistics is also available but the access is slower. A file containing all the FAQs is also available (useful for a printing purpose). You can try also the CERNLIB FAQs. The PAW FAQs are regularly reordered according to the number of accesses each one gets. So, do not add a particular FAQ in your book-marks because it may point to an another FAQ after some time. It is recommended to always use the FAQ page’s search facility to find a particular FAQ.

Contributions
(last update: September 27th 2004)
Here you can find some general interest PAW macros and programs provided by PAW users. If you have such macros or programs send them to Paw.Support@Cern.Ch.

Tutorial
If you are not familiar with PAW and want to learn how to use it, this is probably a place to go. This tutorial has been modified to take into account the latest Ntuple queries processor capabilities. Therefore it is valid only for PAW versions greater or equal to 2.07. The old version of the PAW tutorial, using the
ROOT: http://root.cern.ch

Development Version 4.01/04

The development release of ROOT 4.01/04 is now available.

Tar files for the source, documentation and binaries are available at:

Version 4.01/04 Release Notes (Draft)

The CVS tag for this version is **v4-01-04**.

The AFS versions of 4.01/04 can be found at:

/afs/cern.ch/sw/root/v4.01.04/ch73_slc3/root for Redhat7.3-slc3 gcc3.2.3
/afs/cern.ch/sw/root/v4.01.04/ch73_gcc32/root for Redhat7.3 g++3.2.2
/afs/cern.ch/sw/root/v4.01.04/ch73_gcc296/root for Redhat7.3 gcc2.96
/afs/cern.ch/sw/root/v4.01.04/sun4x_58/root for Solaris5.8 CC5.2

Production Version 4.00/08

The production release of ROOT 4.00/08 is now available.

20/11/2004

12/07/2004
References

• Analysis Techniques in High Energy Physics, Claude A Pruneau, Wayne State University. (http://rhic.physics.wayne.edu/REU/talks/Analysis%20Techniques.ppt)