The Behavioral Dynamics of Price Index Changes of Stock Markets: Physics Approach

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ABSTRACT

We introduce a probability density derived from nonextensive Tsallis statistical mechanics that can be applied in the interpretation of percent price index change for important indices such as NYSE Composite, DJIA, S&P 500, NASDAQ Composite, FTSE 100, NIKKEI 225, Hang Seng, Straits Times and SET index. The percent price index change probability distribution function is necessarily connected with a nonlinear Fokker–Planck equation in order to determine the type of diffusion and diffusion constant. The results of Tsallis' probability density and market observation show the behavior of all indices indicate superdiffusive dynamics. Moreover, an Ito-Langevin equation with a simple time-dependent diffusion coefficient and the nonlinear Fokker–Planck equation can exhibit the investment risk of each price index.

Keywords: Econophysics, Nonextensive statistical mechanics, Fokker-Planck equation.

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Introduction

In financial markets, most securities analysts' association consensuses analyze data by using fundamental statistics, qualitative information and experience in decisions without exactly understanding their dynamics' type which is the price variance of their member stocks in each market index. Therefore, the physics viewpoints are proposed in order to address this financial market dynamics. This is a kind of application of new interdisciplinary subject in the issue of Econophysics science. Though the microscopic interaction among traders which leads to behavior in financial markets is not easily understood, there are many descriptive attempts about the behavior of market dynamics. Ising-like model has been a good candidate for describing this behavior for many years Chowdhury and Stau_erb (1999) and Bornholdt (2001), but until now this method has not obviously been the most satisfying model. The advent of nonextensive Tsallis statistical mechanics by means of maximizing the Tsallis entropy Tsallis (1988) connected with the essence of the nonlinear Fokker-Planck equation which is associated with an underlying Ito-Langevin process Plastino and Plastino (1995) and Borland (1998) can more specify the type of diffusion and also be a good choice for taking into account of this dynamics. The previous works related with this statistics closely resembled markets' observation including currency exchange price changes Mantegna and Stanley 2000, but their algorithms were not able to compare among market indices and were used only well in one hour price change Michael and Johnson (2003).

The purpose of this paper is to develop more accurate microscopic interactive traders models based on preceding daily 20 years data in important markets' indices such as NYSE Composite, DJIA, S&P 500, NASDAQ Composite, FTSE 100, NIKKEI 225, Hang Seng, Straits Times and SET index Yahoo Finance and compare markets' risk altogether in at least unit of day. Moreover, investors and others can instantaneously use comparing the significant parameters in term of the risk of investment in easy way before they make the investment decisions.

Methodology

1. Theory

We choose Tsallis' nonextensive statistical mechanics because it is able to be used to interpret the interactions of complex systems interestingly.

1.1 Nonextensive Statistical Mechanics

Nonextensive Tsallis entropy is written in a different way from statistical mechanics ,but it can be proved to Boltzmann-Gibbs entropy ($S = -\int P\ln P$) by taking limit q → 1.

$$S_q = -\frac{1}{1-q}\left(1 - \int P(x,t)^q dx\right); q \in \mathbb{R} \tag{1}$$

Using constraints for nonextensive entropy

$$\int P(x,t)dx = 1 \tag{2}$$

$$\langle x - \bar{x}(t)\rangle_q \equiv \int [x - \bar{x}(t)]\, P(x,t)^q dx = 0 \tag{3}$$

$$\langle \left(x - \bar{x}(t)\right)^2 \rangle_q \equiv \int [x - \bar{x}(t)]^2\, P(x,t)^q dx = \sigma_q(t)^2 \tag{4}$$

We found that if q (Tsallis nonextensivity parameter is independent of time) is equal to 1, the 'q-variance' is the ordinary variance. Then, these constraints are maximized by fixed q and get the Tsallis probability distribution function.

$$P(x,t) = \frac{1}{Z(t)}\{1 + \beta(t)(q-1)[x - \bar{x}(t)]^2\}^{-\frac{1}{q-1}} \tag{5}$$

where Z(t) is a normalization constant for each time

$$Z(t) = \frac{B(\frac{1}{2}, \frac{1}{q-1} - \frac{1}{2})}{\sqrt{(q-1)\beta(t)}} \tag{6}$$

$$\beta(t) = \frac{1}{2\sigma_q(t)^2 Z(t)^{q-1}} \tag{7}$$

where $B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$ is Euler's Beta function. The ordinary variance by using distribution of Eq. (5) can be derived in Eq. (8).

$$\sigma(t)^2 = \langle \left(x - \bar{x}(t)\right)^2 \rangle_1 = \int [x - \bar{x}(t)]^2\, P(x,t)dx = \begin{cases} \frac{1}{(5-3q)\beta(t)}, & q < \frac{5}{3} \\ \infty, & q \geq \frac{5}{3} \end{cases} \tag{8}$$

Nonextensive statistical mechanics and the finite variance from Eq. (8) must lie within the range $1 \leq q < \frac{5}{3}$. Considering nonlinear Fokker-Planck equation

1.2 Fokker-Planck equation

$$\frac{\partial P(x,t)^{\mu}}{\partial t} = -\frac{\partial}{\partial x}[F(x)P(x,t)^{\mu}] + \frac{D}{2}\frac{\partial^2 P(x,t)^{\nu}}{\partial x^2} \quad (9)$$

where $F(x) = a - bx$ (10) is a linear drift force and D is diffusion constant.

Condition for solving Tsallis probability distribution function is $q = 1 + \mu - \nu$. (11)

We obtain the following 3 equations which are dependent of time.

$$-\frac{\mu}{\mu+\nu}\frac{dZ^{\mu+\nu}(t)}{dt} + 2\nu D\beta(t_0)Z(t_0)^{2\mu} - bZ^{\mu+\nu} = 0 \quad (12)$$

$$\frac{\beta(t)}{\beta(t_0)} = \left(\frac{Z(t_0)}{Z(t)}\right)^{2\mu} \quad (13)$$

$$\frac{d\bar{x}}{dt} = a - b\bar{x} \quad (14)$$

μ must equal to 1 to preserve constraint in Eq. (2) by comparing Eq. (6) and Eq. (13).

Then Eq. (11) - Eq. (13) give

$$\beta(t)^{-\frac{3-q}{2}} = \beta(t_0)^{-\frac{3-q}{2}}e^{-b(3-q)(t-t_0)} - 2Db^{-1}(2-q)[\beta(t_0)Z^2(t_0)]^{\frac{q-1}{2}}(e^{-b(3-q)(t-t_0)} - 1). \quad (15)$$

1.3 Ito-Langevin equation

$$\frac{dx}{dt} = a(x,t) + b(x,t)\xi(t) \quad (16)$$

where $\xi(t)$ is δ-correlated Gaussian noise $(\langle\xi(t)\xi(t')\rangle = \delta(t-t'))$ (17)

Eq. (16) can be proved to be equivalent to Eq. (18) in Section 4.3.4 of Gardiner Gardiner (1997).

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x}[a(x,t)P(x,t)] + \frac{1}{2}\frac{\partial^2}{\partial x^2}[b^2(x,t)P(x,t)] \quad (18)$$

Nonlinear Fokker-Planck equation for μ=1 results from Tsallis probability distribution function

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x}[F(x,t)P(x,t)] + \frac{D}{2}\frac{\partial^2 P(x,t)}{\partial x^2}^{2-q} \quad (19)$$

It can be easily seen that

$$a(x,t) = F(x,t)\,, \quad (20)$$

$$b(x,t) = \sqrt{DP(x,t)^{1-q}} \quad (21)$$

Substitute Eq. (20) and Eq. (21) into Eq. (16) then, we obtain Eq. (22)

$$\frac{dx}{dt} = F(x,t) + \sqrt{DP(x,t)^{1-q}}\xi(t) \quad (22)$$

Considering the diffusion coefficient $DP(x,t)^{1-q}$, first, this is anomalous diffusion correlated in time (memory effects) except for q = 1 that is Brownian motion or normal diffusion.

Second, if q is greater than 1, it is apparently observed that this is superdiffusion and makes the diffusion coefficient large in the next time step. On the contrary, last, if q is less than 1, the value of this equation tends to be small jump and that is subdiffusion.

2. Application to financial market indices

2.1 We choose the daily price indices data such as NYSE Composite, DJIA, S&P 500, NASDAQ Composite, FTSE 100, NIKKEI 225, Hang Seng, Straits Times and SET index about 20 years from September 1989 to January 2010 ,approximately 5,000 days (T=1,2,…,5000). Each price index data is converted into the nonoverlapping percent price index change (x(j)) computed by

$$x(j) = \frac{p(j\times t+1)-p((j-1)\times t+1)}{p((j-1)\times t+1)} \times 100 \quad (23)$$

where p(T) is price index at time T and t is time interval.

2.2 We set the initial time interval (t_0) equal to 1 day and fit (genetic algorithm method) the real market index data with Eq. (5) (by adapting to discrete Tsallis probability distribution function). Consequently, we have the appropriate q parameter (due to discrete Tsallis probability distribution function q parameter, in Eq. (8) is greater than 1 and can also be greater than 5/3) and $\beta(t_0)$ (see Fig. 1A).

2.3 We can find parameter b and D by fitting (genetic algorithm method) the resulting inverse variance ($\beta(t)$) from data which is proportional to the inverse of variance with Eq. (15) (see Fig. 2).

2.4 We solve Eq. (5) for each time interval t by using the inverse variance ($\beta(t)$) and q parameter (see Fig. 1B-1E).

2.5 We fit differential Eq. (14) with mean of the percent price index change real financial market data in order to extract value of parameter a (see Fig. 3).

2.6 Considering Eq. (22) and plotting diffusion coefficient compares with the diffusion coefficient of real market data with their percent price index change distributions (see Fig. 4).

2.7 Plotting Eq. (22) compares with the financial market data (data not shown).

2.8 We discussed each parameter and application to common people such as businessman, investor, entrepreneur, and trader.

Results and discussion

There are 4 important parameters such as Tsallis parameter (q), parameters from Eq. (10) and Eq. (14) (a, b), and diffusion constant (D).

All indices perform the anomalous diffusion in superdiffusive type because q is greater than 1(this value comes from fitting Eq. (5) with percent price change distributions data for t=1 day as shown in Fig. 1). In fact, indices should be superdiffusion because the financial market price indices are dependent on each people decision and interaction among traders. In Fig. 1B-1E , we depict percent price index change in each time interval.

It can be seen that the more the time interval increases, the wider the percent price change distribution performs and the lower the highest point of this distribution shows in all indices.

Parameters a and b from Eq. (14) operate the mean's drift or the fluctuation of average percent price index changes. The values of a and b shown in Table 1 indicate that both differ little from zero. Moreover, the sign of parameter a determines the tendency of mean percent price index change, so that a positive sign shows gradual increase in this mean change as shown in Fig. 3 , and the opposite result for a minus sign.

Diffusion constant sets the percent price index change (see Eq. (9) and Eq. (22)) with time interval evolution and plays a significant role in a time-dependent diffusion coefficient ($DP(x,t)^{1-q}$) modeled by Ito-Langevin process. Therefore, percent price index change dritributions depend only on the most recent probability percent price index change due to a time-dependent diffusion coefficient and Ito-Langevin equation. We also show time interval evolution of diffusion coefficient in Fig 4.

It obviously displays that if time interval increases, diffusion coefficient decreases at the same percent price index change. We also found that the lowest diffusion coefficient increases following the increasing time interval. The results like other indices. The diffusion coefficient can also give the investment risk information for investors and others. In case that the recent percent price index change can predict the possibility percent price index change for next time interval by using the Ito-Langevin process in Eq. (22) which performed the same analysis on market indices data (not shown). This result help investors decision better whether they should invest in each market.

We show how to use table 2 and 3 (some data in only two indices) to explain the tendency of percent price index change in a simple way which does not use the Ito-Langevin process in Eq. (22), which would give more information. Supposing the recent percent price index change equals to -3.33% in last 1 day time interval, diffusion coefficient which equals to 6.20 in SET index. If DJIA price changes -3.33% in last 1 day time interval as well, the diffusion coefficient is 4.28. It tells us that percent price SET index change fluctuates more than percent price DJIA index change according to the risk of investment for next time interval.

Now, we can explain the behavior of financial market dynamics interpreted by the nonextensive Tsallis distributions connected with time evolution according to a nonlinear Fokker-Plank equation underlying Ito-langevin process with a time-dependent diffusion coefficient indicating superdiffusion in all indices. The results well reflect in the interaction among traders in diffusion coefficient term according to the risk of investment that depends on the previous step. We checked on our analysis with the financial percent price indices change data performing closely together as well.

References

Borland, L. (1998). Microscopic dynamics of the nonlinear Fokker-Planck equation: A phenomenological model. *Physical Review E, 57*, 6634-6642.

Bornholdt, S. (2001). Expectation bubbles in a spin model of markets: intermittency from frustration across scales. *World Scientific Publishing Company, 12*, 667-674.

Chowdhury, D., & Stau_erb, D. (1999). A generalized spin model of financial markets. *The European Physical Journal B, 8*, 477-482.

Gardiner C.W. 1997, Handbook of Stochastic Methods, second ed. Springer, Berlin.

Mantegna R.N., Stanley H.E., 2000. An Introduction to Econophysics: Correlations and Complexity in Finance, Cambridge University Press, England.

Michael, F., & Johnson, M.D. (2003). Financial market dynamics. *Physica A, 320*, 525 – 534.

Plastino, A. R., & Plastino, A. (1995). Non-extensive statistical mechanics and generalized Fokker-Planck equation. *Physica A, 222*, 347-354.

Tsallis, C. (1988). Possible generalization of Boltzmann-Gibbs statistics. *Journal of Statistical Physics, 52*, 479-487.

Yahoo Finance url: http://finance.yahoo.com.

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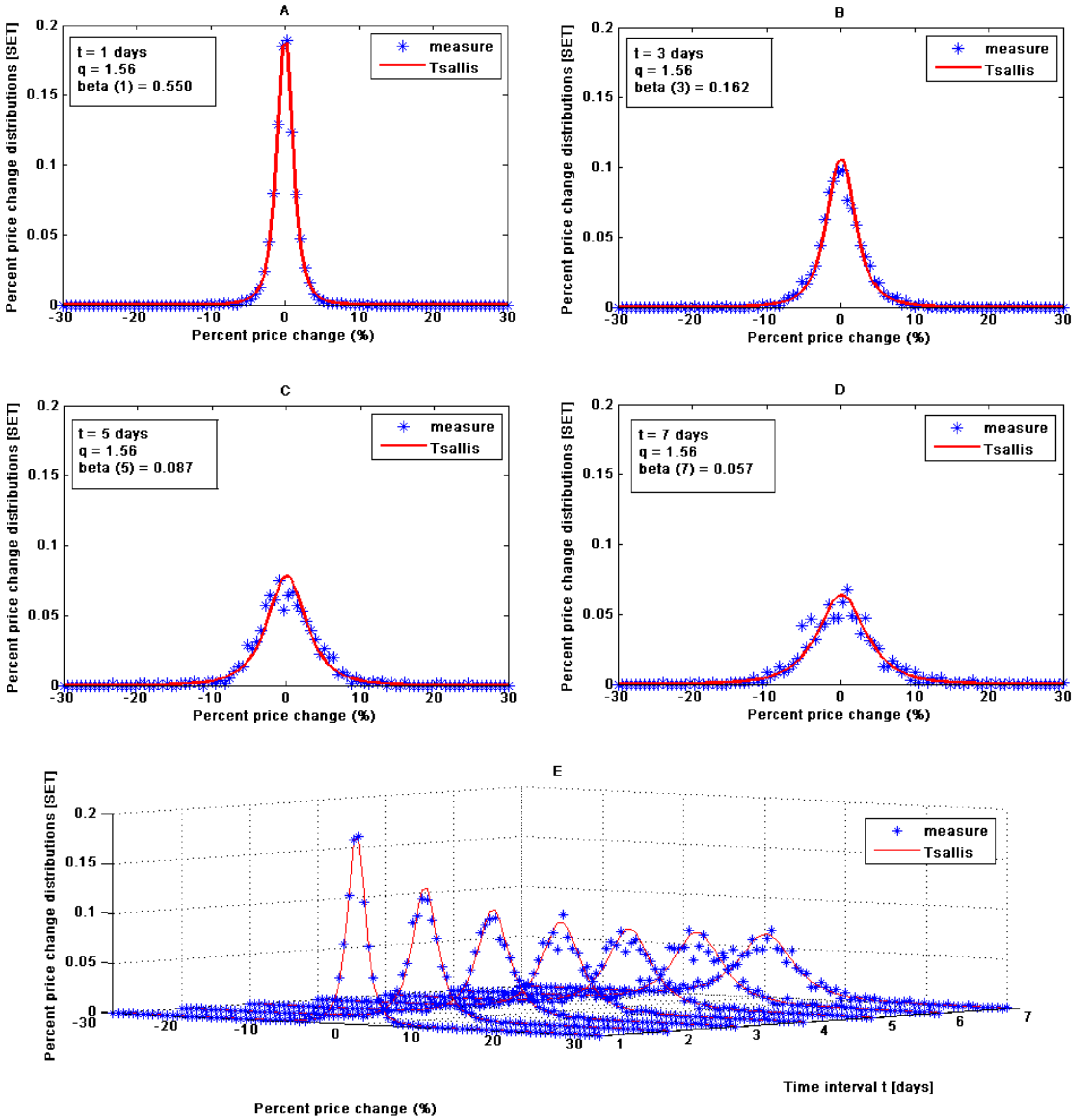

A
t = 1 days
q = 1.56
beta (1) = 0.550
B
t = 3 days
q = 1.56
beta (3) = 0.162
C
t = 5 days
q = 1.56
beta (5) = 0.087
D
t = 7 days
q = 1.56
beta (7) = 0.057
E
measure
Tsallis
Percent price change distributions [SET]
Percent price change (%)
Time interval t [days]

Fig. 1

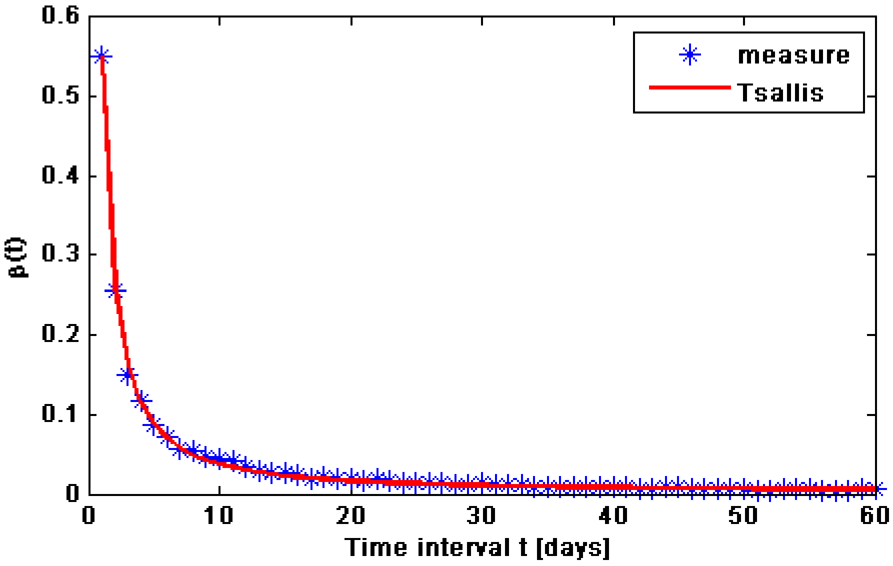
measure
Tsallis
β(t)
Time interval t [days]

Fig. 2

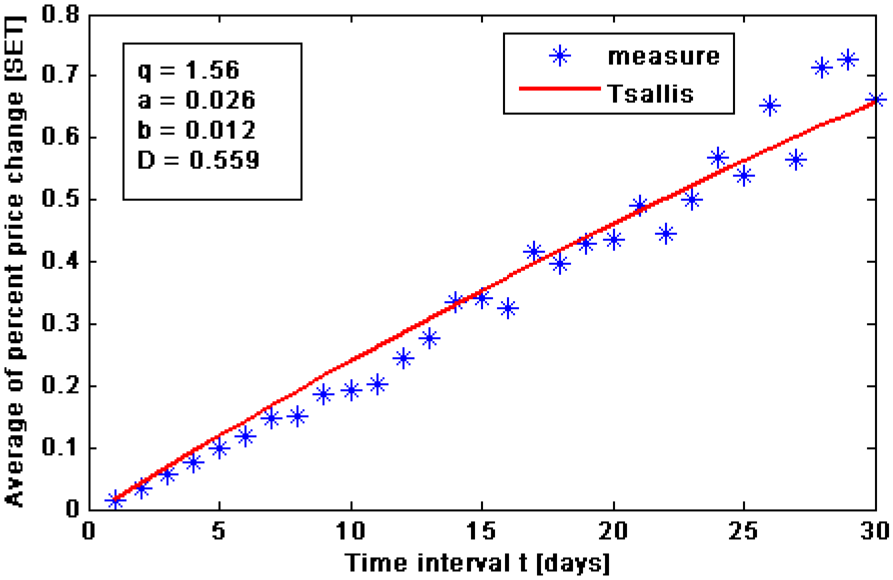
q = 1.56
a = 0.026
b = 0.012
D = 0.559
measure
Tsallis
Average of percent price change [SET]
Time interval t [days]

Fig. 3

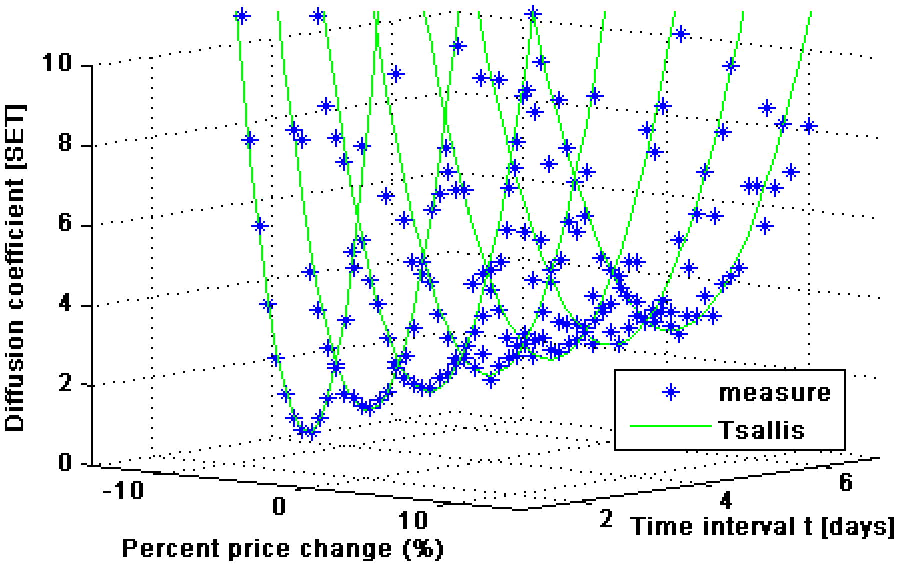
Diffusion coefficient [SET]
10
8
6
4
2
0
-10
0
10
Percent price change (%)
2
4
6
Time interval t [days]
measure
Tsallis

Fig. 4

List of Tables

Table 1

Parameter	America				Europe	Asia			
	NYSE	DJIA	S&P 500	NASDAQ	FTSE 100	NIKKEI 225	HANG SENG	Straits Times	SET
q	1.69	1.60	1.68	1.66	1.40	1.53	1.73	1.59	1.56
a	-0.025	-0.026	-0.022	-0.024	-0.015	0.048	-0.031	0.003	0.026
b	0.020	0.017	0.019	0.016	0.015	0.027	0.023	0.014	0.012
D	0.186	0.194	0.193	0.333	0.243	0.602	0.415	0.362	0.559

Table 2

Diffusion coefficient [SET]		Percent price change (%)																	
		-5.15	-4.55	-3.94	-3.33	-2.73	-2.12	-1.52	-0.91	-0.30	0.30	0.91	1.52	2.12	2.73	3.33	3.94	4.55	5.15
Time interval (days)	1.00	12.84	10.31	8.10	6.20	4.62	3.35	2.40	1.76	1.44	1.43	1.73	2.36	3.29	4.54	6.11	7.99	10.19	12.70
	2.00	8.45	6.97	5.67	4.56	3.63	2.88	2.32	1.94	1.74	1.73	1.91	2.27	2.81	3.53	4.44	5.54	6.82	8.28
	3.00	6.78	5.73	4.80	4.00	3.34	2.80	2.40	2.12	1.98	1.97	2.09	2.34	2.72	3.23	3.87	4.64	5.54	6.57
	4.00	5.94	5.11	4.39	3.76	3.24	2.83	2.51	2.29	2.18	2.17	2.25	2.45	2.74	3.13	3.63	4.22	4.92	5.72
	5.00	5.46	4.78	4.18	3.67	3.24	2.89	2.63	2.44	2.35	2.33	2.40	2.56	2.79	3.11	3.52	4.00	4.57	5.23
	6.00	5.16	4.58	4.07	3.63	3.26	2.96	2.74	2.58	2.50	2.48	2.54	2.67	2.87	3.14	3.48	3.89	4.37	4.92
	7.00	4.97	4.46	4.02	3.63	3.31	3.05	2.85	2.71	2.63	2.62	2.66	2.77	2.94	3.17	3.47	3.82	4.24	4.72

Table 3

Diffusion coefficient [DJIA]		Percent price change (%)																	
		-5.15	-4.55	-3.94	-3.33	-2.73	-2.12	-1.52	-0.91	-0.30	0.30	0.91	1.52	2.12	2.73	3.33	3.94	4.55	5.15
Time interval (days)	1.00	9.73	7.65	5.84	4.28	2.98	1.95	1.18	0.66	0.41	0.42	0.69	1.22	2.01	3.06	4.37	5.95	7.78	9.88
	2.00	6.78	5.37	4.13	3.08	2.20	1.50	0.98	0.63	0.47	0.48	0.67	1.04	1.58	2.31	3.21	4.29	5.54	6.98
	3.00	5.29	4.22	3.28	2.48	1.82	1.29	0.90	0.64	0.52	0.53	0.68	0.97	1.39	1.94	2.64	3.47	4.43	5.53
	4.00	4.41	3.54	2.79	2.14	1.60	1.18	0.86	0.65	0.56	0.57	0.70	0.93	1.28	1.74	2.30	2.98	3.77	4.66
	5.00	3.83	3.10	2.46	1.92	1.47	1.11	0.85	0.68	0.60	0.61	0.72	0.93	1.22	1.61	2.09	2.67	3.33	4.09
	6.00	3.41	2.78	2.23	1.76	1.38	1.07	0.84	0.70	0.63	0.65	0.75	0.93	1.19	1.53	1.95	2.45	3.03	3.70
	7.00	3.10	2.55	2.06	1.65	1.31	1.04	0.85	0.72	0.67	0.68	0.77	0.93	1.17	1.47	1.85	2.30	2.82	3.41