

Cosmic Speed-Up and Dark Energy: String/M Theory Perspectives

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- ❖ Motivations
- ❖ Cosmic Acceleration: Possibilities
- ❖ String/M-Theory: Why in Time-dependent Backgrounds?
- ❖ Flat or Open Space Cosmologies?
- ❖ Eternal Inflation/Acceleration
- ❖ Dark energy and related issues

Dark Energy: Motivations

Why is the present universe accelerating? The answer to this question can be (significantly) different from why inflation got a begin?

Many people (Physicists/Cosmologists/Astrophysicists) are trying to confirm the hypothesis of a dark universe with a negative pressure that drives the universe to accelerate. But one would hold out less hope of understanding the cosmic acceleration (or dark energy) unless or until there is a unified theory that takes it closer to the bedrock of space and time.

Inflation may be linked to some quantum gravity effects, string theory in high-curvature time-dependent backgrounds. While, the late-time cosmic acceleration is attributed to the existence of a dark energy component with negative pressure. There is no unique model of dark energy, and possible candidates include

a positive cosmological constant, a slowly varying scalar field, a phantom (exotic) field or something else, including ghost-condensate!

String or M Theory Cosmology: Motivations

Is it possible to catch two fish using a single fishing rod? Here INFLATION and LATE-TIME COSMIC ACCELERATION are two fish and a dark energy model one would design is the fishing rod.

This motivates us to look for the origin of a dark energy in the framework of the fundamental physics: superstring or M theory

That is to say, the interest in string or M-theory cosmology is two fold

- (1) Can one derive a scalar potential from warped string compactification that has at least one stationary point with $V > 0$.
- (2) Can inflation and/or late-time acceleration of the four dimensional expanding universe arise from time-dependent M-theory compactifications

If the first is closer to reality then "dark energy" is possibly a pure vacuum energy, so called cosmological constant. If the second one is closer, then dark energy is dynamical, like a slowly varying scalar field, which may arise due to a slowly varying size of extra dimensions.

The central question is

Can one derive a dark energy model from string/M theory compactification?

No-go theorem in a language of cosmology?

One of the obstacles for a de Sitter type compactification in supergravity theories is no-go theorem **Gibbons (1984), Maldacena-Nunez (2001)**

The strong energy condition $R_{00}^{(D)} \geq 0$ holds for 10 or 11d supergravities. If one allows extra dimensions to be warped and time-independent, then for a compactified theory this requires

$$R_{00}^{(4)} \geq 0$$

But this does not allow the universe to accelerate! So it must be violated during inflation. Consider a four-dimensional metric

$$ds^2 = -dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega_2^2 \right),$$

where $k = -1, 0, +1$. The time-time component of 4D Ricci tensor is

$$R_{00}^{(4)} = -3 \frac{\ddot{a}(t)}{a(t)}$$

Inflating spacetime means $\ddot{a}/a > 0$ and hence $R_{00}^{(4)} < 0$

No-go theorem in warped string compactifications

How the Gibbons-Maldacena-Nunez “no-go theorem” comes into this business? Consider a D -dimensional metric

$$ds_D^2 = A^2(\mathbf{y}) ds_4^2(\mathbf{x}) + d\Sigma_m^2(\mathbf{y}) \quad (m = D - 4)$$

$d\Sigma_m^2$ is the metric of some compact non-singular m -manifold \mathcal{M} with coordinates y . One computes

$$\mathbf{R}_{00}^{(D)}(\mathbf{x}, \mathbf{y}) = \mathbf{R}_{00}^{(4)}(\mathbf{x}) - \frac{1}{4} A^{-2}(\mathbf{y}) \nabla_y^2 A^4(\mathbf{y})$$

Multiplying by A^2 and integrating over \mathcal{M} we find

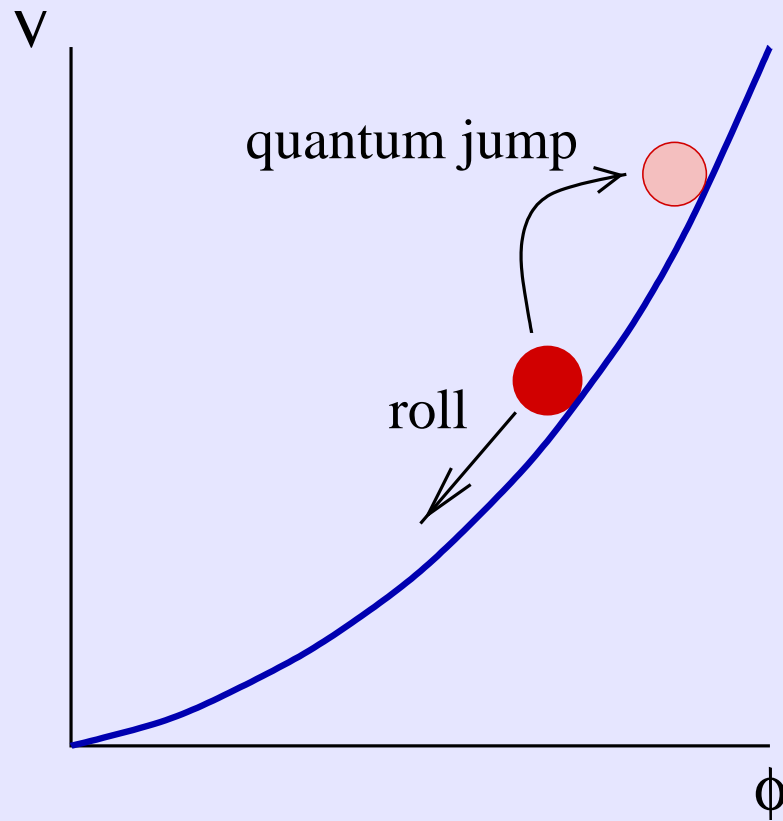
$$\left[\int_{\mathcal{M}} A^2 \right] \mathbf{R}_{00}^{(4)} = \int_{\mathcal{M}} A^2 \mathbf{R}_{00}^{(D)}$$

The result is that

$$\mathbf{R}_{00}^{(D)} \geq 0 \quad \text{only if} \quad \mathbf{R}_{00}^{(4)} \geq 0$$

A contribution of p -form background fields does not affect this result!

Slow-Roll Inflation ?



Roll \implies lower $V \implies$ lower H (Hubble parameter)

Jump \implies higher $V \implies$ higher H

What conditions really violated during inflation?

FRW:

$$\dot{H} = -4\pi G(\rho + p) + \frac{k}{a^2}$$

Inflating spacetime does not mean that $|k| = 0$, rather

$$\frac{k}{a^2} \rightarrow 0 \quad \text{or negligibly small}$$

The condition $\dot{H} > 0$ is too strong! This implies that

$$\frac{\ddot{a}}{a} > H^2 > 0 \Rightarrow \rho + p < 0$$

$$\Rightarrow N^\mu N^\nu T_{\mu\nu} < 0 \Rightarrow w = \frac{p}{\rho} < -1$$

Violation of NEC \Leftrightarrow Phantom Fields? Probably NOT!

In general, cosmic acceleration means that

$$\frac{\ddot{a}}{a} > 0, \quad H > 0 \Rightarrow -1 < w < -\frac{1}{3}$$

Resolution: Hyperbolic Compactification

How To Overcome No-Go Theorem?

- It is possible to explain cosmic acceleration of our four-dimensional universe from supergravity solutions, with or without background fluxes,

if

- 1) one gives up the condition of time-independence of internal space, and in addition,
- 2) the internal space is hyperbolic (a space of constant negative curvature)

—— Townsend and Wolharth [hep-th/0303097]

This is an interesting observation

Similar cosmology was studied by E. Kasner in 1921

Look! This is right after T. Kaluza but before O. Klein

Accelerating Cosmologies: Some Known Examples

- $M_4 \times H_m$: – First example of transient acceleration of a four-dimensional universe from supergravity compactification on hyperbolic spaces –Townsend-Wohlfarth (hep-th/0303097)

- **The solution is**

$$ds^2 = e^{-m\phi(t)} \left(-S^6 dt^2 + S^2 dx_3^2 \right) + r_c^2 e^{2\phi(t)} ds_{H_m}^2$$

$$\phi(t) = \frac{1}{m-1} \left(\ln K(t) - 3\lambda_0 t \right), \quad S^2 = K^{\frac{m}{m-1}} e^{-\frac{(m+2)}{(m-1)}\lambda_0 t}$$

$$K(t) = \frac{\lambda_0 r_c}{(m-1)} \frac{\beta}{\sinh[\lambda_0 \beta |t + t_1|]}, \quad \beta = \sqrt{3 + \frac{6}{m}}$$

- **This solution is obtainable from Space-like brane solutions in zero-flux limit – [hep-th/0303238] .**

- **The proper time τ is defined by $d\tau = S^3(t) dt$. The conditions for expansion and acceleration are $\frac{dS}{d\tau} > 0$, $\frac{d^2S}{d\tau^2} > 0$. **For example, when $m = 7$, the expansion factor is simply****

$$\frac{S(\tau_2)}{S(\tau_1)} = 3.04 \quad \text{Too small for inflation}$$

Other Accelerating Solutions

- $M_4 \times \mathbb{R}_{m_1} \times H_{m_2}$: Suppose we live in a flat $4D$ spacetime, and the internal space is a product of flat and hyperbolic spaces.

The logarithm of the scale factor is

$$\ln(S(t)) = -\frac{(m_1 + m_2 - 4)}{4} \lambda_0 t + \frac{m_1}{2} a(t) + \frac{m_2}{2} b(t)$$

$$a(t) = \alpha_0 t, \quad b(t) = -\frac{m_1}{m_2 - 1} \alpha_0 t + \frac{1}{m_2 - 1} \ln \left(\frac{\beta}{\sinh((m_2 - 1)\beta t)} \right)$$

For example, when $m_1 = 1$ and $m_2 = 6$ and $\lambda_0 = 2\alpha_0$, so the volume factor of \mathbb{R}^1 is unity, then the four-dimensional universe accelerates in the time interval $1.41 > 4\alpha_0 t > 0.14$. The expansion factor is

$$\frac{S(\tau_2)}{S(\tau_1)} = 3.38 \quad \text{Just a small improvement}$$

Hyperbolic-Flux Compactification

- **Kasner type solutions were first studied and generalized in supergravity by A. Chodos and S. Detweiler in 1980 by considering the bosonic part of D-dimensional supergravity with a $(q + 2)$ -form field:**

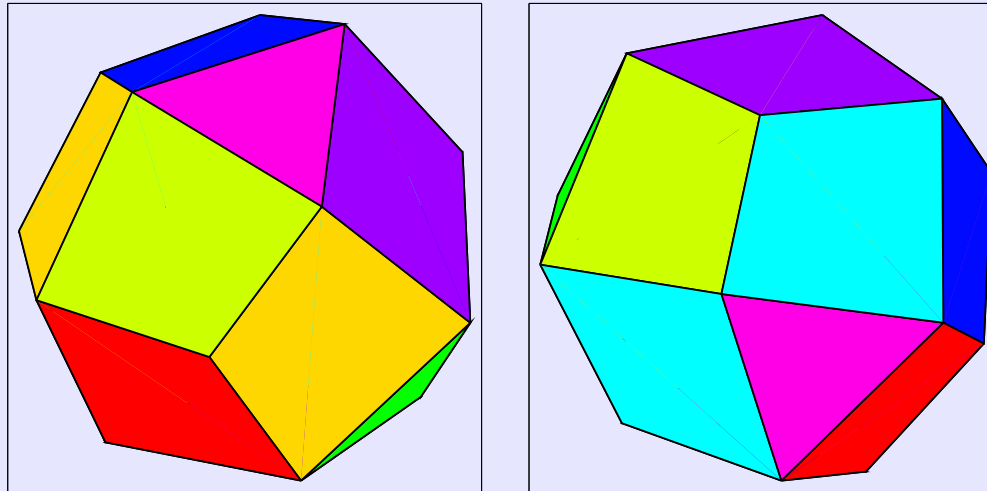
$$\mathcal{L}_D = \frac{1}{16\pi G_D} \sqrt{-g_D} \left(R - \frac{8\pi G_D}{(q+2)!} F_{[q+2]}^2 \right)$$

where $F_{(q+2)} = dA_{(q+1)}$. **The metric in Einstein conformal-frame reads**

$$ds_D^2 = e^{-\frac{2m}{d-2} \phi(t)} \left[-dt^2 + a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega_{d-2}^2 \right) \right] + r_c^2 e^{2\phi(t)} d\Sigma_{m, k_1}^2$$

and $d = q + 2$. The values of $k_i = -1, 0, +1$ correspond to the hyperbolic, flat or spherical space. In $D = 11$ (i.e., $d = 4, m = 7$), one has 4-form anti-symmetric tensor matter fields as required by supersymmetry.

Compact Hyperbolic Manifold



Compact Hyperbolic 3-Space – In Klein Coordinates (using SnapPea)

SnapPea is a computer program developed in order to study compact hyperbolic spaces, as well as compact hyperbolic orbifolds.

SnapPea can compute the volume, fundamental group, symmetry group, homology, Chern-Simon invariant and length spectrum of such spaces

Dimensional Reduction

Upon the dimensional reduction, the d -dimensional Lagrangian density is
([hep-th/0306291](#), [hep-th/0309139](#))

$$\mathcal{L}_d = M_d^2 \sqrt{-g_d} \left(\frac{\mathcal{R}}{2} - \Lambda_d + \mathbf{K} - \mathbf{V}(\phi) \right)$$

where the kinetic and potential terms are

$$\mathbf{K} = -\frac{\lambda}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

$$\frac{\mathcal{R}}{2} - \Lambda_d = k \frac{(d-2)(d-3)}{2a^2} + \frac{(d-1)(d-2)}{2} H^2 + (d-1) \frac{\ddot{a}}{a}$$

$$\mathbf{V}(\phi) = b^2 e^{-\frac{2(d-1)m}{d-2}\phi} - k_1 \frac{m(m-1)}{2r_c^2} e^{-\frac{2\lambda}{m}\phi(t)}$$

with $\lambda \equiv \frac{m(m+d-2)}{(d-2)}$. It turns out that a negatively curved geometry of the internal space (i.e., $k_1 = -1$) gives a positive exponential potential $V(\phi)$ in d -dimensions, even if $b = 0$. In cosmology, practically, one sets $d = 4$.

We should note, on the way to proceed further, that dimensional reduction of any higher-dimensional theory is well defined only if $d \geq 4$, when $k \neq 0$

Gravitational Potential = Dark Energy?

- Is it possible to use M-theory motivated potentials for dark energy?

$$\mathcal{L} = \sqrt{-g} \left[\frac{M_{\text{P}}^2}{2} \mathbf{R} - (\partial\varphi)^2 - 2\mathbf{V}(\varphi) \right]$$

- **The 4d scalar potential arising from hyperbolic-flux compactification is**

$$\mathbf{V}(\varphi) = \frac{M_{\text{P}}^2}{r_c^2} e^{-2c \frac{\varphi}{M_{\text{P}}}} + M_{\text{P}}^2 \frac{f^2}{2} e^{-\frac{6}{c} \frac{\varphi}{M_{\text{P}}}}$$

PS: If one prefers a normalization of φ such that the kinetic part is $\dot{\varphi}^2/2$ and potential is $V(\varphi)$, then this is attained by the substitution $\varphi \rightarrow \varphi/\sqrt{2}$ and $c \rightarrow c/\sqrt{2}$. We will do so later

A canonical scalar φ is related to the original ϕ , that appeared before in the metric Ansatz, slide (12), by the relation

$$\phi = \sqrt{\frac{4}{m(m+2)}} \frac{\varphi}{M_{\text{P}}} + \frac{1}{m+2} \ln \frac{m(m-1)}{4}$$

$$f^2 = b^2 \left(\frac{4}{m(m-1)} \right)^{3/c^2}, \quad c \equiv \sqrt{\frac{m+2}{m}}$$

Potential = Cosmological Constant ?

For all classical (string or Kaluza-Klein) compactifications, only $c > 1$ arises in practice. In particular, for the hyperbolic compactification, since $c = \sqrt{\frac{m+2}{m}}$, one has $1 \lesssim c < \sqrt{3}$ when $m \geq 2$.

In the M theory case $m = 7$, and so $c = 3/\sqrt{7}$, we find

$$V(\varphi) = \frac{M_{\text{P}}^2}{r_c^2} e^{-2\sqrt{9/7} \frac{\varphi - \varphi_0}{M_{\text{P}}}} + \frac{M_{\text{P}}^2 f^2}{2} e^{-2\sqrt{7} \frac{\varphi - \varphi_0}{M_{\text{P}}}}$$

The late-time cosmology, $\varphi \gg \varphi_0$, is almost unaffected by the flux term. The first exponent $\sqrt{9/7} \approx 1.1338$ is within the limit where astronomical data might be relevant, $\lambda \lesssim \sqrt{6}/2 = 1.2247$. The extra dimensions $m = 4$ is marginal for which $\lambda = \lambda_{\text{crit}}$.

The value of r_c is not fixed by field equations. A reasonable value of r_c is $\sim 1 \text{ TeV} \sim 10^{-15} \text{ cm}$ or even less, but bigger than string scale.

M-theory motivated potential may be tuned to the present value of the cosmological constant, that is, $V(\varphi) \sim 10^{-120}$, in 4d Planck units, given that $\varphi - \varphi_0 \sim 91$ and $r_c \sim 1 \text{ TeV}$. By the same token, one has $\delta\rho/\rho \sim 10^{-5}$, if $\varphi_0 - \varphi \sim 80$. And, $\mathcal{N}_e \geq 60$ generally requires $\varphi - \varphi_0 \sim \mathcal{O}(10)$

Four Dimensional Cosmology

For the single hyperbolic space as an internal manifold, we find that the φ equation of motion is (for a while, let me work in the units $M^2 = (8\pi G)^{-1} = 1$)

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{2k_1}{r_c^2} c e^{-2c\varphi} - \frac{3}{c} f^2 e^{-\frac{6}{c}\varphi} = 0$$

while the Friedman equation is

$$3H^2 = \dot{\varphi}^2 - \frac{2k_1}{r_c^2} e^{-2c\varphi} - \frac{3k}{a^2} + f^2 e^{-(6/c)\varphi}$$

The curvature of the spatial part of the metric, k/a^2 is related to the total energy density in the universe, so this term maynot be zero precisely.

Anyway, let us proceed by assuming that the four dimensional universe is spatially flat ($k = 0$). In this case it is convenient to define a new logarithmic time variable τ by

$$d\tau = e^{-c\varphi} dt, \quad \alpha(\tau) = \ln(a(t)) \Rightarrow a(t) = e^{\alpha(\tau)}$$

Examples of Transient Acceleration

In the case of zero flux ($f = 0$) the solution is

$$\sqrt{3} \alpha = \delta_- \ln \cosh \frac{\gamma(\tau + \tau_1)}{r_c} + \delta_+ \ln \sinh \frac{\gamma(\tau + \tau_1)}{r_c} + \mathbf{A}$$

$$\varphi = \delta_- \ln \cosh \frac{\gamma(\tau + \tau_1)}{r_c} - \delta_+ \ln \sinh \frac{\gamma(\tau + \tau_1)}{r_c} + \mathbf{B}$$

$$\delta_{\pm} = \frac{1}{\sqrt{3} \pm c}, \quad \gamma = \sqrt{\frac{3 - c^2}{2}}$$

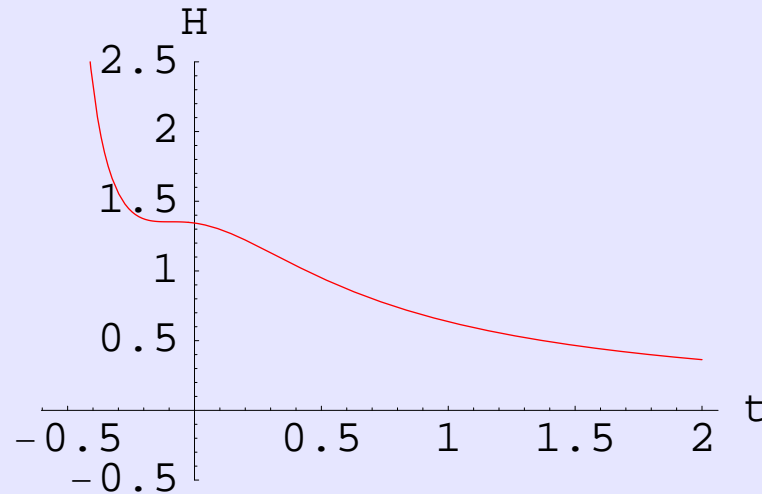
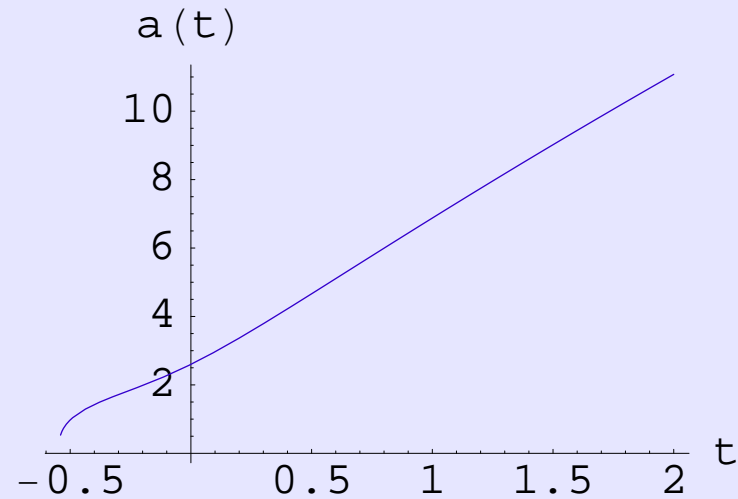
This gives (for simplicity we set $\tau_1 = 0$)

$$\mathbf{H} = \frac{da/dt}{a} = e^{-c\varphi} \alpha'(\tau) > 0$$

$$\frac{\ddot{a}}{a} = e^{-2c\varphi} \frac{2\gamma^2}{r_c^2} \left[\frac{2(c^2 - 1)}{c^2 - 3} + \frac{2\sqrt{3}c (2 \cosh^2(\gamma\tau/r_c) - 1) - c^2 - 3}{3(3 - c^2) (\cosh^2(\gamma\tau/r_c) - 1) \cosh^2(\gamma\tau/r_c)} \right]$$

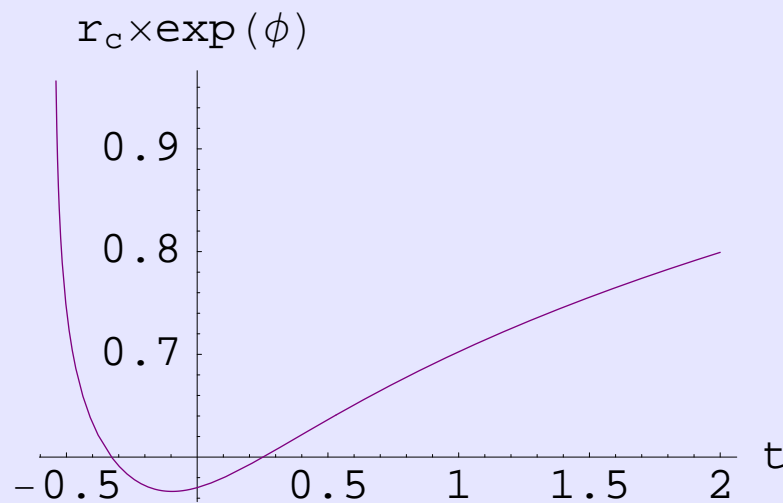
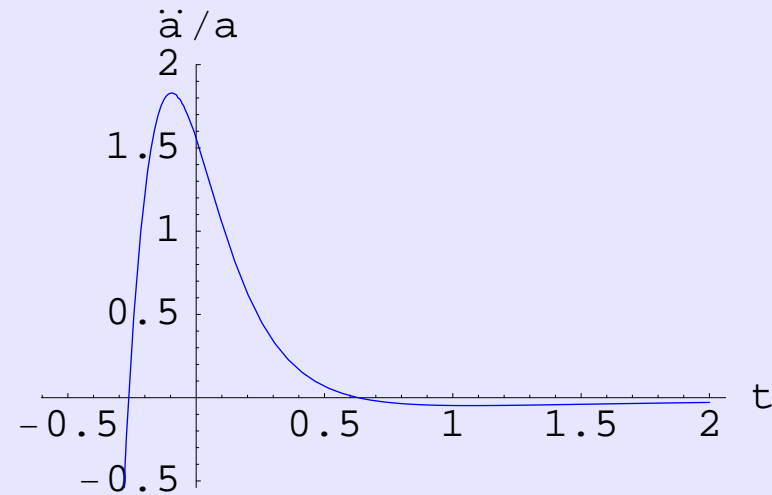
The critical value $c = 1$ separates qualitatively the different cosmologies. Solutions with $c > 1$ are only transiently accelerating, and $c \leq 1$ allows eternally accelerating expansion even for the $k=0$ cosmology. But $c < 1$ is not obtainable from classical compactifications! **M-theory case is $c = 1.13$**

4d scale factor and Hubble parameter for $k=0$ cosmology



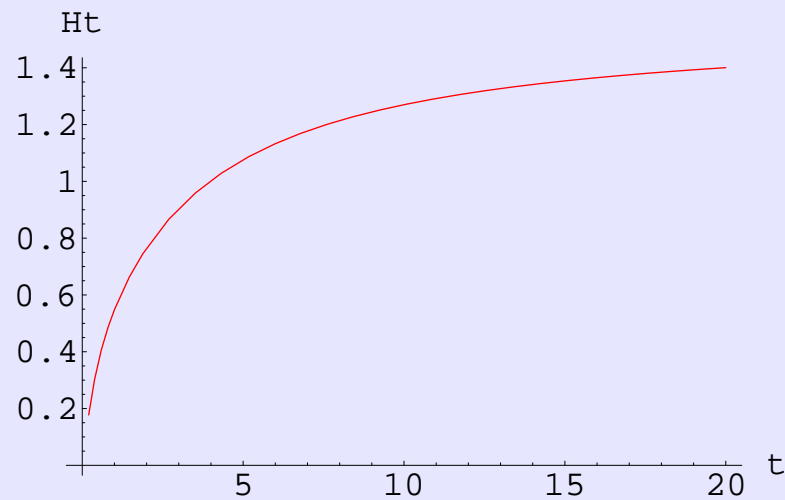
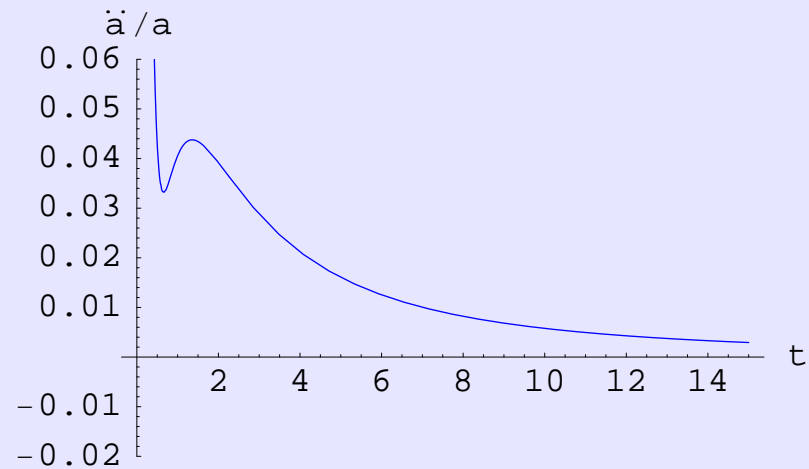
The parameters are fixed at $c = 3/\sqrt{7}$, $r_c = 0.4$, $\tilde{b} = 1$.

Transient acceleration and bouncing of internal volume



Parameter are fixed at $c = 3/\sqrt{7}$, $r_c = 0.4$, $\tilde{b} = 1$, $k = 0$, $k_1 = -1$.

An Example of Eternal Acceleration



The parameter values are $r_c = 0.8$, $\tilde{b} = 4$, $k = 0$, $k_1 = -1$, and $c = 0.8$

Open Universe and Eternally Accelerating Expansion

Next consider case of $k = -1$. In the zero-flux limit, the solution is

$$\mathbf{a}(t) = \frac{c}{\sqrt{c^2 - 1}} t \equiv \mathbf{a}_0, \quad \varphi(t) = \frac{1}{c} \ln \left(\frac{ct}{r_c} \right) \equiv \varphi_0$$

This solution itself is not accelerating since $\ddot{a}(t) = 0$, but, to the lowest order, where a non-zero field strength parameter $b > 0$ serves as a source term, the solution is accelerating when $c < \sqrt{2}$.

To lowest order in cosmological perturbations

$$\mathbf{a}(t) = \mathbf{a}_0 + \mathbf{a}_1, \quad \varphi = \varphi_0 + \varphi_1$$

the solution is

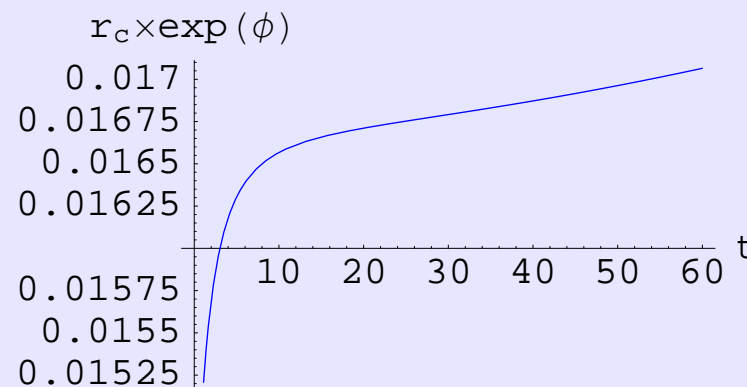
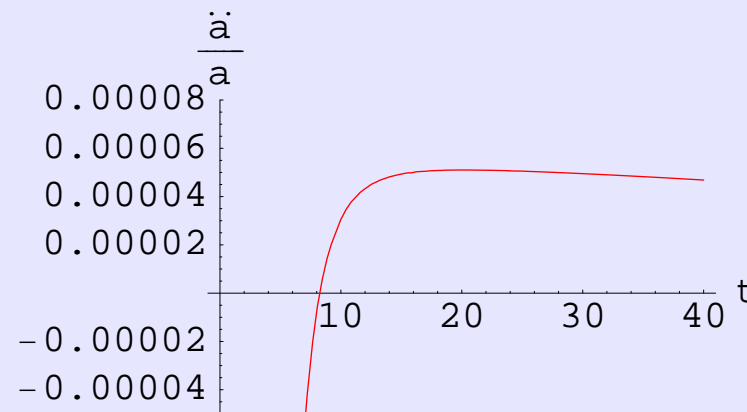
$$\mathbf{a}_1 = \beta t^n, \quad \varphi_1 = \beta \gamma t^{n-1}$$

where β is undetermined (but a small constant) and

$$\gamma = \frac{3(1-n)}{4} \sqrt{c^2 - 1}, \quad n = -\sqrt{\frac{4 - 3c^2}{c^2}}$$

Full classical non-linear equations give an eternally accelerating solution even if $c > 1$! This result holds with a non-trivial background fluxes

Eternally Accelerating Open Universe



We have set $r_c = 0.001$. For $r_c \ll 1$, $r_c e^\phi$ grows very slowly! See the next slide. In fact, for $k = 0$, the scale factor $a(t)$ grows much faster with t as compared to the $k = -1$ case. So $k = -1$ cosmology might tend towards spatial flatness in the subsequent evolution, in order to describe the universe we now observe

Internal space scale factors: Observationally too small

Table 1: The relation between r_c and the scale factors at $t = 10$

r_c	$a(t)$	$r_c e^\phi$	$\varphi - \varphi_0$
10^{-7}	5.4×10^6	0.8×10^{-5}	16.1
10^{-9}	1.8×10^8	2.2×10^{-7}	20.1
10^{-12}	4.0×10^{10}	1.0×10^{-9}	26.3

Table 2: The relation between r_c and the scale factors at $t = 1000$

r_c	$a(t)$	$r_c e^\phi$	$\varphi - \varphi_0$
10^{-7}	1.9×10^8	2.2×10^{-5}	20.5
10^{-9}	7.0×10^9	6.0×10^{-7}	25.5
10^{-12}	1.4×10^{12}	2.5×10^{-9}	30.5

Table 3: The relation between r_c and the scale factors at $t = 10^{10}$

r_c	$a(t)$	$r_c e^\phi$	$\varphi - \varphi_0$
0.1	1×10^9	35	22.5
0.001	4×10^{10}	1	26.5
0.00001	1.4×10^{12}	0.025	30.5

Cosmology with two or more fields

As we know, the enormous moduli space is the most important feature of string theory. So from string or M-theory perspectives, it seems quite unnatural to have a model of inflation with a single inflaton field!

For canonically normalized scalars φ_i , the kinetic term is

$$\frac{1}{2} \sum_i^N \dot{\varphi}_i^2$$

The potential term is

$$V(\varphi_1, \varphi_2) = M_{\text{P}}^2 \left(-\mathbf{k}_1 \Lambda_1 e^{-\mathbf{b} \varphi_1} - \mathbf{k}_2 \Lambda_2 e^{-\mathbf{c} \varphi_1 - \mathbf{d} \varphi_2} + \dots \right)$$

$$\Lambda_i = \frac{m_i(m_i - 1)}{2 r_i^2}$$

where c, d, e, \dots are (inflaton) coupling constants of the order unity.

The equations of motion we need to solve are ($i = 1, 2, \dots, N$)

$$\ddot{\varphi}_i + 3\mathbf{H}\dot{\varphi}_i + \frac{dV}{d\varphi_i} = 0$$

along with the Friedman equation

$$H^2 = \frac{1}{3} (\mathbf{K} + \mathbf{V}) - \frac{k}{a^2}$$

To solve these equations, usually, we need to assign some initial conditions. But it is easy to check that, for a large set of initial conditions, the solution always asymptotes to (here we write only the $k = 0$ solution)

$$a(t) \propto (t/t_0)^\alpha$$

For two-field models (i.e. $N = 2$, we find)

$$\alpha = \frac{c^2}{2} + \frac{(2 - c^2)^2}{2d^2}, \quad H = \frac{\alpha}{t}$$

In one or another way one requires $k_1 < 0$ or $k_2 < 0$ or both, so-called hyperbolic compactification, depending upon the ratio Λ_2/Λ_1 . For $t \gg 0$,

$$V \sim \frac{1}{t^2} \sim \frac{1}{(10^{28} \text{cm})^2} \sim 10^{-56} \text{cm}^{-2} \sim 10^{-47} (\text{GeV})^4$$

So rather asking why the present value of cosmological constant is so small, namely, $\Lambda \sim 10^{-120} M_P^4$, one could ask why the gravitational energy

density always scales with the inverse squared of the physical size (scale factor) of the universe!

Next consider $N = 3$, so that the full spacetime is split as

$$\mathcal{M} \times \mathcal{K}_1 \times \mathcal{K}_2 \times \mathcal{K}_3$$

In this case, the parameters α is improved as

$$\alpha = \frac{2(d^2 + (b - c)^2)}{b^2 d^2} + \frac{2(d(b - e) - f(b - c))^2}{b^2 d^2 g^2}$$

but it is still less than 1. That is, regardless of the initial conditions, the $k = 0$ cosmologies give only a period of transient acceleration.

Concluding Remarks

- If a spatial curvature of the universe is flat and its accelerated expansion is eternal, then such an acceleration may arise from a scalar potential of the form $V \sim \exp(-2c\varphi)$ with the coupling constant $c \leq 1$
- However, for all known classical compactifications of 10 or 11d supergravities on some non-trivial curved internal spaces and/or toroidal spaces with fluxes, only $c \gtrsim 1$ arises in practice. Thus we are led to explore alternatives for Cosmic Acceleration. We find that an eternally accelerating expansion is possible for the coupling $c > 1$ only if the spatial curvature of the universe is negative.
- It is quite possible that the dark energy of the universe is a scalar potential that naturally arises from slowly varying size of extra dimensions, and its magnitude can always be approximated by the inverse square of the physical size (or scale factor) of the universe.
- Finally, a somehow less appreciated but important conclusion is that – the late-time evolution of the universe may be explained by flat space cosmologies, but the spatial curvature k , which must be negative, was significantly important during the early-time inflationary periods.