Price Rising/falling Asymmetry Originating from Different Risk Attitudes of Traders

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Abstract
Price rising/falling asymmetry, which indicates long-term sustaining rising and short-term sudden falling in stock market, is a ubiquitous phenomenon in stock markets throughout the world. Different from widely used time series method, we adopt so-called inverse statistics to depict the asymmetry empirically. To explore mechanism of the price asymmetry, we build a multi-agent model with two kinds of investors, respectively named fundamentalists and chartists. Inspired by Kahneman and Tversky’s claim on people’s asymmetric psychological response to the same amount of gains and losses, we assume that chartists take different risk attitudes to gains and losses, and then employ different trading strategies. The simulation result of the model is consistent with empirical work, which may support our conjecture that chartists’ asymmetric risk attitudes might be one origin of rising/falling asymmetry.

Keywords: Price rising/falling asymmetry; risk attitude; chartist; fundamentalist

1. Introduction
Price rising/falling asymmetry involves the different rates of price change between rising and falling. It can be described as asymmetry of rising/falling ranges with fixed time horizons, or asymmetry of realizing time with fixed rising/falling ranges. Most scholars favored the first description: Cont[1] mentioned large downward price movement but not equally large upward one; Roehner and Sornette[2] paid attention to shape of peaks and bottoms, and induced four vivid peak shape patterns; Veldkamp[3] estimated time reversal asymmetry. In the second description, Simon-sen, Jensen and Johansen[4, 5, 6, 7, 8, 9] made impressive works. They studied optimal investment horizons and gain/loss asymmetry, which was described as different distributions of waiting time needed to obtain same amount positive/negative returns. Particularly Donangelo et al.[4] argued that gain/loss asymmetry was only found for stock index, not for any of the individual stocks which composed the index. Although the first description on asymmetry is more vivid, it can not undermine suspicion of artificially data-choosing easily. To avoid such question, we use inverse statistical method to study the rising/falling asymmetry in this paper.

As a method originally used in turbulence, inverse statistics has also been used to study stocks[5, 6] and exchange rate[7]. The following is the standard procedure to use inverse statistics: at first, a predetermined level of return is fixed; then, the waiting time needed to achieve this return for the first time is recorded; finally, the distribution of waiting time is available. In this paper, we set five times of average daily volatility as the predetermined level of return, which is about 3.7%. As shown in Fig.1, the distribution of waiting time for negative returns (i.e. −3.7%) is marked with red stars, and the distribution of waiting time for positive returns (i.e. +3.7%) is marked with green circles. An interesting feature that can be observed from this figure is the apparent asymmetry between the waiting time distributions for the predetermined return ρ = ±3.7%. In particular, for ρ = −3.7% there is a higher probability, as compared to what is observed for ρ = +3.7%, to find short waiting time, or in other words, draw-downs are faster than draw-ups. From a statistical perspective, this means in short term it is more likely to loss than to gain. That is to say, there exists a gain-loss asymmetry.
Figure 1: Empirical result of inverse statistics for DJIA daily close prices from 1928/10/01 to 2009/10/15. The predetermined return $\rho$ (i.e. rho) is five times of average daily volatility. Circles denote positive returns and stars denote negative returns.


We believe that macroscopic asymmetry should arise from microscopic asymmetry, and therefore conjecture that asymmetric risk attitudes might be one of its sources. Kahneman and Tversky[13, 14] illustrated people’s asymmetric psychological response to the same amount of gains and losses. Particularly, there is a salient character of people’s attitudes to changes of welfare: losses loomed larger than gains. Such asymmetric psychological effect is named as “loss aversion”, and leads to agents’ risk attitudes under various investment circumstances. These arguments spur us to explore the relationship between the agents’ asymmetric perceptions and the stock price fluctuation asymmetry.

Based on the analytical framework of Brock and Hommes[15], we build a heterogeneous agent model to reproduce the rising/falling asymmetry. There are two kinds of agents with heterogeneous expectations, respectively named fundamentalists and chartists. We assume that chartists’ asymmetric psychological response to gains and losses do affect their risk attitudes, while fundamentalists have no such psychological characteristics. Subsequently, chartists’ asymmetric risk attitudes lead to an alternative demand function for stock, and finally lead to rising/falling asymmetry of stock price.

This paper is organized as follows. In Section 2, we present the heterogeneous multi-agent model embedded with chartists’ asymmetry risk attitudes. In Section 3, we compare statistical results of our simulation with empirical findings, and provide some tentative explanations. Section 4 concludes.

2. The Model

In our model, there are two kinds of assets agents can invest: one kind is risk-free, and the other is risky. Risk-free asset is perfectly elastically supplied with a fixed rate of return $r$. The price of risky asset, here corresponding to price index of a stocks market portfolio, is fluctuating all the time.
2.1. Evolutionary process

At the beginning of every period, risky asset price of last period is publicly known but price of this period has not yet formed. Different kinds of agents have different price expectations and trading strategies based asset price of last period. After they make final trading decisions, aggregate excess demand can be calculated. Finally, the price of risky asset in this period is formed. Since agents’ trading volume and historical price are all open, agents’ realized profit in this period can be calculated. Each agent will updated his trading strategies according to the realized profit, which is regarded as a fitness measurement to evolution.

2.2. Heterogeneous beliefs

Two kinds of agents take different price expectations and different trading strategies. Fundamentalists believe that there is a fundamental equilibrium price which asset prices would finally converge to, while chartists form their price expectations in an adaptive pattern.

Let $\alpha$ denote fundamentalists, and $\beta$ denote chartists. Fundamentalists take constant price expectation and variance expectation, as given by

$$E_\alpha(p_t) \equiv \bar{p}_\alpha, V_\alpha(p_t) = \sigma^2_{p_t}.$$  

Fundamentalists buy in when current asset is undervalued and sell out when overvalued.

Chartists also take constant variance expectation, which is

$$V_\beta(p_t) = \sigma^2_{p_t}.$$  

But for price expectation, chartists take the form of adaptive expectation

$$E_\beta(p_t) = p_{t-1} + \Delta p_{t-1}^e, \Delta p_{t-1}^e = \omega(p_{t-1} - p_{t-2}) + (1 - \omega)\Delta p_{t-1,t-2},$$

where $\Delta p_{t-1}^e$ is expected increment, and $\omega$ is the weight of realized increment of last period. As a form of trend extrapolation, adaptive expectation captures some feature of trend chasing, so it is able to represent chartists’ trend-chasing behavior pattern in stock market.

In general framework, the amount of asset purchased by fundamentalists is given as

$$n_{\alpha,t} = \frac{E_{\alpha,t}(p_t) - (1 + r)p_{t-1}}{V_{\alpha,t}(p_t)}.$$  

In most HAM literature, chartists’ excess demand function for risky asset is same as fundamentalists’, which means that chartists and fundamentalists share the same risk attitude. As mentioned, Kahneman and Tversky[13] argued that investors had asymmetric psychological response to the same amount of gains and losses. Behavioral finance theories like disposition effect[16] and overconfidence[17] also provided sound proofs to investors’ similar psychological influence on their trading. Here we suppose that the fundamentalists are more rational than the chartists and such psychological effect is just confined to chartists. So the amount of risky asset purchased by chartists is modified as following

$$n_{\beta,t} = \begin{cases} 
\frac{E_{\beta,t}(p_t) - (1 + r)p_{t-1}}{\lambda \cdot V_{\beta,t}(p_t)}, & (E_{\beta,t}(p_t) - (1 + r)p_{t-1}) > 0 \\
\frac{\frac{1}{2} \cdot V_{\beta,t}(p_t)}{\lambda \cdot V_{\beta,t}(p_t)}, & (E_{\beta,t}(p_t) - (1 + r)p_{t-1}) < 0 
\end{cases},$$

where $\lambda > 0$ denotes risk attitude of chartists. The numerator represents expected future profit. On the premise of $\lambda > 1$, when the numerator is positive, which means likely gains, chartists are willing to bear more risk and lean to buy more shares; when the numerator is negative, which means probable losses, the amount of risk chartists can bear decrease, and they become more risk-aversing as Kahneman and Tversky[13] mentioned. Therefore, chartists prefer reducing their trading volume. This assumption is consistent with the deposit effect.
After agents make their final trading decisions, aggregate excess demand is given by
\[ D_t = \sum_i \eta_{i,t-1} n_{i,t} = \eta_{a,t-1} n_{a,t} + \eta_{f,t-1} n_{f,t}, \]
where \( \eta_{i,t-1} \) is a market fraction of group \( i \) at the end of period \( (t-1) \), and the value is maintained till the beginning of period \( t \). Here a change of the market fraction implies that agents can switch from one group to the other to chase higher realized profit.

Under market-maker framework, price forms according to the following dynamics
\[ p_t = p_{t-1} + \gamma D_t + \epsilon, \]
where \( \gamma \) is adjustment speed of price, while \( \epsilon \) is white noise normally distributed with mean 0 and variance 1.

After current price is formed, each group can calculate their realized profit according to their last trading volume \( n_{i,t} \) as following
\[ \pi_{a,t} = n_{a,t} \cdot \Delta p_t - C, \pi_{f,t} = n_{f,t} \cdot \Delta p_t, \]
where \( C \) is the cost for fundamentalists to gather and analyze information every period and
\[ \Delta p_t = p_t - p_{t-1}. \]

Let \( \eta_{i,t} \) denote market fraction of group \( i \), and it follows discrete choice probability[15] and updates according to current value of realized profit every period
\[ \eta_{i,t} = \frac{\exp(\rho \pi_{i,t}(p_t))}{\sum_i \exp(\rho \pi_{i,t}(p_t))}, \]
where \( \rho \) is a parameter to measure speed of switching, and it also can be regarded as an index of herd effect in the market. Under such market fraction function, no group would be completely driven out of market even in the worst circumstances. That is to say, there are always some agents sticking to their original strategy.

3. Simulation and discussion

We perform simulations on the model with the following parameters
\[ \overline{p}_0 = 100, \sigma_a^2 = \sigma_f^2 = 1, \omega = 0.6, r = 10^{-3}, \lambda = 2.3, \gamma = 0.25, C = 10, \rho = 1, p_0 = \overline{p}_0, \eta_0 = 0.5. \]

Fig.2 illustrates a time series realization of risky asset price, from which we may have a glimpse of rising/falling asymmetry intuitively. To get a more accurate measurement, it is reasonable to investigate rising/falling asymmetry with inverse statistics. Fig. 3 shows the statistical result of the simulated price series. Here the absolute predetermined return \( \rho \) is still five times of average daily volatility. Fig. 3 illustrates three distinct results corresponding to different \( \lambda \) values. When \( \lambda \) is set to 2.3, the asymmetry between positive and negative returns is obvious, as shown in Fig.3(a).

In short term it is more likely to loss than to gain, while in long term probability of gaining predetermined profit is larger than probability of losing the same. This is consistent with empirical work.

Moreover, if we change risk attitude of chartists, let \( \lambda = 1 \), which means that fundamentalists and chartists adopt same risk attitude and same demand function on risky asset, the asymmetry almost disappears as shown in Fig. 3(b). If we push \( \lambda \) further into the new region of \((0,1)\), for instance \( \lambda = 0.5 \), then we get Fig. 3(c), where the relative position of peaks of distributional curves for positive and negative returns is opposite to that in Fig.3(a). Comparing Fig.3(a) with Fig.3(b) and (c), we attribute the price fluctuation asymmetry to risk attitudes of chartists. They bear less risk when facing losses and bear more risk when facing gains. Considering the theoretical analysis, numerical simulation and empirical findings, \( \lambda > 1 \) is more realistic range for risk attitude of chartists.

Without loss of generality for parameters setting, we relax range of values for each parameter. We find, with \( \gamma \in [0.15, 0.27] \), \( \rho \in [0.1, 1.5] \) and \( \omega \in (0, 1) \), the model can reproduce similar inverse statistics results with different value of \( \lambda \) like in Fig.3. We also find that \( \overline{p}_0, \sigma_a^2, \sigma_f^2, r \) and \( C \) have no significant qualitative effects on the results as long as their value is valid. However, the intensity of choice \( \rho \) has large influence on market fraction of fundamentalists and chartists. Since the majority is usually chartists, the market fraction is more volatile with a larger \( \rho \). Moreover, a key parameter in our model is the adjustment speed of price \( \gamma \), and an inappropriate value can lead to asset price divergence, so the adjustment speed of price should be selected carefully.
4. Conclusion

By building a simple multi-agent model, we try to reproduce price rising/falling asymmetry in stock market. We believe that macroscopic asymmetry should arise from microscopic asymmetry, and therefore conjecture that asymmetric risk attitudes might be one of its sources. So the key mechanism to the asymmetry in our model lies in chartists’ asymmetric risk attitudes which are introduced to modify their asset demand function, which gains supports from behavioral finance theories such as prospect theory, disposition effect and so on. The simulation results are consistent with empirical work. We wish this paper can give another path to explanation of the rising/falling asymmetry phenomenon.

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References

Figure 3: Different waiting time distributions with same parameter set of Fig. 2 except for $\lambda$. The values of $\lambda$ in (a) (b) (c) are respectively 2.3, 1 and 0.5.