Will the US Economy Recover in 2010?
A Minimal Spanning Tree Study

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\begin{abstract}
Based on the temporal distributions of clustered segments in the time series of the ten Dow Jones US (DJUS) economic sector indices, we calculated their cross correlations over the period February 2000 to August 2008, the two-year intervals 2002–2003, 2004–2005, 2008–2009, and also over 11 corresponding segments within the present financial crisis. From these cross-correlation matrices, we constructed minimal spanning trees (MSTs) of the US economy at the sector level. We find that the average cross correlation is higher/lower when the market volatility is higher/lower, and the existence of a strongly-correlated sectors that expands and contracts in tandem with changes in the overall market volatility. A core-fringe structure is found in all MSTs, with CY, IN, and NC consistently making up the core, and BM, EN, HC, TL, UT residing predominantly on the fringe. Taking advantage of the high-resolution temporal information available from the clustered segments in each time series, we saw the shocks accompanying volatility movements always start at the fringe, sometimes in conjunction with anomalously high cross correlations here, and propagate inwards to the core of all MSTs of the 11 statistically-stationary corresponding segments. Most of these volatility shocks originate within the domestic fringe sectors, HC, TL, and UT, in the US economy. More importantly, we find that the MSTs can be classified into two distinct, statistically robust, topologies: (i) star-like, with IN at the center, associated with
\end{abstract}

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low-volatility economic growth; and (ii) chain-like, associated with high-volatility economic crisis. When we examined successive corresponding segments within the present housing bubble financial crisis, we find that the latter MST can be obtained from earlier MST through a minimal set of primitive rearrangements, each representing a statistically significant change in the cross correlations of the sectors involved. In contrast, the MST of a corresponding segment within the previous technology bubble financial crisis cannot be obtained from the MST of the preceding corresponding segment through simple rearrangements. We believe this observation suggests that the US economy is much more efficient in responding to volatility shocks now, compared to a decade ago. Finally, we present statistical evidence, based on the emergence of a star-like MST in Sep 2009, and the MST staying robustly star-like throughout the Greek Debt Crisis, that the US economy is on track to a recovery.

Key words: US economic sectors, macroeconomic cycle, financial crisis, economic recovery, financial time series, segmentation, clustering, cross correlations, minimal spanning tree, planar maximally filtered graph

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1. Introduction

In the CBS ‘60 Minutes’ interview televised on 15 March 2009, Ben Bernanke predicted that the recession triggered by the global financial crisis will end in 2009, and the US economy will recover in 2010 [1]. While we will never know whether Bernanke made the prediction based on his gut feelings, or on simulation results from some sophisticated macroeconomic model, what we do know is that the prediction sparked intense public debate on whether the Chairman of the US Federal Reserve was overly optimistic. Given that the financial industry was still reeling from the massive October 2008 slide, reactions to Bernanke’s statement must be especially strong. We also know that the US Federal Reserve does not appear to be behind its Chairman: up till July 2010, the interest rate has not been raised [2], even though there has been calls from within the Federal Reserve system to tighten the money supply [3]. This has led to mounting concerns from economists that the oversupply of government money, in the form of an interest rate that is nearly zero, will cause an inflation when the economy recovers [4, 5]. In fact, a commentator argued that US stimulus money is fueling property bubbles all over Asia, and warned that the global economy will crash once again in 2012 when the Feds rein in their easy money [6].
In January 2004, there was a similar call by economists to raise interest rates [7], when the US economy was showing signs of coming out of the technology bubble crisis. The Federal Reserve responded hesitantly only in June 2004 [2]. We can understand the concern of the US government then, and possibly also now: how do we know that the early signs will lead on to a recovery that will strengthen and stay the course? From these historical and contemporary lessons, we know that a more sensitive and more robust indicator of economic recovery is needed. While much work has been done on developing and validating reliable precursor signatures (also called *leading indicators*) for the onset of financial crises (see for example, Refs. [8, 9, 10, 11, 12, 13, 14], and understanding such economic disasters in general (see for example, Refs. [15, 16, 17, 18, 19, 20], less has been done to find robust indicators of economic recovery (see for example, Refs. [21, 22, 23]. In this work, we hope to address this important gap.

Recently, we adapted the recursive entropic segmentation method [24, 25] developed by Bernaola-Galván and coworkers for biological sequence segmentation, and applied it to financial time series segmentation [26]. Based on our segmentation of the Dow Jones Industrial Average (DJIA) time series between 1997 and 2008, we saw that the US economy, as measured by the DJIA, switched between a high-volatility crisis phase and a low-volatility growth phase. The first crisis phase lasted from mid-1998 to mid-2003, coinciding with the US technology bubble and the ensuing economic recession. The second crisis phase started in mid-2007, coinciding with the US Subprime Crisis and the ensuing global financial crisis. More interestingly, we could also identify a year-long series of precursor shocks prior to the mid-1998 and mid-2007 onsets of two crisis phases, as well as a year-long series of inverted shocks prior to the mid-2003 economic recovery. The series of inverted shocks started with the mid-2002 Dow Jones low, so if we believe the internal dynamics of the US economy had not changed from the previous financial crisis to the present financial crisis, we would naively expect the US economy to recover one year after the March 2009 lows, i.e. the second quarter of 2010, give and take.

Clearly, a single study of a single time series spanning only two financial crises and one growth period is hardly enough statistical evidence in Bernanke’s favor. To enhance the statistical significance of features seen in the segmented DJIA time series, we carried out a cross-section study, comparing the segmented time series of the ten Dow Jones US (DJUS) economic sector indices [27]. By identifying the sequences of onsets in the ten DJUS indices, we find sectors in the US economy going first to last into the present financial crisis in merely two months! While we may or may not have an extended sequence of precursor shocks to work with for
predicting market crashes and financial crisis (see the recent update [28] on the heroic efforts by Sornette and coworkers), when the dominoes are set in motion policy makers will have a month or two to contain the crisis. Since this financial crisis eventually spread globally, we will have to wait for the next potential crisis to find out if containment is at all possible. We do know, however, that the US Federal Reserve acted promptly, announcing the first of a series of interest rate cuts in August 2007. Unfortunately, as detailed in Ref. [27], we saw these rate cuts rapidly losing effectiveness. A critical discussion on the actions taken by the US Federal Reserve can be found in Ref. [29].

In the same comparative study, we also identified the sequence of economic recoveries in the different US economic sectors. The excruciatingly slow complete economic recovery from the previous financial crisis, defined as from consistent growth in the first sector to consistent growth in the last sector, took one and a half years. Given this long time scale, developing robust indicators to detect economic recovery, and thereafter design stimulus packages, should be easier than finding sensitive indicators that would warn us of an impending financial crisis. We would imagine that tracking slow month-to-month indicators should be enough to give us a confident forecast on the start of growth, but all through the second half of 2009 and 2010 to date, we hear commentators mostly urging caution [30, 31, 32, 33, 34, 35, 36, 37]. We believe this cautious outlook can be blamed partly on swings in the stock markets, which always become strong when things are taking a turn for the better or for the worst. Perhaps the way to allay such market-driven fears is to extract convincing signs from the high-frequency stock-market data itself. Based on these signs, policy makers can then tell more confidently that the economy will recover in a matter of months, and start planning measures to further stimulate the recovery.

In this paper, which is organized into six sections, we report a minimal spanning tree (MST) study of the segmented time series of the ten DJUS economic sector indices. In Section 2, we describe the data sets studied, and the statistical methods used to analyze them. We will also review the main results from Ref. [27]. In Section 3, we examine the gross structure of the 10-sector MST over the 2000 to 2009 period, as well as those over the 2002–2003 crisis period, the 2004–2005 growth period, and the 2008–2009 crisis period. We explain the macroeconomic significance of the core-fringe structure of the MSTs, and also suggest why the MSTs organize themselves into a star topology during growth, and into a chain topology during crisis. Then, in Section 4, we construct MSTs within segments associated with distinct macroeconomic phases to study the correlational dynamics within the US economy. We again find that the MST is star-
like in low-volatility segments, and chain-like in high-volatility segments. This
tells us that the star-like MST is a robust and reliable character of economic well
being. By combining temporal information obtained through statistical segmen-
tation and clustering, we show that volatility shocks always start at the fringe and
propagate inwards. Some of the links to leader sectors have anomalously high
cross-correlations. We also check whether such volatility shocks have a more
domestic or more global origin. Finally in Section 5, by examining a nearly con-
tiguous sequence of corresponding segments, we look at how the MST rearranges
in the pre-recovery periods for both the previous and the current financial crises.
We found very violent rearrangements prior to the previous economic recovery.
For the present financial crisis, we can see clear signatures of star-to-chain and
chain-to-star rearrangements, accompanied by the expected changes in market
volatilities and cross-correlations. This suggests that the US market has become
more efficient, as far as processing information is concerned, over the past 5–10
years. After predicting that the US economy will recover in early 2010, we sum-
marize our findings in Section 6.

2. Data and methods

2.1. Data

Tic-by-tic data for the ten Dow Jones US (DJUS) economic sector indices (see
Table 1 for the indexing scheme $i = 1, \ldots, 10$ used) over the period 14 February
2000 to 25 November 2009 were downloaded from the Thomson-Reuters Tick-
history (formerly known as Taqtic) database [38]. These were then processed
into time series $X_i(t) = (X_{i,1}, \ldots, X_{i,t}, \ldots, X_{i,N})$ at fixed time intervals indexed by
$1 \leq t \leq N$. Since financial markets are known to exhibit complex dynamics on
multiple time scales, the data frequency has to be carefully selected. In the fi-
nancial economics literature, intervals ranging from 5 to 60 minutes have been
used for estimating realized or benchmark daily volatilities for foreign exchange
or stock market time series [39, 40, 41, 42, 43]. In general, higher data frequencies
are not employed due to worries about the effects of market microstructures.

We chose to sample the time series at 30-minute intervals. As explained
in Ref. [26], the half-hourly data frequency allows us to confidently identify
statistically stationary segments as short as a day. Higher data frequency was
not used, because in a macroeconomic study such as this, we are not interested
in segments shorter than a day, i.e. the intraday market microstructure. From
the index time series $X_i$, we then prepare the log-index movement time series
Table 1: The ten Dow Jones US economic sector indices.

<table>
<thead>
<tr>
<th>i</th>
<th>symbol</th>
<th>sector</th>
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<tbody>
<tr>
<td>1</td>
<td>BM</td>
<td>Basic Materials</td>
</tr>
<tr>
<td>2</td>
<td>CY</td>
<td>Consumer Services</td>
</tr>
<tr>
<td>3</td>
<td>EN</td>
<td>Energy</td>
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<tr>
<td>4</td>
<td>FN</td>
<td>Financials</td>
</tr>
<tr>
<td>5</td>
<td>HC</td>
<td>Healthcare</td>
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<tr>
<td>6</td>
<td>IN</td>
<td>Industrials</td>
</tr>
<tr>
<td>7</td>
<td>NC</td>
<td>Consumer Goods</td>
</tr>
<tr>
<td>8</td>
<td>TC</td>
<td>Technology</td>
</tr>
<tr>
<td>9</td>
<td>TL</td>
<td>Telecommunications</td>
</tr>
<tr>
<td>10</td>
<td>UT</td>
<td>Utilities</td>
</tr>
</tbody>
</table>

\[ \mathbf{x}_i = (x_{i,1}, \ldots, x_{i,t}, \ldots, x_{i,N-1}), \text{ where } x_{i,t} = \log X_{i,t+1} - \log X_{i,t}. \]

We work with log-index movements, because different indices have different magnitudes, and it is more meaningful to compare their fractional changes.

2.2. Segmentation

Financial time series are well known to be highly nonstationary. In particular, several recent studies revealed that the instantaneous volatility fluctuates about a constant level, before switching over rapidly to fluctuations about a different constant level [44, 45, 46]. Based on these, and similar earlier observations, economics and finance practitioners explored various methods for decomposing a nonstationary time series into stationary segments, which are called regimes or trends in the economics and finance literatures. In these literatures, segment boundaries are referred to as structural breaks, trend breaks, or change points. The earliest works in this field are by Goldfeld and Quandt [47], and by Hamilton [48]. Since these pioneering works, an enormous economics literature on structural breaks and change points has been amassed, a few based on the original Markov switching models [49], and many others based on autoregressive models and unit-root tests [50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64]. In the econophysics literature, apart from our own work, we are only aware of the recent preprint by Tóth et al., who segmented the time series of market orders on the London Stock Exchange, modeling each segment by a stationary Poisson process [65].

As with all model-driven segmentation of time series data, we assume that each economic sector time series \( \mathbf{x}_i \) consist of \( M_i \) segments, and that within segment \( m_i \), the log-index movements \( x_{i,m}^{m_i} \) follows a stationary statistical distribution. From the seminal work by Mantegna and Stanley [66], we know that high-
frequency index movements can be fitted very well to stable Lévy distributions. We also know from the study by Kullmann et al. [67] that the daily log-index movements can be fitted well to a truncated Lévy distribution, when the sample size is small, but becomes normally distributed when the sample size is large. This suggests that the appropriate model for each stationary segment ought to be a Lévy stable process. However, parameter estimation for Lévy stable distributions [68, 69, 70, 71, 72, 73, 74, 75] is a computationally expensive process, and computing the probability density [76, 77, 78, 79, 80] is equally tedious. From our experience segmenting biological sequences, we know that segment boundaries that are statistically very significant can be discovered by any segmentation procedure, no matter what model we assumed for the underlying stationary segments. We believe that the most statistically significant segment boundaries in financial time series would also be equally insensitive to choice of model, or model mis-specification. Indeed, when we compared segments of the 2002–2003 DJIA half-hourly time series obtained assuming that the log-index movements are normally distributed, against those obtained assuming the log-index movements are Lévy stable distributed, the strongest segment boundaries are in good agreement (no more than two days apart) [81]. With this reassurance, we chose to intentionally mis-specify the model, and work instead with the lognormal index movement (LIM) model. In this model, the log-index movements in segment $m_i$ are assumed to follow a stationary Gaussian process with mean $\mu_{i,m_i}$ and variance $\sigma^2_{i,m_i}$. Unlike parameter estimation for the Lévy distribution, maximum-likelihood estimates of the Gaussian parameters $\mu_{i,m_i}$ and $\sigma^2_{i,m_i}$ can be done very cheaply.

To find the unknown segment boundaries $t_{i,m_i}$, which separates segments $m_i$ and $m_i + 1$, we use the recursive segmentation scheme introduced by Bernaola-Galván et al. [24, 25]. In this segmentation scheme, we start with the time series $x = (x_1, \ldots, x_t, x_{t+1}, \ldots, x_n)$, and compute the Jensen-Shannon divergence [82]

$$\Delta(t) = \ln \frac{L_2(t)}{L_1}, \tag{1}$$

where within the log-normal index movement model,

$$L_1 = \prod_{s=1}^{n} \frac{1}{\sqrt{2\pi \sigma^2}} \exp \left[ -\frac{(x_s - \mu)^2}{2\sigma^2} \right] \tag{2}$$

is the likelihood that $x$ is generated probabilistically by a single Gaussian model.
with mean $\mu$ and variance $\sigma^2$, and

$$L_2(t) = \prod_{s=1}^{t} \frac{1}{\sqrt{2\pi\sigma^2_L}} \exp\left[-\frac{(x_s - \mu_L)^2}{2\sigma^2_L}\right] \prod_{s=t+1}^{n} \frac{1}{\sqrt{2\pi\sigma^2_R}} \exp\left[-\frac{(x_s - \mu_R)^2}{2\sigma^2_R}\right]$$

(3)

is the likelihood that $x$ is generated by two statistically distinct models: the left segment $x_L = (x_1, \ldots, x_t)$ by a Gaussian model with mean $\mu_L$ and variance $\sigma^2_L$, and the right segment $x_R = (x_{t+1}, \ldots, x_n)$ by a Gaussian model with mean $\mu_R$ and variance $\sigma^2_R$. In terms of the maximum likelihood estimates $\hat{\mu}, \hat{\mu}_L, \hat{\mu}_R$ and $\hat{\sigma}^2, \hat{\sigma}^2_L, \hat{\sigma}^2_R$, the Jensen-Shannon divergence $\Delta(t)$, which measures how much better a two-segment model fits the time series data compared to a one-segment model, simplifies to

$$\Delta(t) = n \ln \hat{\sigma} - t \ln \hat{\sigma}_L - (n - t) \ln \hat{\sigma}_R + \frac{1}{2} \geq 0.$$  

(4)

Scanning through all possible times $t$, a cut is then placed at $t^*$, for which the Jensen-Shannon divergence

$$\Delta^* = \Delta(t^*) = \max_t \Delta(t)$$

(5)

is maximized, to break the time series $x = (x_1, \ldots, x_n)$ into two statistically most distinct segments $x_L^* = (x_1, \ldots, x_{t^*})$ and $x_R^* = (x_{t^*+1}, \ldots, x_n)$ (see for example, Fig. 1). This one-into-two segmentation is then applied recursively onto $x_L^*$ and $x_R^*$ to obtain shorter and shorter segments (see also Fig. 1, for example). At each stage of the recursive segmentation, we also optimize the segment boundaries using the first-order algorithm described in Ref. [83], where we recompute the optimum position of segment boundary $m$, within the time series subsequence bound by segment boundaries $m \pm 1$. This is done iteratively for all segment boundaries, until they have all converged onto their optimum positions.

As the optimized recursive segmentation progresses, the Jensen-Shannon divergence of newly discovered segment boundaries, as well as the previously discovered segment boundaries, will in general become smaller and smaller. Segment boundaries thus become less and less significant statistically, and at some point, we must terminate the recursive segmentation. There are three ways to do so. In the first approach, the Jensen-Shannon divergences of new segment boundaries are tested for statistical significance against various $\chi^2$ distributions with the appropriate degrees of freedom [24, 25]. The recursive segmentation terminates when no new segment boundaries more significant than the chosen confidence
Figure 1: The Jensen-Shannon divergence spectrum for the DJIA time series from Jan 1997 to Aug 2008 (red). This is a typical spectrum consisting of one very strong peak, in this example, at mid-2003. Also show are the Jensen-Shannon divergence spectra for the left segment (green, 1997 to mid-2003) and the right segment (blue, mid-2003 to Aug 2008) obtained at the second stage of the recursive segmentation. In this example, the two segments have divergence maxima at mid-2002 and mid-2007 respectively.

level $p$ can be found. In the second approach, new segment boundaries are accepted if the information criteria of the two-segment models they imply are larger than the information criteria of the one-segment models we are selecting against [84, 85]. Here, the recursive segmentation terminates when further segmentation does not explain the data better. In the third approach, we define signal-to-noise ratios based on the Jensen-Shannon divergence fluctuations within supersegments that new segment boundaries are supposed to divide [83]. The recursive segmentation terminates when the signal-to-noise ratios of all new segment boundaries fall below a chosen threshold value.

Alternatively, we could also terminate the recursive segmentation when no new optimized segment boundaries with Jensen-Shannon divergence greater than a cutoff of $\Delta_0 = 10$ are found. The short and medium segments produced by this termination criterion are reasonable, but the long segments obtained tend to have internal segment structures masked by their context [86]. We then recursively segment these long segments, by progressively lowering the cutoff $\Delta_0$ until a segment boundary with strength $\Delta > 10$ appears. The final segmentation then consists of segment boundaries discovered through the automated recursive segmentation, as
well as segment boundaries discovered through progressive refinement of overly long segments. Based on the experience in our previous works [26, 27], this semi-automatic recursive segmentation appears to produce acceptable results.

2.3. Segment clustering

After the segmentation is completed, we obtain a large number (typically > 100) of segments for each time series. While successive segments are statistically distinct from each other, segments that are far apart can actually be statistically similar. This observation suggests that the large number of segments may belong to a small number of segment classes. By comparing multiple indicators, economists classified different market periods into four macroeconomic phases or regimes: (i) a growth phase; (ii) a contraction phase; (iii) a correction phase; and (iv) a crash phase. We therefore expect the time series segments to also be organized into roughly four classes. A similar problem arises in biological sequences, which can have thousands of segments that can be organized into tens of segment types that differ in their biological functions. In the ground-breaking paper by Azad et al., the 248 segments of human chromosome 22 was classified into 53 domain types using single-link hierarchical clustering [87]. Inspired by this prospect of reducing the complexity of our segmentations, we performed independent hierarchical agglomerative clusterings on the segments within each US economic sector time series, using the complete link algorithm (see Ref. [88] for details on the complete link algorithm, and also a review on the broad area of statistical clustering).

In this segment clustering, we used the Jensen-Shannon divergences between segments as their statistical distances. Clustering of different periods within a financial time series has been previously investigated [89, 90, 91], in the absence of any segmentation analysis. Analyzing the hierarchical complete-link clustering trees obtained (see the example in Fig. 2), we then selected between four to six clusters of segments for each US economic sector. These clusters represent different macroeconomic phases (differentiated by their market volatilities, see Fig. 3) present in the time series data. Once all segments have been assigned to their respective clusters, we use the heat-map-like color scheme in Table 2 to plot the temporal distributions of clustered segments, an example of which is shown in Fig. 4. All the analyses presented in this paper are based on features identified from the temporal distributions of clustered segments for the ten DJUS economic sector indices.
Figure 2: The complete-link hierarchical clustering trees of DJIA the segments between Jan 1997 and Aug 2008, obtained using the log-normal index movement model. The differentiated clusters are coloured according to their market volatilities: low (deep blue and blue), moderate (cyan and green), high (yellow and orange), and extremely high (red). Also shown at the major branches are the Jensen-Shannon divergence values at which subclusters are merged.
Figure 3: Means and standard deviations of the DJIA segments between Jan 1997 and Aug 2008, obtained using the log-normal index movement model. As we can see, the clusters are differentiated by their standard deviations.

Figure 4: Temporal distributions of the clustered segments for the log-normal index movement model, superimposed onto the DJIA time series. The red solid lines indicate the dates of important market events: (1) July 1997 Asian Financial Crisis; (2) October 1997 Mini Crash; (3) August 1998 Russian Financial Crisis; (4) DJI 2000 High; (5) NASDAQ Crash; (6) start of 2001 recession; (7) Sep 11 Attack; (8) end of 2001 recession; (9) DJI 2002 Low; (10) February 2007 Chinese Correction.
Table 2: Heat-map-like color scheme for the different volatility clusters, and the macroeconomic phases they correspond to. The crisis phase, which consists of the high-volatility (yellow) and very-high-volatility (orange) clusters, is significantly longer than the economic contraction phase accepted by economists. In fact, economic contraction, as determined by successive quarters of contraction in the GDP, typically occurs at the end of a crisis phase.

<table>
<thead>
<tr>
<th>volatility</th>
<th>extremely low</th>
<th>low</th>
<th>moderate</th>
<th>high</th>
<th>very high</th>
<th>extremely high</th>
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<tr>
<td>color</td>
<td>black</td>
<td>blue</td>
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<td>phase</td>
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2.4. Identifying corresponding segments

Of the many features that we can identify from individual temporal distributions, as well as across the panel of temporal distributions, corresponding segments that appear in all or most of the indices are the most striking visually. In the economics and finance literature, a mean or volatility movement that occurs over multiple time series is called comovement [92, 93, 95, 96, 97, 98], common jumps [99, 100, 101], common shocks [102, 103, 104], or common breaks [105, 106, 107, 108, 109, 110]. The consensus that arise from this body of work is that the statistical significance of a change point is amplified by the cross section it occurs concurrently over. In our study, the corresponding segments do not necessarily start at the same time, because our use of high-frequency data allows us to identify the change points that are individually optimum for the ten DJUS economic sector indices. More importantly, our corresponding segments in the various indices do not end at the same time. As discussed in Ref. [27], the durations of each corresponding segment, and the Jensen-Shannon divergence values at the start of these segments, tell us how strongly the shock impacted different sectors in the US economy. Moreover, the different start times of the corresponding segments allow us to roughly map out the progress of the shock.

Because of the different start times and different durations, we mark segments in the ten DJUS economic sector indices as corresponding segments if they (i) have similar volatilities (high and high, or low and low); or (ii) are flanked by volatility movements in the same directions(low-to-high and moderate-to-high, or high-to-low and moderate-to-low). For this, we took advantage of the heat-map-like color scheme in the temporal distributions (see for example, Fig. 5).

2.5. Identifying the onset of the current global financial crisis

Besides the various corresponding segments that we can identify in the ten DJUS economic sector indices, their temporal distributions of clustered segments
Figure 5: Panel of temporal distributions of clustered segments for the ten DJUS economic sector indices. Around Sep 2007, a high-volatility (yellow) segment can be seen in all but two (TC and UT) of the sectors. In TC, a moderate-volatility (green) segment can be found around Sep 2007, flanked by low-volatility (blue) segments before and after. In UT, an extremely-high-volatility (red) segment can be found around Sep 2007, flanked by high-volatility (yellow) segments before and after. These volatility movements surrounding these two segments (low-to-moderate and then moderate-to-low in TC, and high-to-extremely-high and then extremely-high-to-high in UT) are in the same directions as the high-volatility segment identified in the eight sectors. Even though these segments (circled) do not start at the same time, and do not have the same durations, it is highly likely that they are the sectors’ responses to the same shock.

also allow us to identify the macroeconomic onset of the current global financial crisis. When an economic sector enters a crisis phase, its temporal distribution goes from mostly low-volatility segments to mostly high-volatility segments. Since there are always some short high-volatility segments in the midst of the long low-volatility segments, and some short low-volatility segments in the midst of the long high-volatility segments, we choose the end of the last low-volatility segment lasting longer than two months to be the start of the crisis in Ref. [27]. In Fig. 6, we show the temporal distributions of clustered segments for BM, FN and EN. Based on our dating criterion, the start dates of high-volatility phases in BM and FN are 20 June 2007 and 23 July 2007 respectively, consistent with the current global financial crisis starting around July 2007. The EN sector, however,
is an anomaly, because based on our working definition, the high-volatility phase would have started on 24 February 2005. In reality, the market volatility of EN remains moderate between 2005 and 2007, so what we are seeing in Fig. 6(c) is an extremely extended market correction phase, driven by the ever rising oil price (Fig. 6(d)).

Figure 6: Temporal distributions of clustered segments between 14 February 2000 and 31 August 2008, showing the onsets of the present financial crisis in the (a) BM and (b) FN sectors, and also the anomaly in the (c) EN sector. Also shown is the (d) price of crude oil, which started rising sharply after the mid-2003 economic recovery.

The EN anomaly aside, we could easily pinpoint the onsets of the current financial crisis in the rest of the economic sectors using the same dating criterion. Fig. 7 shows the onsets of the current financial crisis in different economic sectors, arranged in the sequence of start dates. From Fig. 7, we see that NC and UT are the leaders in the onset sequence. This is understandable as the present financial downturn was triggered by the Subprime Crisis, and NC contains the
homebuilders and home supply manufacturers. What is harder to comprehend is the fact that HC, IN, and TL, were the next three sectors to become jittery, even before the FN sector, which was singled out by the public as the culprit for creating the crisis. Most surprisingly, we find the US economy went from the first sector to the last sector into the high-volatility phase in merely two months! This is an very short time scale for an entire economy to descend into a financial crisis.

2.6. The market impact of interest rate cuts

From the pattern of clustered segments after the onset of current financial crisis, we notice that most of the important shocks in different economic sectors occur within a day or two of each other, and appear to be exogenously driven by the US Federal Reserve funds rate cuts, as shown in Fig. 8. In BM, TC and TL, we circled brief volatility movements a few days to a week before the interest rate cuts, which tell us that these sectors were in fact anticipating the rate cuts. The Federal funds rate is the rate which banks lend money to each other overnight.
During a financial crisis, the US Federal Reserve lowers the interest rate to increase the money supply, and thereby ensure liquidity in the financial markets [111]. While each interest rate cut always trigger off complex responses from traders and investors, it is probably still fair to say that the Federal funds rate cuts were implemented during the Subprime Crisis to calm the mood of the market, which should be equivalent to lowering the market volatility.

![Figure 8](image-url)  

Figure 8: Federal reserve interest rate cuts during the onset of the present financial crisis, superimposed onto the temporal distributions of clustered segments of the ten US economic sectors between 23 May 2007 and 29 August 2008. Assuming the goal of an interest rate cut is to calm the market down, we find the first two cuts effective, the next two cuts counter-effective, and the last three cuts ineffective. Also shown are circles indicating anticipation of the interest rate cuts on 17 August 2007 and 11 December 2007.

Indeed, the first interest rate cut on 17 August 2007 appears to be highly effective, in the sense that the market volatility fell across a broad spectrum of economic sectors right after the cut. The only exception is NC, which did not respond to the first rate cut. In comparison, the second interest rate cut on 18 September
2007 appears to be slightly less effective. Even factoring in anticipations and lags, BM and EN, which were in the moderate-volatility phase, did not respond to the rate cut. More interestingly, the next two interest rate cuts, on 31 October 2007 and 11 December 2007, turn out to have opposite effect as intended, as the market actually made a transition into a higher-volatility phase in some sectors. Although the emergency cut on 22 January 2007 seems effective, if we look at the segments before and after the rate cut, we will see that it did not induce a persistent lowering of the market volatility. Thus, it has to be considered as an ineffective rate cut, along with the the last two interest rate cuts, on 18 March 2008 and 30 April 2008, which do not coincide with any segment boundaries. As far as we can tell, this is the first time we have been able to examine with high temporal resolution the effects of a series of closely-timed US Federal Reserve rate cuts has on the stock market. The take-home message from this analysis appears to be that interest rate cuts has to be applied sparingly, in order to remain effective.

2.7. Cross-correlations

In performing segmentation and thereafter segment clustering, we have selectively discarded information contained in the ten high-frequency time series to obtain a coarse-grained picture of the macroeconomic dynamics of the US economy. While this picture provides a useful bird’s eye view of the dynamical processes within the US economy, a significant amount of useful information has also been thrown out. To recover more of the information contained in the high-frequency time series, and shed more light on the exciting stories unfolding before our eyes, we compute the normalized cross-correlation matrix $C_i$, whereby the matrix element

$$C_{ij} = \frac{\sum_{t=1}^{T} (x_{it} - \bar{x}_i)(x_{jt} - \bar{x}_j)}{\sqrt{\sum_{t=1}^{T} (x_{it} - \bar{x}_i)^2 \sum_{t'=1}^{T} (x_{jt} - \bar{x}_j)^2}}$$

(6)

is the zero-lag cross-correlation between US economic sectors $i$ and $j$.

Cross-correlations between different stocks, and between different benchmark indices have been widely studied in the finance literature. Such studies have been particularly popular in the bid to understand the meltdown of global financial markets during the present financial crisis [112, 113, 114, 115, 116, 117]. In the econophysics literature, there have been attempts to understand the nontrivial cross-correlations between different financial time series using random matrix theory [118, 119, 120, 121, 122, 123, 124, 125]. In all these studies, the cross-correlations were computed either over the entire data period, or employs sliding windows. In our own study, we not only calculate the cross-correlation matrix.
over the entire duration of the time series, but also over two-year intervals strictly within the growth and crisis macroeconomic phases, and over individual corresponding segments. To compute the cross-correlation matrix over a given corresponding segment, we select the largest interval within which most sectors can be found in a single macroeconomic phase, as shown in Fig. 9.

Figure 9: Interval selected for the computation of the cross-correlation matrix between the ten DJUS economic sector indices. In this figure, the lower and upper limits of the interval are chosen such that the interval covers a single macroeconomic phase for nearly all indices. Exception is made for IN, because its high-volatility segment is too short, and thus the selected interval also covers the preceding moderate volatility segment.

2.8. Minimal spanning trees

Even though our cross-correlation matrices are only $10 \times 10$ in size, the information contained in the 36 independent matrix elements is still not easy for a human to process. To better understand the correlational dynamics between the US economic sectors at different times, we look instead at simplified graphical representations of the cross-correlation matrices. For this study, we work primarily with the minimal spanning tree (MST) representation of the cross-correlation matrix, but also a little with the planar maximally filtered graph (PMFG) repre-
sentation, to understand what kind of cross-correlational structures have been left out in the MST representation.

The minimal spanning tree (also called minimum spanning tree) approach to understanding weighted graphs is frequently credited to Kruskal [126] or Prim [127], although there were studies dating all the way back to 1926. For a good reading on the history of the minimal spanning tree method, see the article by Graham and Hell [128]. In economics, the method is not widely used [129, 130]. However, since its first application in econophysics by Mantegna [131], and shown to be a robust caricature of the underlying correlations [132, 133], the MST has been incorporated into the basic tool suite for statistical analysis of financial market data [134, 135, 136, 137, 138, 139, 140]. In particular, Onnela et al. made extensive use of MSTs to study the dynamics of cross correlations during market crashes [141, 142, 143]. Clustering techniques based on the MST have also been used to discover different sectors in a stock market [144, 145, 146, 147, 148, 149], how the interdependences of the European economies are evolving [150, 151], and how global markets are linked to each other [152, 153, 154]. More recently, Eom et al. used the MST as a means to reduce the $N(N-1)/2$ linkages between $N$ stocks to $N-1$ links, for studying the effects of market factors on the information flow between stocks [155].

To construct an MST representation of the cross-correlation matrix, Mantegna defined the distance metric [131]

$$d_{ij} = \sqrt{2(1 - C_{ij})},$$

which measures the statistical distance between two financial time series $i$ and $i$, whose cross-correlation is $-1 \leq C_{ij} \leq 1$. Applying Kruskal’s algorithm [126], a link is first drawn connecting the pair $(i_1, j_1)$ of time series with the smallest distance $d_{i_1j_1} = \min_{(i,j)} d_{ij}$. Following this, a link is drawn connecting the pair $(i_2, j_2)$ with the next smallest distance $d_{i_2j_2} = \min_{(i,j)\neq(i_1,j_1)} d_{ij}$. This process is repeated with pairs $(i_k, j_k)$ with increasingly larger distances $d_{i_kj_k}$, until all time series are incorporated into the spanning graph. There is one additional constraint: if $(i_l, j_l)$ is the next pair of time series to be linked based on their distance $d_{i_lj_l}$, but will create a cycle in the growing graph in so doing, no link will be drawn between $i_l$ and $j_l$. Instead, we will skip $(i_l, j_l)$ and move on to the pair $(i_m, j_m)$ with the next smallest distance $d_{i_mj_m}$. The spanning graph obtained at the end contains no cycles, hence the name minimal spanning tree. Alternatively, since $d_{ij}$ is a monotonically decreasing function of $C_{ij}$, we can get the same MST, if we start by linking the pair of time series with the largest cross-correlation, and then
progressively linking pairs with smaller and smaller cross-correlations, so long as we ensure the no-cycle constraint is satisfied at all times.

3. Macroeconomic MSTs

Before we move on to our MST analysis, let us first develop an intuitive picture for the sectorial structure of the US economy. As a significant fraction of what the US produces is consumed domestically, the US market is a gigantic consumption market. We therefore expect the noncyclical consumer goods (NC) and consumer services (CY) to be central players in the US economy. Furthermore, CY and NC consume products predominantly generated by the industrials (IN), thus we expect IN, CY and NC (and perhaps also FN, since financing is an important ingredient in US consumerism) to be the core industries of the US economy. In contrast, emerging economic sectors such as telecommunications (TL) and technologies (TC), along with less attractive economic sectors like healthcare (HC) and utilities (UT), contribute less significantly to the GDP, and hence sit at the fringe of the US economy. Finally, the oil & gas (EN) and basic materials (BM) sectors are strongly driven by changes in the global supply and demand cycle, and thus represent the US economy’s connection to the global market.

Table 3: Cross-correlation matrix computed from the half-hourly time series of the ten DJUS economic sector indices over the period February 2000 to August 2008. Also shown are the cross correlations \(\langle C \rangle\) of each economic sector averaged across the rest of the US economy.

<table>
<thead>
<tr>
<th></th>
<th>BM</th>
<th>CY</th>
<th>EN</th>
<th>FN</th>
<th>HC</th>
<th>IN</th>
<th>NC</th>
<th>TC</th>
<th>TL</th>
<th>UT</th>
</tr>
</thead>
<tbody>
<tr>
<td>BM</td>
<td>0.611</td>
<td>0.522</td>
<td>0.568</td>
<td>0.347</td>
<td>0.656</td>
<td>0.556</td>
<td>0.438</td>
<td>0.435</td>
<td>0.458</td>
<td></td>
</tr>
<tr>
<td>CY</td>
<td>0.611</td>
<td>0.320</td>
<td>0.767</td>
<td>0.435</td>
<td>0.815</td>
<td>0.660</td>
<td>0.679</td>
<td>0.600</td>
<td>0.433</td>
<td></td>
</tr>
<tr>
<td>EN</td>
<td>0.522</td>
<td>0.320</td>
<td>0.316</td>
<td>0.261</td>
<td>0.393</td>
<td>0.350</td>
<td>0.254</td>
<td>0.276</td>
<td>0.451</td>
<td></td>
</tr>
<tr>
<td>FN</td>
<td>0.568</td>
<td>0.767</td>
<td>0.316</td>
<td>0.403</td>
<td>0.751</td>
<td>0.616</td>
<td>0.577</td>
<td>0.559</td>
<td>0.440</td>
<td></td>
</tr>
<tr>
<td>HC</td>
<td>0.347</td>
<td>0.435</td>
<td>0.261</td>
<td>0.403</td>
<td>0.436</td>
<td>0.469</td>
<td>0.325</td>
<td>0.342</td>
<td>0.323</td>
<td></td>
</tr>
<tr>
<td>IN</td>
<td>0.656</td>
<td>0.815</td>
<td>0.393</td>
<td>0.751</td>
<td>0.436</td>
<td>0.660</td>
<td>0.775</td>
<td>0.618</td>
<td>0.445</td>
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</tr>
<tr>
<td>NC</td>
<td>0.556</td>
<td>0.660</td>
<td>0.350</td>
<td>0.616</td>
<td>0.469</td>
<td>0.660</td>
<td>0.472</td>
<td>0.497</td>
<td>0.485</td>
<td></td>
</tr>
<tr>
<td>TC</td>
<td>0.438</td>
<td>0.679</td>
<td>0.254</td>
<td>0.577</td>
<td>0.325</td>
<td>0.775</td>
<td>0.775</td>
<td>0.566</td>
<td>0.270</td>
<td></td>
</tr>
<tr>
<td>TL</td>
<td>0.435</td>
<td>0.600</td>
<td>0.276</td>
<td>0.559</td>
<td>0.342</td>
<td>0.618</td>
<td>0.497</td>
<td>0.566</td>
<td>0.382</td>
<td></td>
</tr>
<tr>
<td>UT</td>
<td>0.458</td>
<td>0.433</td>
<td>0.451</td>
<td>0.440</td>
<td>0.323</td>
<td>0.445</td>
<td>0.485</td>
<td>0.270</td>
<td>0.382</td>
<td></td>
</tr>
<tr>
<td>(\langle C \rangle)</td>
<td>0.510</td>
<td>0.591</td>
<td>0.349</td>
<td>0.555</td>
<td>0.371</td>
<td>0.617</td>
<td>0.529</td>
<td>0.484</td>
<td>0.475</td>
<td>0.410</td>
</tr>
</tbody>
</table>

To check if this intuitive picture of the US economy is approximately correct, we computed the cross-correlation matrix of the time series from February 2000 to August 2008. As shown in Table 3, IN and CY are the most strongly correlated,
with $C(IN, CY) = 0.815$, while EN and TC are the least strongly correlated, with $C(EN, TC) = 0.254$. Based on the average cross correlations $\langle C \rangle$, it also appears that IN is most strongly tied in with the rest of the sectors $\langle C \rangle(IN) = 0.617$, while EN is least strongly tied in with the rest of the US economy $\langle C \rangle(EN) = 0.349$. Based on this cross-correlation matrix, we constructed the MST shown in Fig. 10(a). As expected, the core sectors of the US economy, IN, CY and NC, are at the centre of the MST. The sectors EN and BM, which represent the US economy’s connection to the world market, sit on one end of the MST, while the sectors HC, TC, TL, and UT, lies on the fringe of the MST, consistent with their lesser importance to the US economy. Heimo et al. arrived at a similar conclusion, in their MST study of 116 NYSE stocks from 1982 to 2000 [156].

Over the period 2000 to 2009, the US economy went from a crisis phase (mid-1998 to mid-2003) into a growth phase (mid-2003 to mid-2007), and back into a crisis phase (mid-2007 to present). We expect interesting structural differences between the MSTs constructed entirely within the previous crisis (2001–2002, Fig. 10(b)), the previous growth (2004–2005, Fig. 10(c)), and the present crisis (2008–2009, Fig. 10(d)). Indeed, we see two topologically distinct MST structures: a chain-like MST structure which occurs for both crises, and a star-like MST structure which occurs for the growth phase. Even though we only have three data points (two crises and a growth), we believe the generic association of chain-like MST and star-like MST to the crisis and growth phases respectively is statistically robust. Our reasons are two-fold. First, the MST is a representation based on order statistics (ranks of cross correlations). Results derived based on order statistics, which are insensitive to noise, tend to be highly robust statistically. Second, the star-to-chain transition in the MST structure as the US economy goes from growth into crisis cannot be brought about by a fixed quantum increase, nor can it be caused by a proportional increase, in correlations. These two types of correlational changes do not change the ordering of cross correlations among the ten economic sectors, and hence cannot modify the MST. Since noise and global shifts in correlations cannot be responsible for the star-to-chain or the chain-to-star transitions, correlational changes that accompany these transitions are highly significant. Our assessment that the topology change in the MST is statistically significant is further supported by the observations by Onnela et al., who looked at the mean occupation level around the most connected node in their MST, and found the mean occupation level becoming low during market crashes [141, 142, 143]. This is the same phenomenon we see for the star-to-chain evolution, at the microscopic scale of individual stocks. In the next two sections, we will investigate the characters of these correlational changes, and discuss the im-
Figure 10: The MSTs of the ten DJUS economic sectors, constructed using half-hourly time series from (a) February 2000 to August 2008, (b) 2001–2002, (c) 2004–2005, and (d) 2008-2009. The first and the third two-year windows, (b) and (d), are entirely within an economic crisis, whereas the second two-year window, (c), is entirely within an economic growth period.
lications for early detection of true economic recovery based on the chain-to-star transition.

From Fig. 10, we also see that in both the crisis and growth phases, IN is found to be the central industry of the US economy. This is understandable, since the United States is a highly industrialised country with IN driving the rest of the sectors. However, when the US economy went from the mid-2003 to mid-2007 economic growth into the current crisis, the IN star center shed the sectors NC, HC and TL, which shifted to other parts of the MST. In the restructured MST, NC formed the center of another cluster. We believe this is a signature of the trigger role played by NC in the Subprime Crisis. More interesting, the cluster centered around NC consists of HC, TL, and UT, which were part of the five sectors that went first into the crisis phase (see Fig. 7). The last of these five sectors is IN, so it appears that correlational changes within these five sectors in July 2007 is responsible for the main difference between the growth MST (Fig. 10(c)) and the crisis MST (Fig. 10(d)). This gross restructuring of the MST thus provides an interesting way to visualize how the current financial crisis propagated throughout the entire US economy.

4. Segment-by-segment analysis

Even within the macroeconomic growth and crisis phases, the DJUS economic sector time series are highly nonstationary. Both the cross correlations between the ten sectors, and the MSTs they imply, are expected to be highly dynamic. To understand how cross correlations change with time, we extracted the average cross correlations of the ten DJUS economic sectors in 11 corresponding segments within the present financial crisis (see Fig. 11). All four macroeconomic phases are represented in these 11 corresponding segments. Ranking the average cross correlations from highest to lowest in Table 4, we see that IN is always most strongly correlated to the rest of the US economy, whatever the prevailing economic climate, followed by CY and NC. Meanwhile, EN is most weakly coupled to the rest of the US economy, in most of the corresponding segments. This is consistent with our expectation that the oil & gas industry’s strong dependence on global supply and demand makes it less susceptible to movements within the US economy.

In general, we observe a positive correlation between the average market cross correlation $\langle\langle C\rangle\rangle$ and the market volatility. As can be seen from Table 4, higher average market cross correlations are generally associated with higher volatility phases. Specifically, in the low-volatility economic growth phase (B), the average
market cross correlation is low, in the range $0.5 < \langle C \rangle < 0.6$, whereas in the moderate-volatility market correction phase ($G1, G2$), the average market cross correlation is also moderate, in the range $0.6 < \langle C \rangle < 0.7$. In the higher-volatility phases ($Y1, Y2, Y3, Y4; R1, R2, R3, R4$), the average market cross correlation is high, in the range $0.7 < \langle C \rangle < 0.9$. The higher correlations observed during the higher-volatility phases is consistent with the tendency for traders to panic and to buy and sell stocks from different sectors at the same time. Conversely, when the market is calm, stocks from different sectors tend to be bought and sold at different times, explaining the lower correlations observed for the lower-volatility phases. These average market cross correlations are all higher than the average market cross correlations computed over the entire time series, because cross correlations within the US economy has been increasing over the years (see for example, Ref. [157]).
Table 4: Ranks of the ten DJUS economic sectors based on their average half-hourly cross-correlations, over February 2000 to November 2009, as well as over the 11 corresponding segments identified in Fig. 11. The average cross correlations for EN in Y3, and those for BM, EN, and UT in Y4, are anomalously low, even negative. Also shown are the cross correlations $\langle C \rangle$ averaged over all ten sectors, for the entire period from February 2000 to November 2009, as well as the 11 corresponding segments.

<table>
<thead>
<tr>
<th></th>
<th>BM</th>
<th>CY</th>
<th>EN</th>
<th>FN</th>
<th>HC</th>
<th>IN</th>
<th>NC</th>
<th>TC</th>
<th>TL</th>
<th>UT</th>
<th>$\langle C \rangle$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entire</td>
<td>5</td>
<td>2</td>
<td>10</td>
<td>3</td>
<td>9</td>
<td>1</td>
<td>4</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>0.489</td>
</tr>
<tr>
<td>Y1</td>
<td>6</td>
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</tr>
<tr>
<td>G1</td>
<td>6</td>
<td>2</td>
<td>10</td>
<td>5</td>
<td>7</td>
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<td>3</td>
<td>4</td>
<td>9</td>
<td>8</td>
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</tr>
<tr>
<td>B</td>
<td>2</td>
<td>8</td>
<td>10</td>
<td>7</td>
<td>4</td>
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<td>3</td>
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<tr>
<td>Y2</td>
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<tr>
<td>R2</td>
<td>7</td>
<td>3</td>
<td>10</td>
<td>2</td>
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<td>1</td>
<td>6</td>
<td>9</td>
<td>4</td>
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</tr>
<tr>
<td>Y4</td>
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<td>10</td>
<td>2</td>
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<td>1</td>
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<td>0.559</td>
</tr>
<tr>
<td>R3</td>
<td>5</td>
<td>2</td>
<td>10</td>
<td>10</td>
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<td>3</td>
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<td>7</td>
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<td>5</td>
<td>3</td>
<td>10</td>
<td>8</td>
<td>0.709</td>
</tr>
</tbody>
</table>

4.1. MST structures

As expected, changes in the MST can be seen going from one corresponding segment to the next (see Fig. 12). However, the sectors IN, CY and NC remain at the cores of all 11 MSTs, whereas the sectors HC, TC, TL, and UT are mostly found at the fringes of these MSTs. Interestingly, the financials (FN), which is frequently found close to the core, occasionally drifts out to the fringe. While the core-and-fringe structure of the MSTs remains well defined as the market volatility changes, we observe shifting relative importances between the different sectors. We will study these MST rearrangements, which we believe are the US economy’s response to shocks originating within specific economic sectors, in Section 5. Here, let us make the remarkable observation that, through the fluxes of correlational changes, the EN-BM-IN-CY-NC-TC-HC backbone of the MSTs remained relatively unchanged throughout the entire crisis period. This robust correlational structure must therefore be a key to understanding the performance of the present US economy.

In Fig. 12, we incorporate more visual information on the cross-correlation matrix, by varying the widths of the bonds in the MSTs. The thicker the bond between two sectors, the stronger their correlations. As we can then see, sectors at the core are generally more strongly correlated than those on the fringes of the
MSTs. This reinforces our intuitive picture that sectors on the fringe are more detached from the overall economy, whereas those at the core are most important to the US economy. In this representation of the MSTs, a correlational core consisting of thick bonds can also be seen. Even as the core and backbone of the MSTs remain more or less unchanged, the correlational core of thick bonds expands and contracts with time. We can think of the correlational core defining the active participants in the US economy for a given corresponding segment. In the high-volatility phase, the correlational core expands all the way out to the fringes, telling us that fringe sectors become more involved in the US economy during a financial crisis. A similar phenomenon was observed by Onnela et al. in the MSTs of individual stocks across market crashes [141, 142, 143].

### 4.2. MST dynamics

In Ref. [27], we developed a causal tree analogy speculating that exogenous shocks shaking the root of the tree will first be felt strongly by branches closest to the root, and then weakly by branches further from the root. Naturally, now that we have a better picture of the correlational structure of the US economy in the form of an MST, we expect volatility shocks to propagate along the invariant backbone of the MST, since it is along this backbone that we have the strongest cross correlations. To explore this idea, we make use of high-resolution temporal information available from the segmentation/clustering analysis, to identify for each corresponding segment the statistically significant start dates in the ten DJUS economic sectors. We then rank the start dates from earliest to latest, and in Fig. 12 label the sectors according to these ranks, omitting those sectors for which the start date cannot be identified. From the 11 corresponding segments identified within the present financial crisis, we find that shocks always originate from the fringe of the MST, and propagate inwards. However, contrary to our naive expectations, shocks do not necessarily propagate along the MST. For example, in Fig. 12(h), we see that the corresponding segment Y4 started in TL, propagated to EN (which is not directly connected to TL in the MST), and then onto TC and FN (both of which are not directly connected to TL or EN), before propagating into the core of the MST. This inward propagation of volatility shocks is seen even when the MST is anomalous. For example, in Fig. 12(e), where TC is at the center of the MST, the corresponding segment R1 started first in FN, which has moved to the fringe of the MST, then in TL, then in EN, and BM, before propagating into the core of the MST. In no case was a shock found to start at the core of the MST.
Figure 12: MSTs of the ten DJUS economic sectors for the corresponding segments (a) Y1, (b) G1, (c) B, (d) Y2, (e) R1, (f) Y3, (g) R2, (h) Y4, (i) R3, (j) R4, (k) G2, within the present financial crisis. In this figure, thicker bonds represent stronger cross correlations, whereas the number besides each sector indicates the order with which the sector made the transition into the given corresponding segment.
Figure 12: (continued) MSTs of the ten DJUS economic sectors for the corresponding segments (a) Y1, (b) G1, (c) B, (d) Y2, (e) R1, (f) Y3, (g) R2, (h) Y4, (i) R3, (j) R4, (k) G2, within the present financial crisis. In this figure, thicker bonds represent stronger cross correlations, whereas the number besides each sector indicates the order with which the sector made the transition into the given corresponding segment.
Table 5: Ranks of identifiable start dates in the ten DJUS economic sectors, from earliest to latest, for each of the 11 corresponding segments between May 2007 and November 2009.

<table>
<thead>
<tr>
<th></th>
<th>BM</th>
<th>CY</th>
<th>EN</th>
<th>FN</th>
<th>HC</th>
<th>IN</th>
<th>NC</th>
<th>TC</th>
<th>TL</th>
<th>UT</th>
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<tbody>
<tr>
<td>Y1</td>
<td>3</td>
<td>7</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>9</td>
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<td>10</td>
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<td>6</td>
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<tr>
<td>G1</td>
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<td>B</td>
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<td>Y2</td>
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<td>R1</td>
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<td>Y3</td>
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<tr>
<td>R2</td>
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<tr>
<td>Y4</td>
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<td>R3</td>
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<td>R4</td>
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<td>G2</td>
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</table>

Looking at the leading sectors more closely, we find a mix between shocks starting in EN and BM, and shocks starting in the fringe domestic sectors. In Table 5, we rank the start dates in the ten sectors from earliest to latest, for each of the 11 corresponding segments. In cases where we have joint leaders, for example, EN and FN in G1, we split the count between them. In this way, we find that out of the 11 volatility shocks, only two and a half originated from EN and BM. The remaining eight and a half shocks originated in fringe domestic sectors which are effectively not coupled to the global market. This suggests that the US economy experiences internal feedbacks that are stronger than its coupling to the global economy. More interestingly, we find in Fig. 12 anomalously high cross correlations at the fringe for some corresponding segments, for example, the HC-NC link in Y1, the TC-IN link in B, and the TL-CY link in Y4. As we can see from Table 5, Y1 started in HC, B started in TC, and Y4 started in TL. This suggests that fringe cross correlations frequently become anomalously high in the leading sector of a volatility shock. This is opposite to what we saw for the previous crisis, where there is a pronounced ‘distancing-the-leader’ effect, i.e. the cross correlations between the leader sector and all other sectors are smaller than the typical cross correlations within the other sectors [157].

4.3. Comparison between MST and PMFG

The planar maximally filtered graph (PMFG) was introduced by Tumminello et al. to extract a representative subgraph of the cross-correlation matrix containing more information than the MST [158]. Since then, the method has been applied for sector identification [159], to develop hierarchically nested factor models.
to understand the time horizon dependence of equity returns, in portfolio optimization, and to understand the network structure of cross correlations among the world market indices. More recently, Pozzi et al. computed the MSTs and PMFGs for the daily returns of 300 of the most-capitalized stocks on the NYSE for different window sizes between 2001 and 2003, and found that the center is always populated by stocks from the financial sector, whereas other sectors share the peripheral. This conclusion is different from what we arrived at based on the DJUS economic sector indices between 2002 and 2003 (near the end of the previous financial crisis), where IN remains central, and FN sits on the periphery of the chain-like MST.

Because our main interest in this study is the present financial crisis, we did not construct the sectorial PMFG for the 2002–2003 period. Instead, we constructed the PMFGs for the three corresponding segments at the start of the Subprime Crisis (see Fig. 13). These PMFGs reveal secondary centers in sectorial dynamics of the US economy. For example, if we adopt the very simplistic criterion of having five or more links to be a center in the PMFG, we see that BM, CY, IN, NC, and TC are all PMFG centers within the corresponding segment Y1, CY, IN, NC, and TC are the PMFG centers within the corresponding segment G1, while BM, CY, IN, NC, and HC are the PMFG centers within the corresponding segment B. In particular, the PMFG structure of corresponding segment B suggests that IN and HC are the two epicenters of trading activities in October 2007. Since IN is most strongly linked to growth sectors (BM, CY, EN, FN, NC, TC) in the US economy, while HC is most strongly linked to quality sectors (TL, UT), we believe we are seeing the signatures of a ‘flight to quality’ in the early stages of the Subprime Crisis. Unlike the ‘flights to quality’ studied by economists (see for example, the recent works by Phillips and Yü, who tracked the massive flow of funds from US technology stocks to the US property market to commodities to the bond market, each time generating a bubble that crashed when the funds leave), the phenomenon we are seeing is within the same asset class.

5. MST rearrangements

Up till this point, we understood from our combined segmentation/clustering and cross-correlational analyses that the MST of the ten DJUS economic sectors presents a star-like topology during economy growth, and a chain-like topology during financial crisis (see Fig. 10). These two limiting MSTs, along with those with intermediate topologies, can also be seen at the mesoscopic scale of corre-
Figure 13: Planar maximally filtered graphs (PMRGs) of the corresponding segments Y1, G1, and B identified in Fig. 11. In this figure, solid links are strong links making up the MSTs, while dashed links are weaker links neglected in the MSTs.

responding segments within the present financial crisis (see Fig. 12). For each corresponding segment, we then looked at the sectorial distribution of strong cross correlations, and the temporal order in which sectors made the transition, to find that strong cross correlations are frequently found at the fringe of the MST, where the volatility shocks always originate. In this section, we address the most natural question that follows: what are the natures of the correlational changes, visualized as MST rearrangements, that accompany these transitions?

5.1. Minimal MST rearrangements

If we treat the MST like a molecule, the MST rearrangements that occur from one corresponding segment to the next can be described using the chemical lan-
guage of bond breaking and bond formation. This analogy is useful, because it allows us to focus on identifying the minimal set of primitive rearrangements that occur in the MST, an example of which is shown in Fig. 14. Between the successive corresponding segments Y1 and G1 identified in Fig. 11, we first note that the EN-BM-IN(-TC)-CY(-FN)-NC-HC backbone remains unchanged. We then note that TL, which is bonded to IN in Y1, and UT, which is bonded to CY in Y1, are both bonded to NC in G1. This tells us that the minimal set of primitive rearrangements necessary to get from the Y1 MST to the G1 MST consists of the breaking of the TL-IN and UT-CY bonds, and the formation of the TL-NC and UT-NC bonds. We also see from Fig. 14 that, as expected, all MST cross correlations decreased going from Y1 to G1. In fact, all cross correlations decreased going from Y1 to G1. Therefore, to have the above rearrangements, we need $C(TL, NC)$ to weaken slower than $C(TL, IN)$, or have $C(TL, IN)$ weaken faster than $C(TL, NC)$. Similarly, we need $C(UT, NC)$ to weaken slower than $C(UT, IN)$, or have $C(UT, CY)$ weaken faster than $C(UT, NC)$. In any case, we need at least one cross correlation within the (TL-IN, TL-NC) and (UT-CY, UT-NC) pairs of cross correlations to be anomalous, for the rearrangement to occur.

![Figure 14: The MST of the corresponding segment G1 can be obtained from the MST of the corresponding segment Y1 preceding it, by breaking the TL-IN and UT-CY bonds, and forming new bonds between TL-NC and UT-NC.](image)

With this ‘chemical’ understanding of minimal MST rearrangements, we now proceed to investigate the cross-correlational changes going from corresponding segments G1 to B to Y2, as shown in Fig. 15. Accompanying the Y1 to G1 tran-
Figure 15: The primitive MST rearrangements going from G1 to B to Y2. The MST went from chain-like in G1 to star-like in B, to an intermediate topology in Y2.

sition, we saw a chain-like MST rearranging into another chain-like MST. For the G1 to B to Y2 transitions, we see the more interesting MST rearrangements from chain-like to star-like, and then to a topology intermediate between a chain and a star. As expected, more primitive rearrangements are needed to bring about the chain-to-star transition going from the moderate-volatility G1 to the low-volatility B. Ignoring the change in sector directly bonded to IN within the CY-FN pair, we see that three bonds have to be broken and reformed. These three bonds are significant, because NC is nearly a star center in the G1 MST, but loses the status after the three bonds are broken. Of course, the bonds reformed around IN, making it the star center of the B MST. A similar interaction between NC and IN occurs again for the B to Y2 transition, where NC becomes central again, with the breaking of the UT-IN and HC-UT bonds to reform around NC. Since these
corresponding segments are right after the start of the Subprime Crisis, it is no wonder that NC features so prominently in the MST rearrangements.

Quantitatively, we expect cross correlations to fall generically between all sectors, when the US economy progressed from the moderate-volatility G1 segment to the low-volatility B segment. This can be seen easily from the thick bonds in the G1 MST compared to the thin bonds in the B MST in Fig. 15. In fact, the drop in average cross correlations of CY is anomalously large, from $\langle C \rangle_{G1} = 0.80$ to $\langle C \rangle_{B} = 0.45$ in B. In addition, when all other cross correlations were falling, that between UT and HC increased slightly. This bucking of the trend makes the correlational changes between UT and HC highly significant statistically. Subsequently, when the US economy progressed from the low-volatility B segment to the high-volatility Y2 segment, the cross correlation between UT and HC decreased, when the cross correlations between all other sectors increased.

5.2. Early detection of economic recovery

Speaking of ‘green shoots’ of economic revival that were evident at the time, Federal Reserve chairman Ben Bernanke predicted that “America’s worst recession in decades will likely end in 2009 before a recovery gathers steam in 2010” [1]. After learning that the MST of the ten DJUS economic sectors is star-like and chain-like during the low-volatility economic growth phase and high-volatility economic crisis phase respectively, we look out for a star-like MST in the time series data of 2009 and 2010. Star-like MSTs can also be found deep inside an economic crisis phase. However, within the crisis phase, these star-like MSTs very quickly unravel to become chain-like MSTs. On the other hand, the star-shape topology is extremely robust and stable within the growth phase. Therefore, a persistent star-like MST, if it can be found, may be interpreted as the statistical signature that the US economy is firmly on track to full recovery (which may take up to two years across all sectors).

More importantly, the number of primitive rearrangements needed to transform the MST of a given period into a star-like MST indicates how close we are to the actual recovery. We can use this feature of the prerecovery MST for the early detection of economic recovery. This should be possible whether the star-like MST is a cause, in the sense that such a correlational structure within the US economy promotes growth, or an effect, in the sense that economic growth naturally results in this MST topology. Indeed, when we inspect the MST structure of the moderate-volatility G2 segment in Sep 2009, we find that it is already star-like, with IN as the star center. From Fig. 16, we see that it is two to three primitive rearrangements away from the growth MST of 2004–2005. Therefore, based on
the time series data up till 25 Nov 2009, the statistical evidence summarized in the MST suggests that the US economy was already in the pre-recovery stage, and Bernanke might be prophetic to call for an actual economic recovery in 2010.

![MST diagrams](image)

Figure 16: Comparison of the MSTs for (a) the 2004–2005 growth period, and (b) the moderate-volatility segment around September 2009.

5.3. Comparison between previous and present recoveries

Since the time series data we have covers the recovery periods for both financial crises, we wanted also to compare the sequence of pre-recovery MSTs for the previous crisis against the ones we see for the present crisis. We immediately encountered two problems. First, volatility movements in the various sectors between 2002 and 2004 are much less coordinated than those we find in the present financial crisis, and thus it is difficult to find corresponding segments. Second, for successive corresponding segments that we can find for the 2002–2004, successive MSTs are structurally very different from each other, suggesting very violent rearrangements within the MSTs. In Fig. 17, we show the MSTs of four successive corresponding segments identified before and after the 11 Sep 2001 attack, from August 2001 to December 2001. These corresponding segments, an August 2001 moderate-volatility segment before the 11 Sep 2001 attack, a two-week extremely-high-volatility segment right after the attack, a October 2001 high-volatility segment following this, and a November–December 2001 moderate-volatility segment afterwards, are amongst the most well-defined ones that we can identified through the previous financial crisis. As we can see, it is impossible to assign a small number of bonds that must be broken and re-formed to go from one MST to the next. Throughout the violent rearrangements, IN remained at the center of the MSTs. We also see that the MST is chain-like before and after the 11 Sep 2001 attack, became briefly star-like in the October...
2001 high-volatility segment, before unravelling again to a chain-like MST for the rest of the year.

![MSTs for successive corresponding segments: (a) moderate-volatility segment before the 11 Sep 2001 attack on the World Trade Center; (b) extremely-high-volatility segment right after the 11 Sep 2001 attack; (c) high-volatility segment following; and (d) moderate-volatility segment following.](image)

### 6. Conclusions

To summarize, we performed a cross-section analysis on the high-frequency time series of the ten DJUS economic sector indices between February 2000 and November 2009, to discover statistical signatures that can be used to forecast economic recovery. The half-hourly time series of these indices are first segmented individually using a recursive entropic segmentation scheme. The segments of each economic sector are then hierarchically clustered into between four and seven clusters, representing the growth, crisis, correction, and crash macroeconomic phases. In our previous study [27], we compared the temporal distributions of clustered segments across all ten economic sectors, to see that the US economy emerged from the previous technology bubble financial crisis starting mid-2003, enjoyed a four-year period of growth, and then succumbed to the present property bubble financial crisis starting mid-2007. From this cross-section of temporal distributions of clustered segments, we also see the US economy taking one and a half years to completely recover from the previous financial crisis, but only two
months to completely enter the present financial crisis. More interestingly, for the present financial crisis, we find the volatility dynamics within the US economic sectors to be strongly driven by interest rate cuts by the Federal Reserve. Of the seven interest rate cuts made over 2007 and 2008, the first two lowered market volatilities, the next two raised market volatilities, while the last four had no permanent effect on market volatilities.

In this paper, we extended the cross-section analysis, by constructing the cross-correlation matrices and therefrom the MSTs of the ten DJUS economic sectors first over February 2000 to August 2008, and the two-year intervals 2002–2003, 2004–2005, 2008–2009, as well as the 11 corresponding segments identified in Fig. 11. In general, we find stronger cross correlations when the market volatility is high, and weaker cross correlations when the market volatility is low. We also find evidence that cross correlations within the US economy have been increasing over the years. In all MSTs, we find a core-fringe structure, with CY, IN, and NC forming the core, and HC, TL, UT residing on the fringe. In spite of the supposed market turmoil we expect throughout the present financial crisis, a highly conserved EN-BM-IN-CY-NC-TC-HC backbone can be identified in most of the MSTs. Through an enhanced visualization scheme for the MSTs, we see a dynamic core of strongly-correlated sectors, which expands and contracts in tandem with changes in the overall market volatility. In addition, for all 11 corresponding segments studied, we find the volatility shocks starting always at the fringe, frequently accompanied by anomalously high cross correlations here, and propagating inwards towards the core of the MST. These volatility shocks originate mostly from the US domestic fringe sectors, which are weakly coupled to the world market, instead of coming from EN and BM, which are most strongly coupled to the global supply and demand cycles.

More importantly, we see that the MSTs of the ten DJUS economic sectors can be classified into two distinct topologies: star-like and chain-like. The MST is robustly star-like during economic growth, with IN at the center, and robustly chain-like within an economic crisis. For the present financial crisis, the MST of a corresponding segment can be obtained from the MST of the preceding corresponding segment through a small set of primitive rearrangements. In contrast, very violent rearrangements are implied going from the MST of one corresponding segment to the MST of the next corresponding segment within the previous, mid-1998 to mid-2003, financial crisis. This suggests that the US economy has become more efficient in processing information arising from volatility shocks. Combining these two observations, we postulated that the star-like MST seen in the Sep 2009 G2 corresponding segment indicates that the US economy was in
the early stages of economic recovery.

After this study was completed, US market volatilities remained moderate to high until the start of May 2010, when investor confidence was again tested, first by the glitch in the NYSE electronic trading platform, and then by the unfolding Greek Debt Crisis. Market volatilities skyrocketed, and even after the European Union announced their bailout plan for Greece, the atmosphere of economic uncertainty remains to the present day. When interviewed in July 2010 on NBC’s “Meet the Press” programme, US Treasury Secretary Timothy Geithner acknowledged the slow recovery of the US economy, but added that it is gradually gaining strength [166]. A commentary that appears the same day Geithner’s interview was aired complicates the mood, by citing economists who warn that recent gains in the stock market need not be an indicator of economic recovery [167]. In fact, on 24 Aug 2010, world stock markets fell over concerns that the yen is too strong for the good of the Japanese economy, and also over more bad news anticipated from the US economic reports due to be released the same week [168].

![Figure 18: MSTs for four segments straddling Greek debt crisis. Gr2 and iPad launch in the US by Apple Inc.](image)

To check if the US economic recovery might have been derailed by the Greek Debt Crisis, we segmented the DJUS economic sector time series from January to 39
July 2010, and constructed MSTs for three corresponding segments. The MSTs for these three corresponding segments, the extremely-high-volatility Gr1 segment (21 Jan–15 Feb 2010) and moderate-volatility Gr2 segment (1–31 Mar 2010) before the Greek Debt Crisis, and the extremely-high-volatility Greek Debt Crisis Gr3 segment itself (1 May–15 Jul 2010), are shown in Fig. 18. As we can see, even though market volatilities are high, the MST presented a very robust star shape in all three corresponding segments. While they are nervous, it appears that investor sentiments during the Greek Debt Crisis are distinctly different from those seen over 2008 and the first half of 2009. Judging from the increasingly star-like MST, it appears that the US economy is staying its course to the long-awaited economic recovery.

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References


