Altruistic punishment works in promoting cooperation

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Abstract：Altruistic punishment is a puzzle in biology and human evolution. We study the role of punishment in promoting cooperation from the social norm level in a three-level evolution framework. In such framework, reputation is the instantaneous result of a player's action under certain social norm; players update their strategy after they can really perceive the payoff difference of strategies; and social norm evolves according to the average payoff of all members of the society in a longer term. We model the strategy-level dynamics under different social norm such as no-punishment social norm, punishment-optional social norm and punishment-provoking social norm. We find that altruistic punishment can enlarge the attraction domain of cooperative evolutionary stable state (CESS) and increase the convergence rate to CESS.

Key words: altruistic punishment; cooperative evolutionary stable state; attraction domain; convergence rate

1 Introduction

Altruistic punishment draws considerable interest in the study of human cooperation. As a form of indirect reciprocity, altruistic punishment implies a loss for the punished person as well as a cost of the punisher. There is a dispute about the effectiveness of altruistic punishment on promoting cooperation. Lab experiments and evidence from the field show that humans ordinarily punish defectors at a personal cost [1,2]; theoretical model of Ohtsuki etc. [3] suggests that there is only a small parameter region where costly (altruistic) punishment leads to an efficient equilibrium, and efficient strategy for indirect reciprocity is to withhold help for defectors rather than punishing them.

This paper argues that Ohtsuki's analysis only focuses on the cooperative evolutionary stable state (CESS) and their resistance to invasion; and if turning to the process of approaching CESS, we will find that altruistic punishment plays an important role in promoting cooperation. By explicitly modeling the dynamics of Ohtsuki's models and then compare the strategy frequency dynamics under different social norm such as no-punishment social norm, punishment- optional social norm and punishment-provoking social norm. We show that altruistic punishment works in at least two ways: (1) enlarges the attraction domain of CESS, (2) increases the velocity of convergence to CESS.

2 Model

2.1 Donor recipient game

At each small time interval, Δt, a fraction $2\Delta t$ of players is randomly sampled from an infinitely large population to form Δt pairs. In each pair, one player acts as a donor and the other player as a recipient. The donor has two basic behavioral choices: cooperation (C), defection (D). Cooperation involves a cost, c, for the donor and a benefit, b, for the recipient. Defection has no cost and yields no benefit. A donor may also have the choice of punishment in some social norm. Punishment has cost, α, for the donor and cost, β, for the recipient. Here b, c, α and β are all positive real number.

Each individual is endowed with a binary reputation, which is either good (G) or bad (B). The donor can base his decision on the recipient's reputation. After each interaction, the reputation of the donor is updated according to the 'social norm' of the population, while the reputation of the recipient remains the same. The reputation update process is susceptible to errors. With probability μ, where $0 < \mu < \frac{1}{2}$, an incorrect reputation is assigned. With probability $1-\mu$ the correct reputation is assigned. All individuals come to the same conclusion; there are no private lists of reputation.

2.2.1 Strategies

A donor can base his action on the recipient's reputation. Each player has an action rule (or strategy), s, which depends on the recipient's reputation. A player with an action rule, s, takes the action s(G) toward a good recipient, and the action s(B) toward a bad one. Each of s(G) and s(B) can be either C, D, or P. There are 2^2 = 4 possible action rules: s(G)s(B) = CC, CD, DC, DD in basic model. For social norms with punishment available, there are 3^2 = 9 possible action rules: s(G)s(B) = CC, CD, CP, DC, DD, DP, PC, PD, and PP. In the present work, we only study four of these strategies, CC, CD, CP and DD, but not DC, DP, PC, PD and PP, because it is odd and not feasible for study the emergence of cooperation.

2.2.2 Social norms

A social norm, n, is used for updating the reputations of players. A donor who has taken the action X (X = C, D, P) toward a recipient whose reputation is J (J = G, B), is assigned the new reputation n(J,X) (= G or B) by the social norm n. There are $2^3 \times 2$ = 64 possible social norms. Social norms of this type are based on 'second-order assessment', and they depend on both the action of the donor and the reputation of the recipient [4]. Figure 1 gives four typical social norms we will study in this work with the related ordinary strategies.

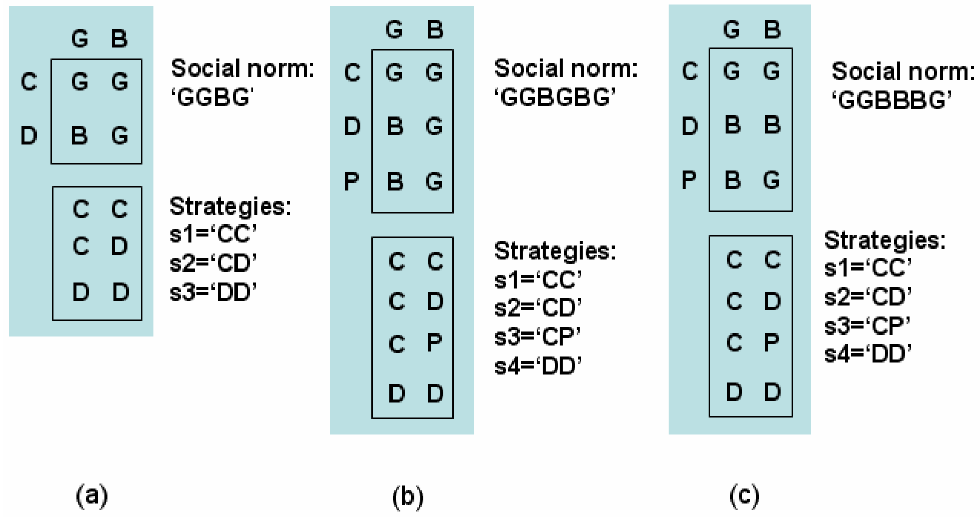


Figure 1 Typical social norms with the related ordinary strategies

In social norm 'GGBG' as figure 1(a), there are no action options of punishment. Donor can only cooperate or defect, because the social norm assigns good reputation to all cooperator and defector to bad recipient and bad assigns bad reputation to defector to good recipient. The state in which all players takes strategy 'CD' is an evolutionary stable state, but the state in which all players takes strategy 2 'DD' is also an evolutionary stable state, as following analysis will reveal. The set of all initial strategy frequencies in feasible domain that converge to a cooperative evolutionary stable state (CESS, all players taking CD strategy in this norm) is the attraction domain of the CESS. We will investigate the ratio of the extent of the CESS attraction domain to that of the entire feasible domain.

In social norm 'GGBGBG' as figure 1(b), one donor has the choice of punishment, and the punishment to a bad recipient will gain a good reputation. But the punishment is just an optional action to bad recipient, because donors defect to a bad recipient without any cost is also assigned a good reputation. We will show that even punishment just as an optional action, it can widen the attraction domain of the CESS (all players taking CD strategy in 'GGBGBG' norm).

In social norm 'GGBBBG' as figure 1(c), a donor defects to a bad recipient is assigned a bad reputation, this encourage a donor to take either cooperation or defection action to bad recipient. And it is a more punishment provoking social norm. We will show that the CESS in 'GGBBBG' norm, all players taking CP strategy, has the largest attraction domain. Even some initial strategy frequencies converge to CESS in both 'GGBGBG' norm and 'GGBBBG' norm, the convergence rate of punishment-provoking 'GGBBBG' norm can be larger.

2.2 Evolutionary dynamics of strategy frequency

Consider a three-level evolution process. Social norm is a social choice often made by the leaders, government s or elites as the form of laws, rules, convention, morality and etc. It evolves very slowly according to the global benefit of all members, measured as the average payoff in a considerably long period of time. Strategy is the individual choice made by bounded rational agents according to their personal payoff in a short period of time. Reputation is updated instantaneously once a donor takes an action according to the social norm.

2.2.1 Stable reputation frequency

A stable reputation frequency can be derived given that the frequency of strategies taken by all players in the whole society is fixed.

In 'GGBG' norm, a fraction x_1 ($0 \le x_1 \le 1$) of players take strategy 'CC', and x_2 and x_3 ($0 \le x_2, x_3 \le 1$) of players take 'CD' and 'DD' strategy. Here $x_1 + x_2 + x_3 = 1$.And the frequency of good reputation ones in 'CC' , 'CD' and 'DD' players is denoted by g_1, g_2 and g_3 respectively. Thus the frequency of good reputation ones in total players is $g = g_1 + g_2 + g_3$. Figure 2 gives the reputation dynamics of 'CC', 'CD' and 'DD' players. A 'CC' player has 1/2 chance to be a donor, and takes cooperation action no matter what reputation the recipient has, and this tends to make him a good reputation. Due to the assignment error, he gets a good reputation with a probability $1-\mu$ and bad reputation with probability μ . The 'CC' player also has 1/2 chance to be a recipient; his reputation does not change and remains as the current frequency g_1. So the new frequency of good reputation 'CC' players is $g_1^{'} = \frac{1}{2}g_1 + \frac{1}{2}(1-\mu)$.

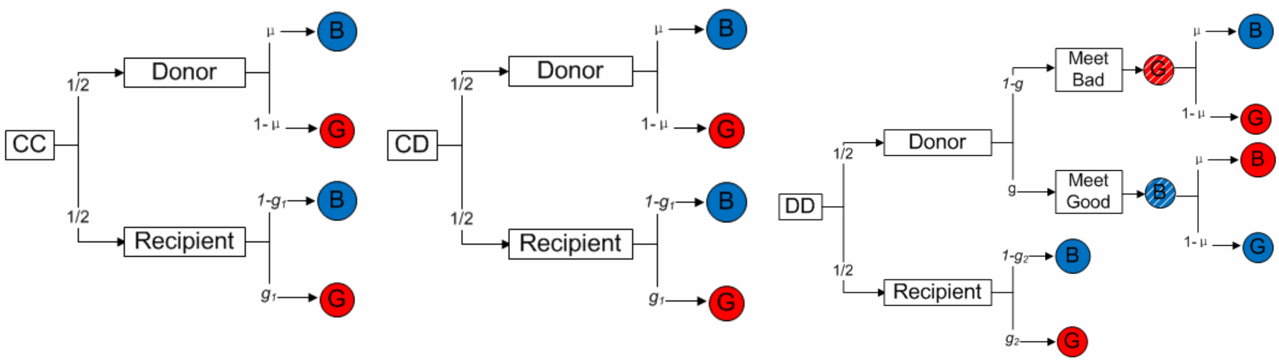


Figure 2 Reputation dynamics of strategies in 'GGBG' norm

Similarly we get the new frequency of good reputation 'CD' and 'DD' players are $g_2^{'} = \frac{1}{2}g_2 + \frac{1}{2}(1-\mu)$ and $g_3^{'} = \frac{1}{2}g_3 + \frac{1}{2}(1-g)(1-\mu) + \frac{1}{2}g\mu$. Since $g = x_1g_1 + x_2g_2 + x_3g_3$, we can solve the linear recursion and get the stable reputation frequency of each strategy $g_1^{*} = g_2^{*} = 1-\mu$,

$g_3^* = (1-\mu)\left[1 - \frac{1-2\mu}{1+(1-2\mu)x_3}\right]$. And the total good reputation frequency is $g^* = \frac{1-\mu}{1+(1-2\mu)x_3}$.

For 'GGBGBG' norm, we can also get the stable reputation frequency of 'CC' 'CD' and 'CP' players $g_1^* = g_2^* = g_3^* = 1-\mu$, and stable reputation frequency of 'DD' players $g_4^* = (1-\mu)\left[1 - \frac{1-2\mu}{1+(1-2\mu)x_4}\right]$. The total good reputation frequency is $g^* = \frac{1-\mu}{1+(1-2\mu)x_4}$.

For 'GGBBBG' norm, the stable reputation frequency of 'CC' and 'CP' players are $g_1^* = g_3^* = 1-\mu$, and that of 'CD' players is $g_2^* = \mu + (1-2\mu)g^*$ and 'DD' players. The total good reputation frequency is

$$g^* = \frac{(1-\mu)(x_1+x_3)+\mu(x_2+x_4)}{1-(1-2\mu)x_2}.$$

2.2.2 Fitness measurements of strategies

The pace of agents' strategy updating is much lower than reputation dynamics. Players will update their strategy only after they can actually conceive the payoff of different strategies by a sufficiently long time during which players' reputation distribution converges and stays in a stable state. So we calculate a strategy's expected payoff in the stable reputation distribution as the fitness measurements.

Figure 3 illustrates the calculation expected payoff of 'CC', 'CD' and 'DD' strategy in 'GGBG' norm. For a 'CC' player, he have 1/2 chance to be a donor and cooperate with a cost c. With another 1/2 chance being a recipient, he meets a 'CC' , 'CD' and 'DD' player with probability x_1, x_2 and x_3 and get b, $b(1-\mu)$ and 0 revenue respectively. So the expected revenue of strategy 'CC' is $p_1 = \frac{1}{2}(-c) + \frac{1}{2}[bx_1 + bx_2(1-\mu)]$. Similarly, the expected revenue of strategy 'CD' and 'DD' are calculated as $p_2 = \frac{1}{2}g(-c) + \frac{1}{2}[bx_1 + bx_2(1-\mu)]$ and $p_3 = \frac{1}{2}(0) + \frac{1}{2}[bx_1 + bx_2 g_3]$.

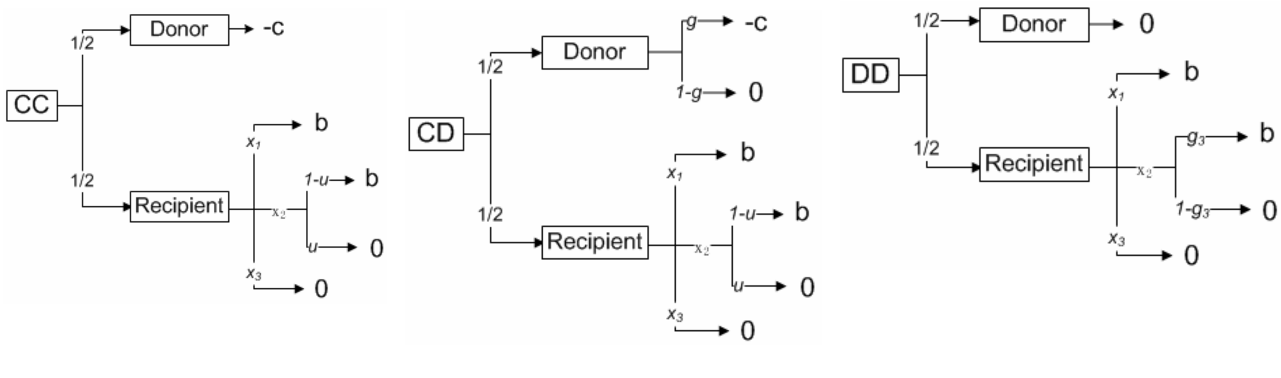


Figure 3 The calculation expected payoff strategies in 'GGBG' norm

For 'GGBGBG' norm, we can also get the expected revenue of strategy 'CC', 'CD', 'CP' and 'DD' are

$$
\begin{aligned}
p_1 &= \frac{1}{2}(-c)+\frac{1}{2}x_3\mu(-\beta)+\frac{1}{2}\left[bx_1+b\left(x_2+x_3\right)(1-\mu)\right] \\
p_2 &= \frac{1}{2}g(-c)+\frac{1}{2}x_3\mu(-\beta)+\frac{1}{2}\left[bx_1+b\left(x_2+x_3\right)(1-\mu)\right] \\
p_3 &= \frac{1}{2}g(-c)+\frac{1}{2}(1-g)(-\alpha)+\frac{1}{2}x_3\mu(-\beta)+\frac{1}{2}\left[bx_1+b\left(x_2+x_3\right)(1-\mu)\right] \\
p_4 &= \frac{1}{2}\left[bx_1+b\left(x_2+x_3\right)g_4\right]+\frac{1}{2}x_3(1-g_4)(-\beta)
\end{aligned}.
$$

For 'GGBBBG' norm, we can also get the expected revenue of strategy 'CC', 'CD', 'CP' and 'DD' are

$$
\begin{aligned}
p_1 &= \frac{1}{2}(-c)+\frac{1}{2}x_3\mu(-\beta)+\frac{1}{2}\left[bx_1+b\left(x_2+x_3\right)(1-\mu)\right] \\
p_2 &= \frac{1}{2}g(-c)+\frac{1}{2}x_3(1-g_2)(-\beta)+\frac{1}{2}\left[bx_1+bg_2\left(x_2+x_3\right)\right] \\
p_3 &= \frac{1}{2}g(-c)+\frac{1}{2}(1-g)(-\alpha)+\frac{1}{2}x_3\mu(-\beta)+\frac{1}{2}\left[bx_1+b\left(x_2+x_3\right)(1-\mu)\right] \\
p_4 &= \frac{1}{2}\left[bx_1+b\left(x_2+x_3\right)\mu\right]+\frac{1}{2}x_3(1-\mu)(-\beta)
\end{aligned}.
$$

2.2.3 Replicator dynamics of strategy frequency

We model the strategy frequency dynamics as the replicator equation[5], given by $\dot{x}_i = x_i\left(p_i - \overline{p}\right)$, were $\overline{p}$ is the average payoff in the entire society, defined as $\overline{p} = \sum x_i p_i$. Here i=1,2,3 for GGBG norm and i=1,2,3,4 for 'GGBGBG' and 'GGBBBG' norm. These differential equations are defined on the simplex $S_3 = \left\{\left(x_1, x_2, x_3\right) \middle| x_1 + x_2 + x_3 = 1, x_i \geq 0\right\}$ for 'GGBG' norm and $S_4 = \left\{\left(x_1, x_2, x_3, x_4\right) \middle| x_1 + x_2 + x_3 + x_4 = 1, x_i \geq 0\right\}$ for 'GGBGBG' norm and 'GGBBBG' norm. Because only the relative size of payoff matters in this dynamics, so we can shift the expected payoff value additively without altering the dynamics at all.

For 'GGBG' norm, $p_1' = p_1 - p_3; p_2' = p_2 - p_3$, and $\overline{p} = x_1 p_1' + x_2 p_2'$. So the dynamical system is $\begin{cases}\dot{x}_1 = x_1(p_1' - \overline{p}) \\ \dot{x}_2 = x_2(p_2' - \overline{p})\end{cases}$. For 'GGBGBG' norm and 'GGBBBG' norm, $p_1' = p_1 - p_4; p_2' = p_2 - p_4; p_3' = p_3 - p_4$, and $\overline{p} = x_1 p_1' + x_2 p_2' + x_3 p_3'$. So the dynamical system is $\begin{cases}\dot{x}_1 = x_1(p_1' - \overline{p}) \\ \dot{x}_2 = x_2(p_2' - \overline{p}) \\ \dot{x}_3 = x_3(p_3' - \overline{p})\end{cases}$.

3 Analyses

3.1 CESS of three social norms

In all social norms, the state that all players take ‘DD’ strategy ($x_1 = 0$, $x_2 = 0$ for ‘GGBG’ norm and $x_1 = 0$, $x_2 = 0$, $x_3 = 0$ for ‘GGBGBG’ and ‘GGBBBG’ norm) is the evolutionary stable state. This can be proven by calculating the Jacobian matrix of each the dynamical system on this point, and the negative eigenvalues of the matrix implies that ‘DD’ is the stable equilibrium.

In ‘GGBG’ norm, the state that all players take ‘CD’ strategy ($x_1 = 0$, $x_2 = 1$) is an evolutionary stable state on the condition that $\frac{1}{1-2\mu}\frac{c}{b} < 1$. In ‘GGBGBG’ norm, the state that all players take ‘CD’ strategy ($x_1 = 0$, $x_2 = 1$, $x_3 = 0$) is an evolutionary stable state on the condition that $\frac{1}{1-2\mu}\frac{c}{b} < 1$. In ‘GGBBBG’ norm, the state that all players take ‘CP’ strategy ($x_1 = 0$, $x_2 = 0$, $x_3 = 1$) is an evolutionary stable state on the condition that $\frac{1}{1-2\mu}\frac{(1-\mu)\alpha + \mu c}{\alpha + \beta + b - c} < 1$. These results can be proven by calculating the Jacobian matrix of the dynamical system on this point, and letting two eigenvalues of the matrix less than zero.

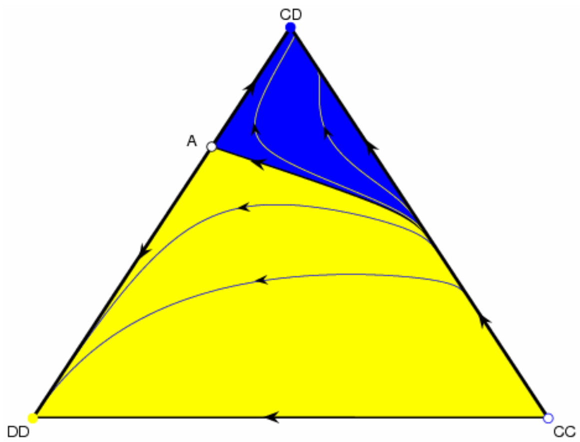
3.2 Attraction domain of CESS

For the lack of analytical tools, we rely heavily on the numerical method to calculate the extent of the attraction domain of cooperative evolutionary stable states.

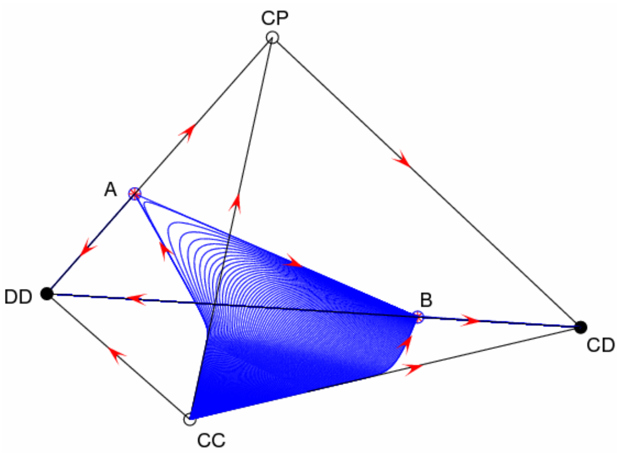
Figure 4 gives the phase portraits of three norms with the same typical parameter setting $\mu = 0.02$, $b = 3$, $c = 2$, $\alpha = 1$, $\beta = 4$.

In the phase portrait of ‘GGBG’ norm (figure 4(a)), the point CD (representing the state that all players takes ‘CD’ strategy, $x_1 = 0$, $x_2 = 1$)) and the point DD (representing the state that all players takes ‘DD’ strategy, $x_1 = 0$, $x_2 = 0$)) are both stable equilibrium as mentioned above. And CD is the cooperative evolutionary state. Point A ($x_1 = 0$, $x_2 = \frac{1}{1-2\mu}\frac{c}{b}$) is a saddle node whose unstable manifold is along the CD-DD line, and the stable manifold constitutes the separatrix line dividing the plane into two regions. The blue region is the domain of attraction of cooperative evolutionary state CD, points lying in this region converges to CD points. The ratio of CESS attraction domain is the extent of blue area over that of the entire feasible domain, i.e. the area of triangle CC-CD-DD. Under this parameter setting, the ratio of CESS attraction domain is about 0.15.

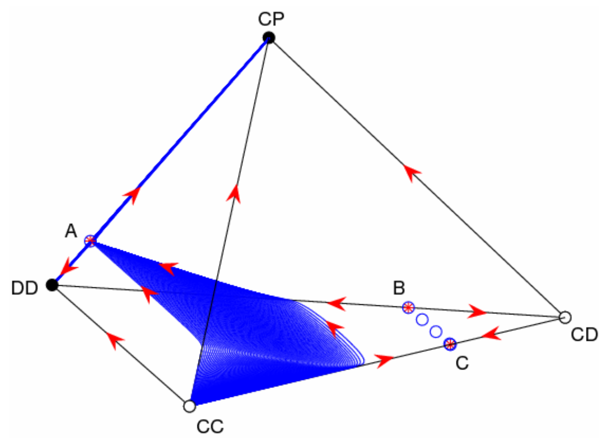
The phase portrait of ‘GGBGBG’ norm in a simplex-4 is given in figure 4(b). CD and DD are the stable equilibrium and CD is the CESS. Points A ($x_1 = 0$, $x_2 = 0$, $x_3 = \frac{1}{1-2\mu}\frac{(1-\mu)(\alpha+c)}{(1-\mu)(b+\beta)+\alpha}$) is a saddle node with a one-dimensional stable manifold and a two-dimensional unstable manifold. Points B ($x_1 = 0$, $x_2 = \frac{1}{1-2\mu}\frac{c}{b}$, $x_3 = 0$) is a saddle node with a one-dimensional unstable manifold along CD-DD line and a two-dimensional stable manifold which constitutes the separatrix surface dividing the simplex into two parts. The region over the surface is the domain of attraction of cooperative evolutionary state CD which is about 0.60 under this parameter setting.



(a) 'GGBG'norm



(b) 'GGBGBG' norm

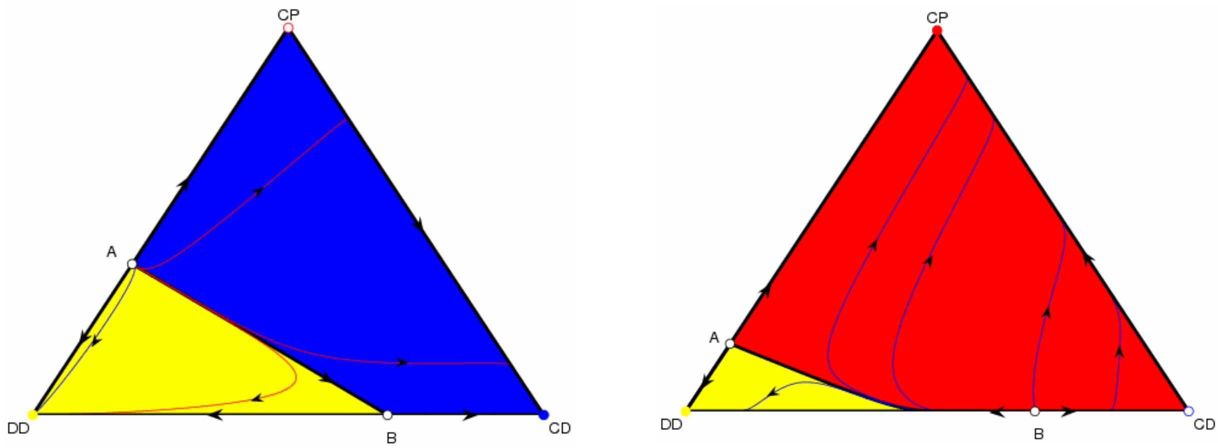


(c) 'GGBBBG' norm

Figure 4 Phase portrait of three social norms ($\mu = 0.02$, $b = 3$, $c = 2$, $\alpha = 1$, $\beta = 4$)

The phase portrait of 'GGBBBG' norm in a simplex-4 is given in figure 4(c). CP ($x_1 = 0$, $x_2 = 0$, $x_3 = 1$) and DD ($x_1 = 0$, $x_2 = 0$, $x_3 = 0$) are the stable equilibrium and CP is the CESS. Points A ($x_1 = 0$, $x_2 = 0$, $x_3 = \frac{1}{1-2\mu}\frac{(1-\mu)\alpha+\mu c}{\alpha+\beta+b-c}$) is a saddle node with a one-dimensional unstable manifold along CP-DD line and a two-dimensional stable manifold which constitutes the separatrix surface dividing the simplex into two parts. The region over the surface is the domain of attraction of cooperative evolutionary state CP which is about 0.81 under this parameter setting. In the facet of CC-CD-DD, there is a line between B($x_1 = 0$, $x_2 = \frac{1}{1-2\mu}\frac{c}{b}$, $x_3 = 0$) and C($x_1 = 1 - \frac{1}{1-2\mu}\frac{c}{b}$, $x_2 = \frac{1}{1-2\mu}\frac{c}{b}$, $x_3 = 0$) consists of equilibriums which are Lyapunov stable.

Figure 5 gives the CD-CP-DD facets for 'GGBGBG' and 'GGBBBG' norm with no 'CC' players i.e. $x_1 = 0$. The blue region in figure 5(a) is the attraction domain of CD state in 'GGBGBG' norm and the red region in figure 5(b) is the attraction domain of CP state in 'GGBBBG' norm.



(a) 'GGBGBG' norm 'GGBBBG' norm

Figure 5 CD-CP-DD facets ($\mu = 0.02$, $b = 3$, $c = 2$, $\alpha = 1$, $\beta = 4$)

Putting two facets over each other, the simplex-3 is divided into three regions, as in figure 7. The lower region C-D-DD is in the attraction domain of non-cooperative evolutionary state DD in both norms. The middle region A-B-C-D is in the attraction domain of non-cooperative evolutionary state DD in 'GGBGBG' norm and attraction domain of cooperative evolutionary state CP in 'GGBBBG' norm. From the same initial points S1, the trajectory in 'GGBGBG' norm converges to DD (yellow), and the trajectory in 'GGBGBG' norm converges to CP (red). The upper region is in the attraction domain of cooperative evolutionary state CD in 'GGBGBG' norm and attraction domain of cooperative evolutionary state CP in 'GGBBBG' norm.

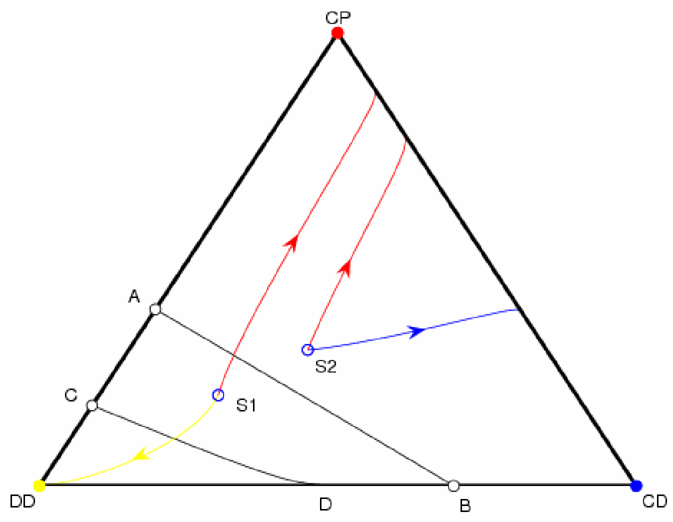
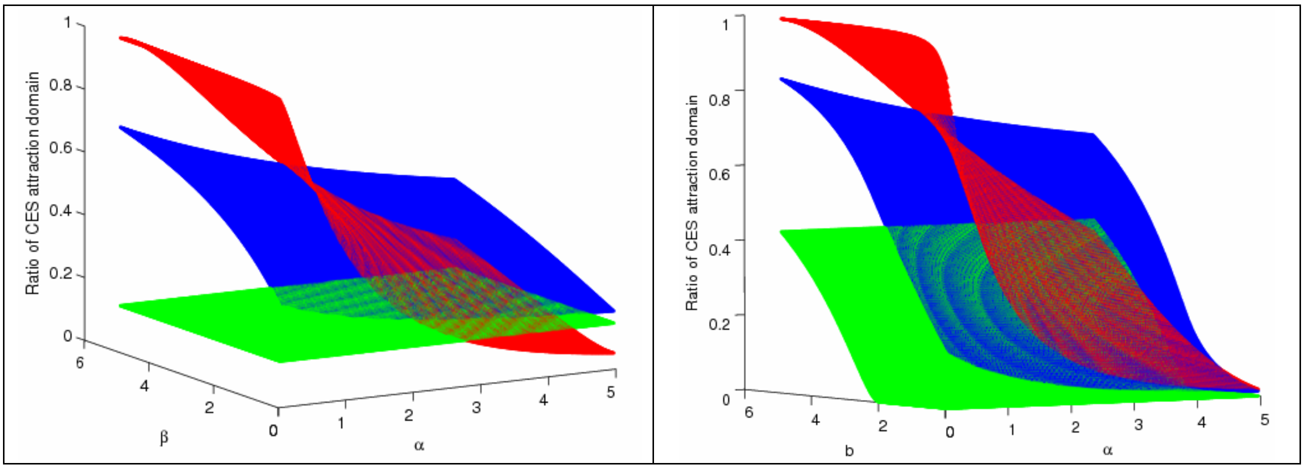


Figure 6 Typical trajectories under 'GGBGBG' and 'GGBBBG' norm

To get the overview of effect of parameters (b, c, α, β) to CESS attraction domain, we calculate the ratios of CESS attraction domain under different parameter settings by plenty of numerical computation and presented the results in figure 7. The red points are the ratios of CESS attraction domain in 'GGBBBG' norm, blue for 'GGBGBG' norm and green for 'GGBG' norm. In most normal parameter settings, the ratios of CESS attraction domain in punishment-provoking 'GGBBBG' norm is largest, followed by punishment-optional 'GGBGBG' norm and the non-punishment 'GGBG' norm smallest. Only in some abnormal situation such as that the punishment cost α is very large or the cooperation cost is much small that the ratio of CESS attraction domain in punishment-provoking 'GGBBBG' norm is not the largest.



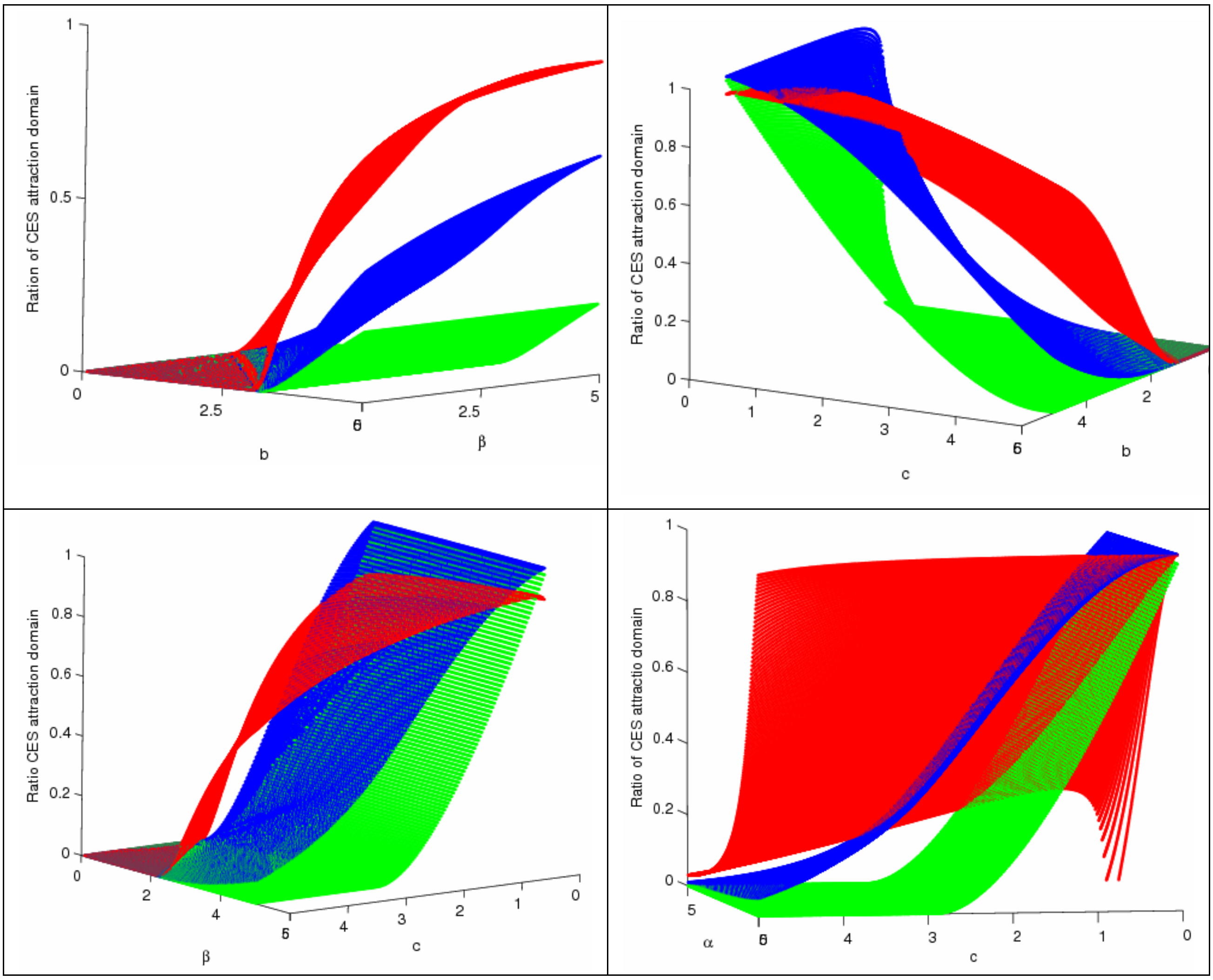


Figure 7 Ratios of CESS attraction domain of different social norm under different parameter settings

($b=3$, $c=2$, $\alpha=1$, $\beta=4$ if not specified)

3.3 Convergence rate to CESS

Even the initial state is in both attraction domain of 'GGBGBG' and 'GGBBBG' norm, it converge to the CESS with different rate in two social norms. We use an example to illustrate the difference in converge rate of two norms. From the same initial points S2 in figure 6, we calculate the frequency of cooperative strategies (CC,CD,CP) in whole population, figure 8 gives evolution of the cooperation ratio, the horizontal axis is the time, the vertical axis is the cooperation ratio. We find that in 'GGBBBG' norm which is more punishment-provoking, cooperation ratio increases more rapidly than 'GGBGBG' norm. This implies that if the players are not patient enough, altruistic punishment is a better choice.

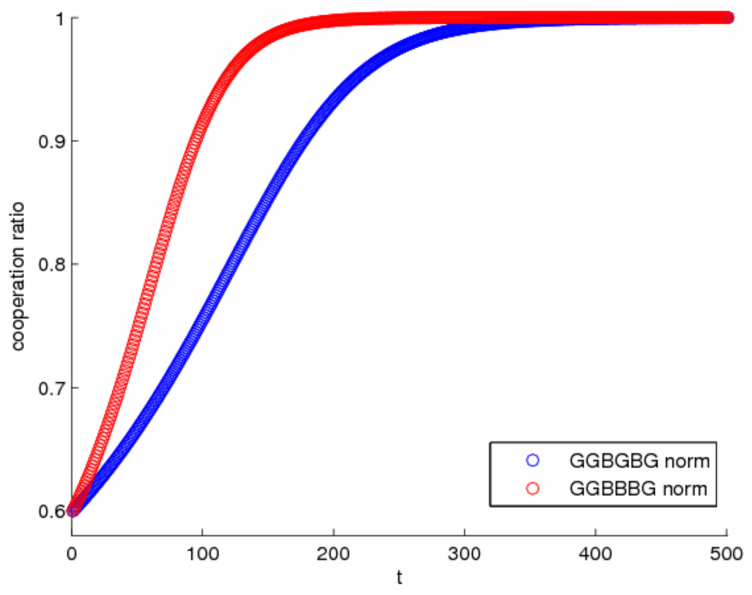


Figure 8 Converge rate from the same initial point in different social norm

4 Conclusion

We provide an explanation to the frequently witnessed altruistic punishment although it is not the most efficient stable state. We find that the attraction domain ratio of CESS in ‘GGBG’ social norm without punishment is very small, that of ‘GGBGBG’ norm with punishment as option is larger and that of ‘GGBBBG’ norm which provokes punishment is the largest. From the same initial points, punishment-provoking social norm can converge to cooperative evolutionary state more rapidly than punishment-optional social norm. Altruistic punishment works in two situations: (1) when there are too much defectors, the population can only struggle out of defection by punishment; (2) when the population is less patient, punishment increases the velocity of convergence to CESS.

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