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MARKET CROWD'S TRADING CONDITIONING AND ITS MEASUREMENT

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ABSTRACT

In this paper, we study market crowd's learning and psychological behavior by correlation analysis, using high frequency data in China stock market. We introduce a notion of trading conditioning for the first time in terms of operant conditioning in psychology and use transaction volume probability in a transaction volume-price probability wave equation to measure the intensity of market crowd's trading conditioning. We find that there is, in general, significant positive correlation between the rate of mean return and the change in the intensity of trading conditioning. They behave notably disposition effect in stock selling and herd behavior in stock buying with expectancy on return. Specifically, "the herd" have significant stronger expectancy on price momentum than its reversal. Second, there is also a significant negative correlation between them in a subdivided term. We explain their trading behavior by conditioning.

Key words: behavioral finance, econophysics, crowd's behavior, trading conditioning, transaction volume-price probability wave

JEL Classifications: G12, D03, D83

1. INTRODUCTION

Shiller (2006) summarized that it seem experience two distinct revolutions in the history of financial theory over the last half century. The first was the neoclassical finance beginning in the 1960s, and the second was the behavioral finance emerging around 1980.

The neoclassical financial theory is based on three assumptions: (1) price volatility behaves independent and random (Bachelier, 1900; Samuelson, 1965); (2) investors are rational and make decision in terms of maximizing expected utility (Samuelson, 1937; Von Neumann and Morgenstern, 1944); and (3) market responding to information is efficient (Fama, 1970). Of them, the expected utility maximum hypothesis that represents people as maximizing the present value of utility subject to a present-value budget constraint solves extreme value problem on utility function by variation method and plays an very important role in deriving a series of normative mathematical financial models, a big span from macro to micro and from descriptive to quantitative in finance research.

However, neoclassical financial theory cannot explain reasonably a variety of anomalies in financial market, for example, leptokurtosis (more peaked and heavy tailed) and cluster in return distribution (Mandelbrot, 1963), i.e., scaling behavior that appears very large and very small return from time to time (Mantegna and Stanley, 1995), significantly different from normal distribution in independent random walk. Other examples are excessive price volatility, financial bubble, market over- and under-reaction, excess trading (volume), and disposition effect and herd behavior in decision making etc. With these anomalies, advocators use a series of ARCH models (Engle, 1982; Bollerslev, 1986) to modify return distribution, a three-factor model to identify common risk factors for average return on stocks (Fama and French, 1993), and incomplete information to explain (Friedman, 1979; Merton, 1987). The traditional dominant theory completely ignores both market crowd interaction and their emotional impact on the market, and, not surprisingly, is unable to deal with financial crisis, a “small” probability event that jeopardizes economy largely and widely in a county, region, and even globe, for examples, financial bubble burst in Japan in the early of 1990s, financial crisis storm in Southeast Asian countries in 1997, and the most spectacular financial crash in globe in 2008.

Therefore, many scholars are skeptical of it even at its early stage (Kahneman and Tversky, 1972; Tversky and Kahneman, 1973, 1974; and Shiller, 1981), incorporate human psychological and interacting behavior into financial market research (DeBondt and Thaler, 1985), and bring the second behavioral revolution. Sewell (2008) defined that behavioral finance is the study of the influence of psychology on the behavior of financial practitioners and the subsequent effect on markets. It helps explain why and how markets might be inefficient.

Today, behavioral finance has made prominent advance in two aspects (Barberis and Thaler, 2003). One of the biggest successes is a series of theoretical papers showing that in an economy where rational and irrational traders interact, irrationality can have a substantial and long-lived impact on prices (De Long at et., 1990; Shleifer and Vishny, 1997). The other is the extensive experimental evidence complied by cognitive psychologists on the biases that arise when people form beliefs, and on people’s preferences, or on how they make decisions, given their beliefs. It is how people exactly deviate from homogeneous and complex expected utility maximum rule (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992).

Behavioral finance has faults at this beginning stage. First of all, because the expected utility maximum hypothesis and the normative models derived from it are inconsistent with empirical results and facts, there has been an explosion of work on so-called non-Expected Utility theories, for examples, weighted-utility theory (Chew and MacCrimmon, 1979), regret theory (Bell, 1982), time-inconsistent preference model (Loewenstein and Prelec, 1992; Laibson, 1998), and other normative models describing market psychological behavior. These models replace Samuelson's utility function with a similar one and, therefore, cannot get ride of the faultiness in his original assumption, difficult to distinguish between them and traditional rational models because of mathematical and predictive similarities (Brav and Heaton, 2002). They end up doing an unsatisfactory job at both normative and descriptive goals (Barberis and Thaler, 2003).

Accordingly, even some of great thinkers and practitioners deny normative model and theory in finance. Tversky and Kahneman (1986) concluded that the normative and the descriptive analyses cannot be reconciled, and no theory of choice can be both normatively adequate and descriptively accurate. Based on the conclusion, Barberis and Thaler (2003) wrote that normative approaches are doomed to failure, because people routinely make choices that are simply impossible to justify on normative grounds in that they violate dominance or invariance. Soros (1987, 2010a) contended that economic phenomena can not be predicted by universally valid laws and modeled by a derivable equation like that in theoretical physics because the thinking of the participants introduces an element of uncertainty into the course of events in social science which is absent in natural phenomena. So, it is impossible for economics to become a science…

Second, behavioral finance models typically capture something about investors' beliefs, or their preference, or the limits to arbitrage, but not all of three; Third, there are obviously competing behavioral explanations for some of the empirical facts at present (Barberis and Thaler, 2003). Forth, behavioral finance should have a unified paradigm and theory which includes two extremely conditional behaviors, both rational and irrational, respectively, to strengthen its explanation (Dong, 2009).

Natural science advances often in spiral, so does social science. Inquisitive mind never stops exploring, studying, and discovering financial market behavior by critically logical and normative deduction.

We observe that there have been main trends and new characteristics in financial studies in the past 10 years. They are summarized as: from macro and qualitative description to micro and quantitative analysis using high frequency data; from homogeneous rational decision making, price independent random walk, and critical mathematical method to heterogeneous bounded rational choice, market crowd interacting and coherent probability wave, and multidisciplinary study; and from price behavior alone to trading volume, the joint behavior of trading volume and price, and psychological behavioral implication in the volume. We will ride on the main trends and undertake both theoretical and empirical study on behavioral finance using a transaction volume-price probability wave equation that can describe market crowd interaction and coherence.

There are many factors that could influence stock price and its volatility, for example, management in listing company, macro economy, news announcement, and psychological behavior etc. They impact on price more or less if and only if there is trading (volume), indirect action on it through trading (volume). Therefore, the relation between price volatility and all of the factors is credited to that between price volatility and trading (volume).

Studied the relation between price volatility and accumulative trading volume in stock market using econophysics, Shi (2006) established a mathematical expression among transaction (accumulative trading) amount energy, price volatility energy, and transaction (accumulative trading) volume distribution energy, i.e. a transaction energy relation hypothesis, because the amount of transaction has constraint on price volatility and transaction volume. Then, he use a variation equation to express a "price first and time first" trading rule in stock market. Its price is paired in terms of "price first and time first", the least trading price volatility principle, in a full competitive and continuous price biding market. It states that actual trading price path is given by a transaction energy equation functional to minimize its wave function with respect to price variations. Based on these two facts, Shi (2006) derived a time-independent transaction volume-price probability wave equation and got two sets of analytical transaction volume distribution function over a trading price range (abbreviated as a "the volume distribution function," if not specified). One is the abstract of zero-order Bessel eigenfunction if the sum of momentum force (action force) and supply-demand restoring force is equal to an eigenvalue constant over a trading price range, i.e. the sum of momentum force, restoring force, and interaction force is equal to zero, and the interaction force is equal to a constant. In this case, there are coherence and stationary equilibrium in stock market. The other is the abstract of a multi-order eigenfunction which includes an exponent eigenfunction if supply-demand restoring force is equal to a constant and is direct proportional to transaction energy constant over a trading price range (interaction force is equal to zero). By empiric test, he demonstrated the equation and the volume distribution functions validity at this early stage. The volume-price behavior resembles a probability wave.

The volume distribution functions can describe not only leptokurtosis, scaling, and cluster behavior in return distribution when market crowd are interacted, which is consistent with empirical results, but also exponent and uniform behavior when they are independent and random in a particular case. In addition, the transaction volume probability is used to describe the intensity of price volatility and the eigenvalues are observable and measurable in the volume distribution functions. They can be tested by high frequency data. For example, the leptokurtosis, scaling, and cluster behaviors in the volume distribution are the consequence of market crowd coherence in stock market. The larger the eigenvalue is, the stronger the coherence is in the market, and the more significant the behaviors. Otherwise, the smaller the eigenvalue is, the less significant the behaviors. They are closer to a normal distribution[①].

The transaction volume-price probability wave equation can describe two extremely conditional behaviors, both rational and irrational, respectively. If supply-demand trading price is always consistent with fundamental value, market crowd behavior is rational, for example, bond exchange in terms of its net present value. If supply-demand trading price is totally inconsistent with fundamental value, market crowd behavior is irrational, for example, some speculative option trading when it has no any value at all. Obviously, market crowd behave between them in most scenarios. They are bounded rational. Unlike the neoclassical finance theory it assumed arbitrarily that investors are rational and trade in terms of homogeneously maximizing expected utility, the transaction volume-price probability wave theory justifies whether investors are rational or irrational in terms of relationship between security trading price and its fundamental value, allowing heterogeneous decision making for everyone.

Therefore, we use the "price first and time first" rule—the least price volatility principle, no

[①]According to the law of large number in probability and statistics, if total transaction volume is much greater than the volume of every trading, then, transaction volume probability is approximately equal to trading frequency.

more the expected utility maximum dogma that is inconsistent with reality, when we study the volume-price behavior normatively in stock market. By calculus of variation, we incorporate a variety of factors that could influence price volatility by trading (volume) into a unified and normative transaction volume-price probability wave equation and have made new advance in econophysics.

Based on aforementioned achievement, we attempt to study behavioral finance further, annotate market crowd's psychological behavior in transaction volume probability in the transaction volume-price probability wave equation, and use the volume probability to measure their psychological behavior. It is a key for us to study market crowd's psychological behavior by price volatility and return (reinforcement and punishment).

Soros (1987, 2010b) studied reflexivity from abstract philosophical thinking at beginning, gradually found that it is correlatied with stock price, and propounded a descriptive reflexivity theory to investigate interaction between stock fundamental value and its trading price. It is that there is a feedback loop between stock fundamental value and its trading price by participants' cognition and decision making, the third reinforcement (punishment) in operant conditioning (Coon and Mitterer, 2007). He makes deep insights on that it is the best place for us to study reflexivity in stock market.

In this paper, we introduce a notion of trading conditioning for the first time in terms of classical conditioning (conditioned reflex) in physiology and operant conditioning in psychology, use transaction volume probability in the probability wave equation to represent the intensity of market crowd's trading conditioning, and annotate their psychological behavior in the volume probability. We study their learning and psychological behavior in stock market by analyzing correlation between the rate of price volatility mean return (abbreviated as "the rate of mean return" if not specified) and the change in the intensity of trading conditioning in terms of a set of equations, using every trading high frequency data in China stock market from its bubble growth, burst, and shrink until market reversal again, a whole course that is paralleled with the collapse of sub-prime bubble in the United States. We find that there is, in general, significant positive correlation between them in any two consecutive trading days. It evidences that the rate of mean return significantly changes trading frequency and has a trading conditioning reinforcement or punishment value for market crowd. They behave notably disposition effect in selling and herd behavior in buying with expectancy on return in stock market. Specifically, "the herd" have significant stronger expectancy on price momentum than its reversal. Second, there is also a significant negative correlation in a subdivided time interval, and third, there is insignificant positive correlation in two terms right before and just after bubble burst. We explain market crowd trading behavior by conditioning.

The remainder of the paper is organized as follows. Section 2 is relevant literature reviews on classical conditioning, operant conditioning, and psychological behavioral study on excessive trading (volume); section 3 presents briefly the transaction volume-price probability wave equation and its analytical transaction volume distribution functions; in section 4, we introduce a notion of trading conditioning, find a way to measure the intensity of trading conditioning, annotate market crowd psychological behavior in transaction volume probability in the probability wave equation, and explain dynamic mechanism and principal for the relation between the rate of mean return and the change in the intensity of trading conditioning in any two consecutive trading days in terms of a set of equations; section 5 tests transaction volume distribution by the volume distribution model and analyzes correlation between the rate of mean return and the change in the intensity of trading conditioning in any two consecutive trading days, using Huaxia SSE 50ETF every trading high frequency data; section 6 discusses empirical results, which are: 1) stationary equilibrium theory in stock market, its psychological explanation, and validity on transaction volume-price probability wave equation; 2) a reinforcement or punishment value for market crowd by the rate of mean return, and their learning and psychological behavior; and 3) potential application. Final are summaries and conclusions.

2. LITERATURES

In order to annotate market crowd's psychological behavior in transaction volume probability in the volume-price probability wave equation, we introduce a notion of trading conditioning for the first time in terms of classical conditioning, operant conditioning, and existing behavioral finance.

Pavlov (1904), a Russian physiologist and Nobel laureate in physiology or medicine in 1904, proposed conditioned reflex (classical conditioning) for the first time when studied dog's behavior using the animal saliva volume to measure the intensity of conditioned reflex. Conditioned reflex is a physiological response to expect that a physiological unconditioned stimulus will follow whenever a conditioned stimulus is present.

Thorndike (1913) is a pioneer in operant conditioning study. Later, Skinner (1938) invented a conditioning chamber, called as a Skinner box, to study it continuously, and found that a rat settles into a smooth pattern of frequent bar pressing, after it gets foods (reinforcement) several times by doing so. Today, psychologists define an operant reinforcement as any event that follows a response and increases its probability.

Like classical conditioning, operant learning is also based on information and expectancies. Operant conditioning is that in the presence of a discriminative stimulus the reinforcement will occur if and only if the operant response occurs (Dragoi, 1997), or operant response occurs in the presence of certain stimuli and is always followed by certain consequences (Irons and Buskist, 2008). Skinner regarded the relation among discriminative stimulus, operant, and reinforcement as a three-term contingency. Pierce and Cheney (2004) defined operant conditioning as that a discriminative stimulus sets the occasion for operant behavior, which is followed by a consequence. In operant conditioning, reinforcement is used to alter the frequency of responses. Reinforcement produces very high operant response rate and tremendous resistance to extinction if it follows the uncertain number of operant times (variable ratio) and time interval (variable interval) (Coon, 2007).

Intra-cranial stimulation (ICS) involves direct stimulation of "pleasure centers" in brain. It is one of the most unusual and powerful reinforcement (Olds and Fobes, 1981). Some rats press a bar thousands of times per hour, much higher than the frequency of bar pressing for foods, to obtain the stimulation in experiment, completely ignoring food, water, and sex in favor of the bar pressing.

Coon (2007) classified operant reinforcement into three categories: primary reinforcement, secondary reinforcement, and feedback. Primary reinforcement is natural, or unlearned. They are usually rooted in biology and produce comfort, end discomfort, or fill an immediate physical need. Money, praise, approval, affection, and similar rewards, all serve as learned or secondary reinforcement. Secondary reinforcement that can be exchanged for primary reinforcement gain their value more directly. Printed money obviously has little or no value of its own, neither eaten nor drunk. However, it can be exchanged for foods and services, and perhaps is the most important source of economic reinforcement or conditioned reinforcement (Pierce and Cheney, 2004). For example, chimpanzees were taught to work for tokens in research (Cowles, 1937). Feedback is a process that an operator makes an (input) adjustment on his behavior after receiving the consequence of his response (output).

Trading conditioning is a kind of operant conditioning. Its discriminative stimulus is information on price volatility and return, its operant class is trading, and its reinforcement or punishment are return, in which positive return (money making) is reinforcement while negative return (money losing) is punishment. Unlike the operant conditioning that mainly studies animal behavior previously, e.g. dog, rattle, chimpanzee, and bee etc., trading conditioning is used to study humans' learning and psychological behavior in stock market, a major consideration in this paper.

It has been long that literature in finance focuses much more attention on price and return and less on trading volume, even completely ignoring it. In the past 10 years, however, there is a changing trend that academics have increasing minds on the information contained in trading volume. In neoclassical finance framework, Lo and Wang (2006) derived an intertemporal asset pricing model of multiple assets in the spirit of ICAPM (Merton, 1973) and explored its implications for trading volume and asset returns. In newly emerging behavioral finance, we have associated trading volume with investors' emotion, belief, and preference. Behavioral finance is the study of the influence of psychology on the behavior of financial practitioners and the subsequent effect on markets (Sewell, 2008). It helps explain why and how markets might be inefficient. Lee and Swaminathan (2000) showed that past trading volume provides an important link between "momentum" and "value" strategies and these findings help to reconcile intermediate-horizon "underreaction" and long-horizon "overreaction" effects. Benos (1998) and Odean (1998) hypothesized that overconfidence produces excessive trading in stock market. Odean (1999) explained why those actively trade in financial markets to be more overconfident than the general population by three reasons: selection bias, survivorship bias, and unrealistic belief, and tested overconfident trading hypothesis by investigating whether the trading profits of discount brokerage customers are sufficient to cover their trading costs. Barber and Odean (2000) documented further evidence that active trading results poor performance and is hazardous to wealth. Such irrational behavior can be explained only by overconfidence. Graham et al. (2009) found that investors who feel competent trade more, and thus explained that overconfidence leads to higher trading frequency. Tested the trading volume predictions of formal overconfidence models, Statman et al. (2006) found that share turnover is positively related to lagged returns in both market-wide and individual security for many months. They are interpreted as the evidence of investor overconfidence and the disposition effect. Barber et al. (2009) demonstrated that psychological biases that lead investors to systematically buy stocks with strong recent performance, to refrain from selling stocks with a loss, and to be net buyers of stocks with unusually high trading volume, likely contribute to the evidence that the trading of individuals is highly correlated and persistent. Grinblatt and Keloharju (2001) evidenced that past return, reference price effect, tax-loss selling, and the size of the holding period capital gain or loss etc. affect trading. Overconfident investors and sensation seeking investors trade more frequently (Grinblatt and Keloharju, 2009). Hong and Stein (2007) found that trading volume appears to be an indicator of sentiment. In other words, when prices look to be high relative to fundamental value, disagreement on price is strong, and volume is abnormally high. Thus, they proposed a disagreement volume model which allows speaking directly to a joint behavior between stock price and trading volume.

Whatever an excessive trading (volume) hypothesis is of overconfidence, sensation seeking, and disagreement, it always converges to a point that trading volume reflects the intensity of investors' emotion, belief, and preference. We will use the conclusion in existing behavioral finance to study trading conditioning in this paper.

3. TRANSACTION VOLUME-PRICE PROBABILITY WAVE EQUATION

In this paper, we will annotate market crowd's psychological behavior in transaction volume probability in the volume-price probability wave equation. Therefore, let us have a brief introduction on the equation and its analytical volume distribution functions.

3.1 The Probability Wave Equation and Its Volume Distribution Functions

There are two independent variables, price and transaction volume, in stock market. The amount of transaction is a constrain condition on them. According to classical mechanics (Greenwood, 1977), the degree of freedom is equal to that the number of independent variables minus the number of constrain conditions. The degree of freedom is one in stock market.

Osborne (1977), a pioneer in econophysics, found that price as a function of volume does not exist empirically and explained why the volume is the function of price and this is not invertible. McCauley (2000) proved Osborne's finding mathematically. Therefore, we chose price as an independent variable and transaction volume as a dependent variable in our study.

Studied the transaction volume-price joint behavior, Shi (2006) documented that there exists stationary equilibrium widely in stock market. The stationary equilibrium is defined as: a trading price is volatile to an equilibrium price upward and downward constantly and the equilibrium price jumps from time to time. A stationary equilibrium price is the price to which transaction volume kurtosis is corresponding. It is a dynamic equilibrium.

According to that the amount of transaction M has constraint on transaction volume v and price p,

$$M = pv, \tag{1}$$

Shi (2006) established a relation among transaction amount energy E, transaction volume distribution energy, and price volatility energy $W(p)$, i.e., transaction energy hypothesis, as

$$-E + p\frac{v_t^2}{V} + W(p) = 0, \tag{2}$$

and

$$E = pv_{tt}. \tag{3}$$

where v_t and v_{tt} are transaction momentum (action momentum) and transaction impulse, respectively; V is total transaction volume over a trading price range, and $W(p)$ is price volatility energy or a stationary equilibrium term.

In terms of "price first and time first" trading rule, a trading price least volatility principle, Shi wrote

$$\delta\int F(p,\psi)dp = 0, \tag{4}$$

where

$$F(p,\psi) = (W - E)\psi^* \psi + \frac{B^2}{V} p\left(\frac{d\psi^*}{dp}\right)\left(\frac{d\psi}{dp}\right). \tag{5}$$

From equation (2) and (4), Shi derived a transaction volume-price probability wave equation,

$$\frac{B^2}{V}\left(p\frac{d^2\psi}{dp^2} + \frac{d\psi}{dp}\right) + [E - W(p)]\psi = 0. \tag{6}$$

If market crowd accept a price most in given informational and bounded rational decision making scenarios, then, selling volume will increase and buying volume will decrease when trading price is higher than the price. Supply quantity will be more than demand quantity. Trading price will drop. On the other hand, when trading price is lower than the price, selling volume will decrease and buying volume will increase. Supply quantity will be fewer than demand quantity. Trading price will rise. The most acceptable price is a stationary equilibrium price in the time interval. The equilibrium item that drives price upward and downward can be expressed by

$$W(p) = A(p - p_0) \approx A(p - \overline{p}), \tag{7}$$

where p_0 is a stationary equilibrium price, $\overline{p}$ is a volume weight price mean value, $-A$ is supply-demand restoring force in which the minus sign indicates that the force is always toward to equilibrium price.

Substituting equation (7) into equation (6) and using natural boundary conditions

$$\psi(0) = 0,\ \psi(p_0) < \infty, \text{ and } \psi(+\infty) \to 0, \tag{8}$$

we obtain two sets of analytical solutions. One is a set of zero-order Bessel eigenfunctions. They are written by

$$\psi_m(p) = C_m J_0[\omega_m(p - p_0)], \qquad (m = 0,1,2,\cdots), \tag{9}$$

and

$$\omega_m^2 = v_{tt} - A = \frac{v}{V}v_{tt}, \qquad (\omega_m > 0) \qquad (m = 0,1,2,\cdots) \tag{10}$$

where ω_m are a set of eigenvalue constants, C_m are normalized constants, $J_0[\omega_m(p - p_0)]$ are zero-order Bessel eigenfunctions, $-\frac{v}{V}v_{tt}$ is interaction force in which the minus sign shows that the force is always toward to a stationary equilibrium price. The absolute of functions (9), $|\psi_m(p)|$, is transaction volume probability at price p over a trading price range (reference to figure 1). Here, there is coherence that the sum of momentum force v_{tt} and restoring force $-A$ is equal to an eigenvalue constant over a trading price range. In another word, the sum of momentum force, restoring force, and interaction force is equal to zero, and the interaction force is equal to a constant over a trading price range. The trading system is stationary equilibrium.

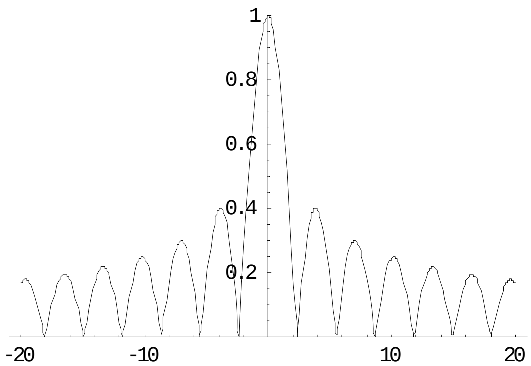


Figure 1: The absolute of zero-order Bessel eigenfunctions

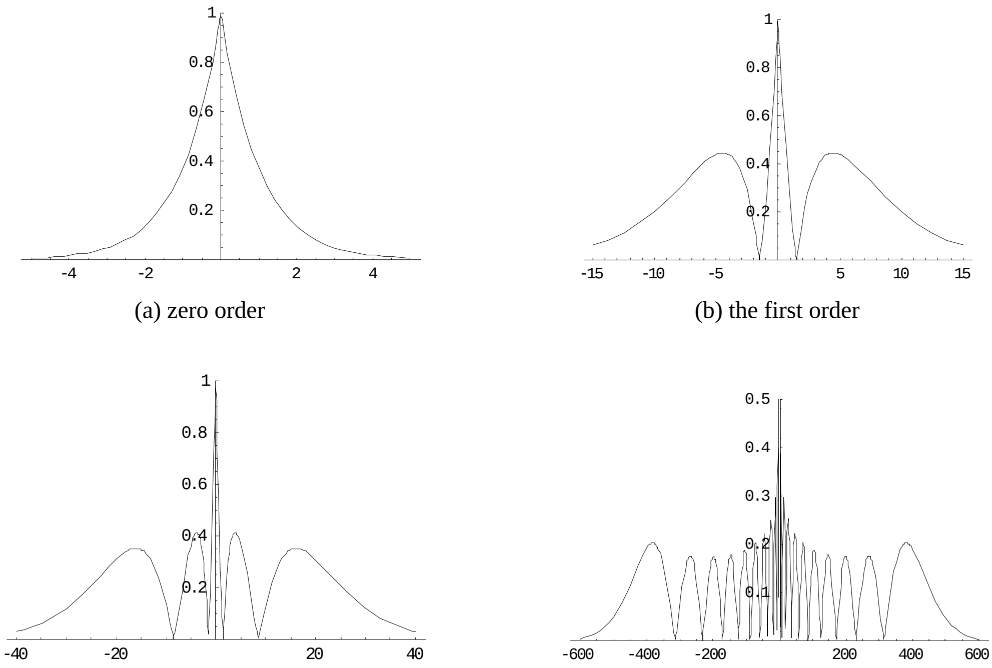


Figure 2: The absolute of the multi-order eigenfunctions

The other is a set of multi-order eigenfunctions. They are as follows

$$\psi_m(p) = C_m e^{-\sqrt{A_m}|p-p_0|} \cdot F\left(-n,1,2\sqrt{A_m}|p-p_0|\right), \quad (n,m = 0,1,2,\cdots) \quad (11)$$

and

$$\sqrt{A_m} = \frac{E_m}{1+2n} = const. > 0, \qquad (n,m = 0,1,2,\cdots) \quad (12)$$

where $F\left(-n,1,2\sqrt{A_m}|p-p_0|\right)$ are a set of n order confluent hypergeometric eigenfunctions or the first Kummer's eigenfunctions (reference to figure 2). Here, the magnitude of the restoring force is an eigenvalue constant and is direct proportional to transaction amount energy over a trading price range (the interaction force is equal to zero). The absolute of functions (11), $\left|\psi_m(p)\right|$, is transaction volume probability, too (reference to figure 2).

X-coordinate is price, y-coordinate is transaction volume, and origin is a stationary equilibrium price in figure 1 and 2,

There are several different characteristics between figure 1 and figure 2. First, the volume distribution is attenuated in wave with price departing from its equilibrium price in figure 1, while it is almost uniform in wave except for the zero-order distribution that is exponent in figure 2. The higher order the volume distribution function is, the better uniform the volume distributes over a trading price range. Second, unlike a closed distribution in figure 2, the distribution is open in tail in figure 1. It can describe a pulse of trading which pairs far away from a stationary equilibrium price. Third, the magnitude of eigenvalue in figure 2 is about two orders of magnitude larger than that in figure 1 except for exponent distribution.

However, both can fit exponent distribution very well.

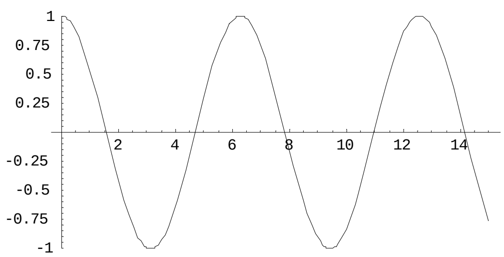
3.2 Eigenvalue, Probability Wave, and Stationary Equilibrium

Let us have brief explanation on eigenvalue and probability wave for readers who are not majored in physics.

An eigenvalue is different from a parameter in a distribution function in that an eigenvalue is observable and measurable. Thus, it is possible for us to understand an eigenfunction (a distribution function with eigenvalue) mechanism and test it validity. For example, there are two sets of eigenvalues in the volume distribution. One is tested and measured by equation (10). The volume distribution is the consequence of a kind of coherence that the sum of momentum force and supply-demand restoring force is equal to an eigenvalue constant over a trading price range. It can describe market crowd interaction and coherence behavior in stock market. The other is tested and measured by equation (12). The volume distribution is the result of that the supply-demand restoring force is direct proportional to transaction energy and equal to an eigenvalue constant over a trading price range. It can describe exponent and uniform random distribution behavior in stock market. Here, market crowd interaction force is equal to zero. However, we usually use a distribution function with a parameter to analyze statistic and uncertain behavior in an event in a great majority of scenarios if we know its distribution in a certain extent, but do not understand its mechanism. In this approximate analysis, we do not clear what information is contained in the parameter.

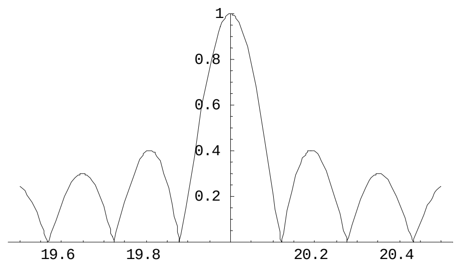
Probability wave is a kind of wave in which we use volume probability rather than its amplitude to describe its intensity. For example, we use transaction volume probability rather than the amplitude of price volatility to describe price volatility intensity in a transaction volume-price probability wave. Now, let us contrast and compare a probability wave with a classical wave. First, x-coordinate represents price and y-coordinate stands for accumulative trading volume probability in a probability wave in stock market, while x-coordinate is time and y-coordinate is its amplitude in a classical wave. Second, we use the volume probability to describe its intensity in a probability wave, whereas we use the amplitude to describe its intensity in a classical wave. Third, the intensity of a probability wave is equal to and larger than zero, while that of a classical wave can be positive, negative, and zero. Forth, the intensity of a probability wave shows a wave change with an independent variable. Large amplitude and volatility is not equal to strong intensity in a probability wave. In a classical wave, the larger the amplitude is, the stronger the intensity does be. Fifth, probability wave is the consequence of coherence among many individual trading volume in stock market or many-bodies in physics. It can not exist independently. Classical wave can exist independently. Sixth, we have not yet found that there is a periodicity and time cycle in a probability wave. It implies that there is uncertain for price prediction. In classical wave, we can measure the speed of a wave. Thus, there is periodicity and time cycle.

However, there are some commons in both. For examples, there exists coherence in both probability wave and classical wave, and there is repeat change and swing reference to an equilibrium point (see figure 3).



$$y(t) = y_0 \cos(kx - \omega t)$$

(a) Classical wave



$$|\psi_m(p)| = C_m |J_0[\omega_m(p - p_0)]|$$

(b) Probability wave

Figure 3: Classical wave and probability wave

Shi (2006) tested the volume-price probability wave equation and stationary equilibrium theory by the absolute of zero-order Bessel eigenfunction regression model, using intraday every trading high frequency data in China stock market. If the volume distribution shows significance over a trading price range, it is interpreted that the volume-price behavior is stationary equilibrium. Stationary equilibrium price is the price to which the volume kurtosis is corresponding. We can identify the equilibrium price from the test report directly. If the volume distribution lacks significance over a trading price range, the volume distribution had more than two of kurtosis over a trading price range. It states that equilibrium price jumps on the sampling trading day. He documented that stationary equilibrium exists widely[①] and the equilibrium price jumps from time to time in stock market[②]. The volume-price probability wave equation and the distributions are valid.

4. PSYCHOLOGICAL ANNOTATION ON THE PROBABILITY WAVE EQUATION

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① Shi (2006) fitted and tested 618 volume distributions over a trading price range. Although there are different in price, price volatility path and direction, they show the same characteristic that accumulative trading volume exhibits kurtosis near the price mean value.

② Unlike Poisson-diffusion jump (Ait-Sahalia, 2004) and Levy jump (Li et al., 2008), stationary equilibrium price jump has its clear mechanism. For example, if there is sudden increasing buying volume to break stationary equilibrium, trading price will be volatile upward and downward from a stationary equilibrium price to another by its jump. We can measure its jump behavior in terms of volume distribution over a trading price range (Shi, 2006).

Let us consider a rigid trading system first in which there is no subjective and cognitive change in trading process. The transaction volume is the same at any same time interval when price is volatile. If stationary equilibrium is broken by incremental buying volume and stationary equilibrium price jumps in the trading system, then, there is one-to-one correspondence between incremental amount of transaction and the equilibrium price jump because total transaction volume is the same in two time intervals fore-and-aft its jump.

In a real financial market, there are a great variety of factors that could influence investors' subjective cognition, supply and demand quantity, and transaction volume. In our previous study, we have already incorporated all of these factors into a unified and normative transaction volume-price probability wave equation. Therefore, it is a key for us to study market crowd cognitive and learning behavior quantitatively by price volatility and return (reinforcement and punishment) how to annotate psychological behavior in transaction volume probability in the equation and measure the intensity of market crowd trading conditioning subject to price volatility and return.

4.1 Trading Conditioning

It is the fact that people have physiological demand for clothes, foods, and services in life. Their physiological response to them is a kind of unconditioned reflex. Printed money obviously has little or no value of its own, neither eaten nor drunk. However, when we associate money, asset, and return with the necessities of life and services tightly through exchange in commodity exchange economy, we have been conditioned and will produce the same physiological response to them, i.e. conditioned reflex or classical conditioning. Pierce and Cheney (2004) wrote that money can be exchanged for foods and services, and perhaps is the most important source of economic and conditioned reinforcement.

Stock holders experience increase or decrease in asset if price rises or drops in stock market. It is classical conditioning if they produce the same physiological response when stimulated by information on price volatility and return as they do for the necessities of life and services, and it is operant conditioning if they trade with expectancy on return (profit or loss) in the future after they analyze, judge, and have decision making in the presence of information on price volatility and return.

The operant conditioning is trading conditioning. Its discriminative stimulus is information on price volatility and return, its operant class is trading, and reinforcement or punishment is return. There are several characteristics in trading conditioning. First, it does not loss reinforcement value as quickly as primary reinforcement does, because money and return is hardly satisfied. Second, return not only has a reinforcement value but also has a punishment value. Positive return, profit, has a reinforcement value (market reward to stock holders) whereas negative return, loss, has a punishment value (market punish to stock holders). People buy if they expect stock price rise. Otherwise, they sell if they expect stock its drop. It takes them to trade with expectancy on return no matter whether price will rise or drop because of trading conditioning. Third, positive return (reinforcement) occurs in an uncertain time interval after an investor holds stock because no time cycle has been known in the behavior of a probability wave so far. It makes tremendous resistance to extinction in trading conditioning, similar to variable interval (VI) schedules in operant conditioning experiment (Coon, 2007). Forth, an investor receives feedback from the consequence of his trading. It could inspire his emotion, influence his judgment, change his expectancy on return, and promote his trading again. For examples, a loss stop seller is likely to buy stock again because he changes his expectancy, stimulated by rising price right after his selling. A speculator might sell his stock for a little profit or even do it in loss in a very short term because of his expectancy change. Therefore, both expectancy on return and its change adjusted by feedback could promote one's trading again in his decision making.

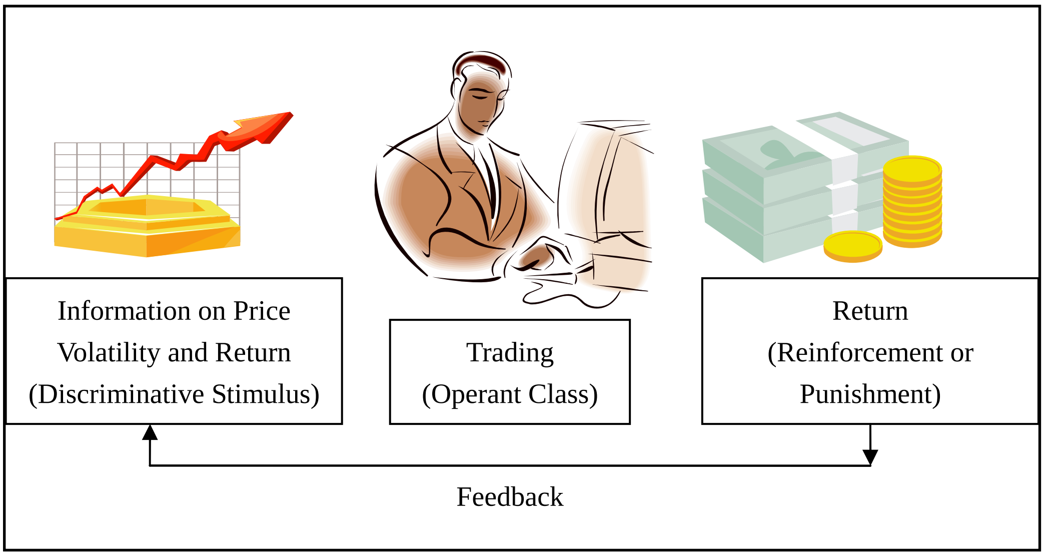

Figure 4: Three-term contingency in trading conditioning

Based on Skinners' three-term contingency and operant conditioning (Dragoi, 1997; Pierce and Cheney, 2004; Irons and Buskist, 2008), we define trading conditioning in stock market as: in the presence of information on price volatility and return (discriminative stimulus), one trades (operant class) with expectancy on return (reinforcement and punishment) in the future after he analyzes, judges, and has decision making; He receives feedback from the consequence of his trading which could inspire his emotion, influence his judgment, change his expectancy on return, and promote his trading again (see figure 4).

In our research, we prefer "discriminative stimulus" to "certain stimulus" because there are a great variety of factors that could influence trading behavior.

4.2 The Intensity of Market Crowd Trading Conditioning

Whereas transaction volume-price probability wave equation describes market crowd interaction and coherence in stock market (see Section 3), we attempt to annotate their psychological behavior in the volume probability in the equation, measure the intensity of market crowd trading conditioning, and study their learning and psychological behavior by price volatility and return. Here, market crowd specify trading crowd in the market. The number of market crowd is measured by the volume of trading rather than the number of participants. A unit of trading volume stands for an individual who might be heterogeneous in decision making. The number of market crowd who made of a fewer institutional investors may be much greater than the number of market crowd who made of a large number of individual investors in an event. A fewer institutional investors could produce much stronger impact on price than many individual investors do (Nofsinger and Sias, 1999).

According to trading conditioning, we know that information on price volatility and return sets an occasion, market crowd trade with expectancy on return. The higher volume the market crowd trade, the higher frequency they do[①]. Thus, They have stronger expectancy on return and show stronger intensity of trading conditioning, and vice versa. The higher the transaction volume is, the larger the buying volume is. Market crowd have stronger expectancy on positive return (reinforcement). The higher the transaction volume is, the larger the selling volume is, too. Market crowd have stronger expectancy on negative return (punishment). Therefore, we can use transaction volume probability to represent the intensity of market crowd trading conditioning.

Pavlov (1904) used saliva volume to measure the intensity of a dog for foods in classical conditioning experiment. Cognitive psychologists used bar pressing frequency to quantify the intensity of a rattle for pleasure in operant conditioning test (Olds and Fobes, 1981). Today, behavioral finance scholars use trading frequency and volume to denote the intensity of investors' emotion, preference, and trading. So, what we use transaction volume probability to measure the intensity of market crowd's trading conditioning in stock market is consistent with the measurement they have done in research.

4.3 The Rate of Mean Return and the Change in the Intensity of Trading Conditioning

We have used transaction volume probability distribution to measure the intensity distribution of market crowd trading conditioning over a trading price range. Here, information on price volatility and return sets an occasion, market crowd trade with expectancy on return (reinforcement and punishment). If we use the amplitude of stationary equilibrium price jump in two consecutive trading days to stand for the rate of mean return, we can use the rate of change in total transaction volume in the two days to quantify the change in the intensity of market crowd trading conditioning subject to the rate of mean return. It is

$$\Delta I = \frac{V' - V}{V}, \tag{13}$$

where ΔI is the change in the intensity of market crowd trading conditioning subject to the rate of mean return, V and V' are total transaction volume on fore-and-aft two trading days, respectively. Obviously, ΔI could be positive, negative, or zero.

Let us illustrate the dynamic mechanism and principle regarding the relation between the rate of mean return and the change in the intensity of market crowd's trading conditioning. According to transaction volume-price probability wave equation (6), if stationary equilibrium price is p_0 on

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[①]According to the law of large number in probability and statistics, if total transaction volume is much greater than the volume of every trading, then, transaction volume probability is approximately equal to trading frequency. In our study, total daily trading volume is about 360,000,000 shares in average. We can use transaction volume probability to represent trading frequency. The higher the transaction volume is at a price, the higher the trading frequency. Although there is difference in information contained in a single large trading volume which is equivalent to total volume of several small trades, transaction volume probability is still approximately equal to trading frequency. The abnormal volume distribution disturbed by large volume trading reveals that stationary equilibrium is easily broken by capital advantage speculators. We will study it in the future.

T trading day, we know that the volume-price behavior is governed by

$$\frac{B^2}{V}\left(p\frac{d^2\psi}{dp^2}+\frac{d\psi}{dp}\right)+\left[E-A(p-p_0)\right]\psi=0\,. \qquad (14)$$

If there is sudden change in supply-demand and transaction impulse v_{tt}' on T+1 trading day, i.e. put a sudden change of transaction amount energy M' into Hamilton in equation (14), then, we have

$$\frac{B^2}{V}\left(p\frac{d^2\psi}{dp^2}+\frac{d\psi}{dp}\right)+\left[E-A(p-p_0)+M'\right]\psi=0\,. \qquad (15)$$

Because the probability wave equation retains its validity after Hamilton is added by a sudden change M', we can simplify equation (15) on T+1 trading day as

$$\frac{B^2}{V'}\left(p\frac{d^2\psi}{dp^2}+\frac{d\psi}{dp}\right)+\left[E'-A'(p-p_0')\right]\psi=0\,, \qquad (16)$$

where $p_0'=p_0+\Delta p$ is a stationary equilibrium price on T+1 trading day, Δp is the amplitude of stationary equilibrium price jump, A' and E' are the magnitude of restoring force and the transaction amount energy on T+1 trading day, respectively. Therefore, the rate of mean return $\bar{r}$ between two consecutive trading days is

$$\bar{r}=\frac{\Delta p}{p_0}\,. \qquad (17)$$

Equation (15) is stationary equilibrium price jump model for fore-and-aft trading day. A set of joint equations (14) and (16), together with equation (3), is a trading conditioning model subject to the rate of mean return. By trading conditioning model, we can calculate the rate of mean return in terms of equation (17) and figure out the change in the intensity of market crowd trading conditioning according to equation (13).

Actually, when we study market crowd's psychological behavior by trading conditioning model, we do not use the joint equations of (14) and (16) to calculate the rate of mean return and the change in the intensity of trading conditioning. We get total transaction volume from every day trading data directly and stationary equilibrium price from test reports using a transaction volume distribution regression model. Then, we figure out the rate of mean return and the change in the intensity of trading conditioning by equations (13) and (17), analyze their correlation, and study market crowd's learning and psychological behavior in decision making.

5. EMPIRICAL TESTS

In this section, we use the amplitude of stationary equilibrium price jump to stand for the rate of mean return and the change of total transaction volume to represent the change in the intensity of trading conditioning in any two consecutive trading days. We choose Huaxia SSE 50ETF (510050) every trading high frequency data, test transaction distribution by the absolute of zero-order Bessel eigenfunction regression model, and get stationary equilibrium prices on every day from test reports. For those that show significance, we can determine a stationary equilibrium price from test reports directly. Otherwise, we choose the volume weight price mean value in our analysis. In this way, we can figure out the rate of mean return in any two consecutive trading days approximately and study the correlations between the rate of mean return and the change in the intensity of trading conditioning.

5.1 Data

Our data is from HF2 database, provided by Harvest Fund Management Co., Ltd. The sampling is from April 2, 2007 to April 10, 2009, in which there are about 740 days and 495 trading days in total, i.e., there are 495 volume distributions in our test.

We process the data in two steps. First, we reserve two places of decimals in price by rounding-off method and add the volume at a corresponding price (original data reserves three places of decimals). Second, transaction volume at a price is divided by total transaction volume over a trading price range. Thus, we acquire transaction volume probability (distribution) over a trading price range on every trading day.

5.2 The Volume Distribution Test and Stationary Equilibrium Price

In stationary equilibrium, we have theoretical transaction volume-price distribution function as

$$|\psi_m(p)| = C_m |J_0[\omega_m(p - p_0)]|, \qquad (m = 0,1,2\cdots) \tag{18}$$

where C_m, ω_m, and p_0 are a normalized constant, an eigenvalue constant and a stationary equilibrium price, respectively. They are three constant coefficients to be determined by its nonlinear regression model,

$$|\psi_m(p_i)| = C_m |J_0[\omega_m(p_i - p_0)]| + \varepsilon_i, \qquad (i = 1,2,3\cdots,n) \tag{19}$$

in which n is the number of prices over a trading price range in a trading day; ε_i is random error subject to $N(0,\sigma^2)$; $|\psi_m(p_i)|$ is an observed transaction volume probability at a price, while $C_m|J_0[\omega_m(p_i - p_0)]|$ is a theoretical transaction volume probability. We use Origin 6.0 Professional software, in which Levenberg-Marquardt nonlinear least square method is used, to fit the volume distribution, get the values of C_m, ω_m and p_0, and determine theoretical volume distribution eigenfunctions (see Figure 6 (a)).

We use F statistic to test significance. The coefficient of determination R^2 is as follow:

$$R^2 = \frac{ESS}{TSS} = \frac{TSS - RSS}{TSS}, \tag{20}$$

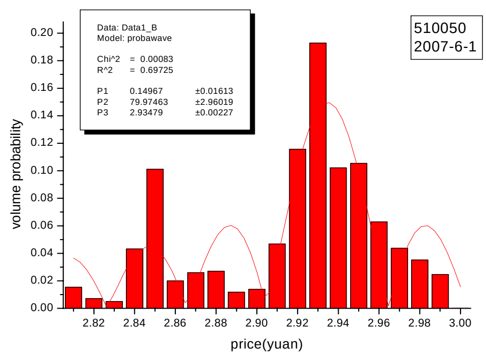
where $ESS = \sum_{i=1}^{n}\left(\hat{Y}_i - \bar{Y}\right)^2$, $RSS = \sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2$, and $TSS = \sum_{i=1}^{n}\left(Y_i - \bar{Y}\right)^2$ are the explained sum of squares, the residual sum of squares, and total sum of squares, respectively. And,

$$F = \frac{ESS/k}{RSS/(n-k-1)} \tag{13}$$

where n and k is sample size and the number of explanatory variables, respectively. If $F > F_{0.05}$ or

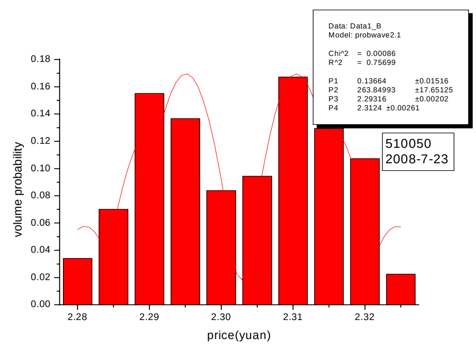
$$R^2 > R_{crit}^2 = \frac{k \cdot F_{0.05}}{k \cdot F_{0.05} + (n-k-1)}, \tag{14}$$

the regression model (11) holds true at 95% significant level. Here, $k = 1$.



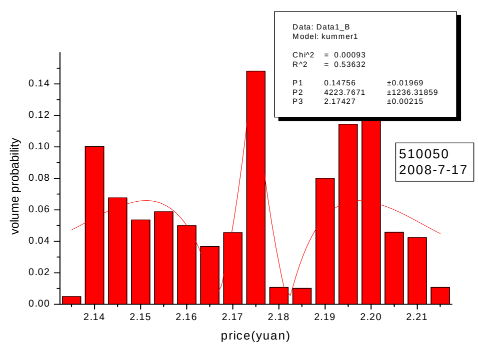
(a) A typical sample is fitted by function (10)

($R^2 = 0.697 > R_{crit}^2 = 0.50$)



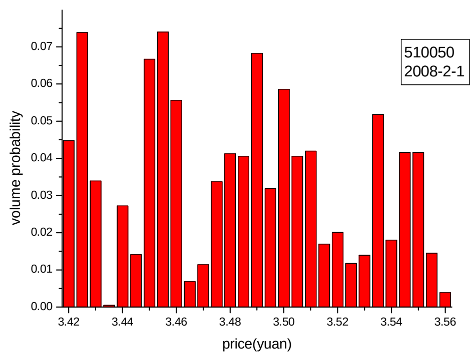
(b) A typical sample is fitted by function (16)

($R_2^2 = 0.757 > R_{2crit}^2 = 0.575$)



(c) A typical sample is fitted by function

($R^2 = 0.536 > R_{crit}^2 = 0.232$)



(d) One of five insignificant samples

Figure 6: The volume distribution test reports in samples [①]

① In figures, P1, P2, and P3 are a normalized constant, an eigenvalue, and a stationary equilibrium price, respectively. P4 is a stationary equilibrium price, too.

Our test results show that 380 out of 495 distributions show significance from April 2, 2007 to April 10, 2009 (76.77%). The remainders (23.23%) lack significance.

There are two notable characteristics among the distributions without significance. First, the number of trading prices is fewer or the sample size is not large enough in prices for statistic test. It is partly credited by previous data process, in which we reserved two places of decimals in price by rounding-off three places in original data. The data process directly results in some information loss in the distributions.

To solve the problem, we add 0.005 in three places of decimals in price and subdivided volume at corresponding prices. As a result, 28 volume distributions show significance, modeled by absolute zero-order Bessel function. Thus, there are total 408 (380+28) volume distributions, around 82.42%, showing significance. We can get a stationary equilibrium price in any one of these distributions from test report.

Second, the remainders of 87 volume distributions show at least two of kurtosis over a trading price range. If abrupt change takes place in supply and demand quantity on a trading day, for example, there is a continuous large buying volume, original stationary equilibrium is broken. Price volatility is going to be adjusted to a new equilibrium price. Trading price is volatile upward and downward from original one to new one. Stationary equilibrium price jumps on day. In this case, the volume distribution function is the linear superposition of function (18), that is,

$$|\psi_m(p)| = \sum_n C_m \left| J_0\left[\omega_{m,n}(p - p_{0n})\right]\right|, \qquad (n = 1,2\cdots) \quad (23)$$

where n is the number of stationary equilibrium prices. We fit them with a double stationary equilibrium price regression model as

$$|\psi_m(p_i)| = \sum_i \sum_{n=1,2} C_m \left| J_0\left[\omega_{m,n}(p_i - p_{0,n})\right]\right| + \varepsilon_i \qquad (i = 1,2\cdots) \quad (24)$$

where $n = 2$.

We test the significance ($R_2^2 > R_{2crit}^2$, here $k = 2$). Of 87 distributions, 59 distributions (11.92% in total) show significance at 95% level (see figure 6 (b)).

For the rest of 28 transaction volume distributions, we fit them by the second set of distribution functions. It is

$$|\psi_m(p)| = C_m e^{-\sqrt{A_m}|p-p_0|} \cdot \left| F\left(-n,1,2\sqrt{A_m}|p-p_0|\right)\right| . \quad (25)$$

In convenience, we choose $n = 1$. It is

$$|\psi_m(p)| = C_m e^{-\sqrt{A_m}|p-p_0|} \cdot \left| F\left(-1,1,2\sqrt{A_m}|p-p_0|\right)\right|$$

$$= C_m e^{-\sqrt{A_m}|p-p_0|} \cdot \left|1 - 2\sqrt{A_m}|p-p_0|\right| . \quad (26)$$

Our test result is that 23 distributions (about 4.65% in total) show significance at 95% level by this regression model (see figure 6 (c)). The rest 5 distributions still lack significance, which show very unstable on the trading days (see figure 6 (d)).

In figure 6 are some typical test results fitted by transaction volume distribution functions (18), (23), and (26), respectively.

5.3 Correlation and Significant Test

There are 408, about 82.42% in total, transaction volume distributions that show significance in our test by the absolute of zero-order Bessel eigenfunction regression model. We can get the stationary equilibrium prices from test reports directly. For the rest (87 distributions), we choose the volume weight price mean values. In this way, we can figure out the rate of mean return in any two consecutive trading days and study the correlation between the rate of mean return and the change in the intensity of trading conditioning. Here, the change in the intensity of trading conditioning is approximately equal to the rate of change in total transaction volume in two days. It is determined by equation (13).

Correlation coefficient $r_{X,Y}$ is given by

$$r_{X,Y} = \frac{\operatorname{cov}(X,Y)}{\sigma_X \sigma_Y}, \tag{27}$$

where σ_X and σ_Y are the standard deviations of variable X and Y, $\operatorname{cov}(X,Y)$ is covariance. We use t-statistic to test significance. If we have

$$H_0 : \rho = 0, \quad H_1 : \rho \neq 0, \tag{28}$$

then,

$$t = \frac{|r-\rho|}{\sqrt{(1-r^2)/(n-2)}}, \tag{29}$$

where r and n are correlation coefficient and sample size, respectively. For $\alpha = 0.05$, if $t > t_{crit} = t_{0.05/2}(n-2)$, then, original hypothesis is rejected. Correlation coefficient is significant not equal to zero at 95% level.

We subdivide two year high frequency data into 5 time intervals from bubble growth, burst, and shrink until market reversal again in China, a whole course that is paralleled with the collapse of sub-prime bubble that originated in the United States in 2008 and set off a chain reaction worldwide (reference to Table). The first is from April 2, 2007 (SSE Composite Index at 3252.59 points) to June 29, 2007 (SSE Composite Index at 3820.70 points), the first half before bubble burst in China. The second is from July 2, 2007 (SSE Composite Index at 3836.29 points) to October 31, 2007 (SSE Composite Index at 5954.77 points), the second half before bubble burst in China. The third is from November 1, 2007 (SSE Composite Index at 5954.77 points) to April 40, 2008 (SSE Composite Index at 3693.11 points), the first half after bubble burst in China. The forth is from May 5, 2008 (SSE Composite Index at 3761.01 points) to October 31, 2008 (SSE Composite Index at 1728.79 points), the second half after bubble burst in China. And the last is from November 3, 2008 (SSE Composite Index at 1719.77 points) to April 10, 2009 (SSE Composite Index at 2444.23 points), price reversal time interval after a year deep drop (reference to Table).

There is advantage for this because we can study market crowd cognitive and learning behavioral change with environment (and time). In our test, we use Eviews 6.0.

Table: Test Reports on Correlation and Its Significance

	Terms	Number of Distributions	SSE Composite Index (1A0001)	Correlation Coefficients and Its Significant Test Results
A	2007.4.2—2009.4.10	494	3252.59—2444.23	0.1391 （t=3.115>t_{crit}=1.960）
B	2007.4.2—2007.6.29	59	3252.59—3820.70	-0.2567 （t=2.006> t_{crit} =2.001）
C	2007.7.2—2007.10.30	83	3836.29—5954.77	**0.0729 （t=0.6583< t_{crit} =1.990）**
D	2007.11.1—2008.4.30	122	5914.28—3693.11	**0.1026 （t=1.130< t_{crit} =1.980）**
E	2008.5.5—2008.10.31	123	3761.01—1728.79	0.1963 （t=2.202> t_{crit} =1.980）
F	2008.11.3—2009.4.10	107	1719.77—2444.23	0.4766 （t=5.556> t_{crit} =1.983）

Notes:

1) Correlation specifies the correlation between the rate of mean return and the change in the intensity of trading conditioning in any two consecutive trading days;
2) Here, t_{crit} is $t_{0.05/2}(n-2)$; If $t > t_{crit}$, then, the correlation coefficient is significantly not equal to zero; On the other hand, we can not reject original hypothesis that the correlation coefficient is equal to zero;
3) The lack of significance is printed in bold and red;
4) SSE Composite Index is measured by closing point.

By empiric results, we have several main findings. First, there is significant positive correlation in general between the rate of mean return and the change in the intensity of trading conditioning (reference to line A in Table). Second, correlation coefficient varies in 5 subdivided periods. For examples, (a) they lack significance in spite of positive correlations in two time intervals right before and just after bubble burst (reference to line C and D in Table); (b) it shows positive significance in the second half after bubble burst; (c) there is positive correlation during price reversal time interval after a year time deep drop. Its correlation coefficient is 0.4766, the highest one (reference to line F in Table); (d) particularly, there exists significant negative correlation (correlation coefficient is -0.2567) when SEE Composite Index is rising during bull market (reference to line B in Table); We will discuss them in next section

6. DISCUSSIONS

In this section, we will discuss some main concerns, based on empirical test results. They are: (1) stationary equilibrium theory, its psychological explanation, and validity on transaction volume-price probability wave equation; (2) reinforcement and punishment values by the rate of mean return, and market crowd psychological behaviors; and (3) potential applications.

6.1 Stationary Equilibrium Theory and Psychological Explanation

Studied transaction volume-price joint behavior in econophysics, Shi (2006) found that there exists stationary equilibrium universally in stock market. The volume-price behavior resembles a probability wave and follows a normative transaction volume-price probability wave equation.

Stationary equilibrium specifies that trading price is volatile upward and downward to an equilibrium price constantly and the equilibrium price has a step jump from time to time. The equilibrium is dynamic. It satisfies that the sum of momentum force, supply-demand restoring force, and interaction force is equal to zero, and the interaction force is equal to either an eigenvalue constant or zero over a trading price range in a stationary equilibrium state.

In stationary equilibrium theory, price volatility is decomposed of two parts. First, trading price is volatile upward and downward to a stationary equilibrium price constantly, a price to which transaction volume kurtosis corresponds over a trading price range. We use equation (7) to express price volatility energy, equation (10) to measure market crowd's interaction and coherence, and equation (12) to indicate market crowd's independent behavior. In conclusion, we use a transaction volume-price probability wave equation (6) to describe the volume-price joint behavior.

Let us explain why the volume kurtosis emerges at stationary equilibrium price from a viewpoint of market crowd and their psychological behavior for return. In a given informational and bounded rational scenario, market crowd accept a price most. When trading price is higher than the price, selling volume will increase and buying volume will decrease. Supply quantity will be more than demand quantity. Trading price will drop. On the other hand, when trading price is lower than the price, selling volume will decrease and buying volume will increase. Supply quantity will be fewer than demand quantity. Trading price will rise. The most acceptable price is a stationary equilibrium price. At the price, it takes the longest time for trading. So, we can find the highest trading frequency, the largest transaction volume probability, and the maximum intensity of trading conditioning at it.

Second, a stationary equilibrium price jumps from time to time. The restoring force is weak and easily be overcome in a stationary equilibrium state in stock market. For example, if there is abrupt change in supply-demand quantity by a series of large buying volume, then trading price is going to adjust to be volatile upward and downward to a new stationary equilibrium price. The stationary equilibrium price jumps. We use the joint equations of (14) and (16) to illustrate jump behavioral mechanism and principle, and the amplitude of jump to represents price volatility mean return in market.

We use every trading high frequency data in Huaxia SSE 50ETF from April 2, 2007 to April 10, 2009, in which there are nearly 740 days and 495 trading days in total, i.e., the number of volume distribution over a price range is 495. We apply the absolute of zero-order Bessel eigenfunction regression model, equation (19), to fit every volume distribution and test significance in our samples. There are 408 (82.42% in total) distributions with significance. We can get stationary equilibrium prices from our test reports directly. It characterizes that there exist stationary equilibrium universally in stock market. For distributions without significance, they display two or more than two of kurtosis over trading price range. It states that a stationary equilibrium price jumps in these samples.

Our empirical results further demonstrate the validity of transaction volume-price probability wave equation, consistent with early findings (Shi, 2006).

6.2 Return Reinforcement (Punishment) Value and Market Crowd Psychological Behaviors

In Section 5, empirical test shows that there is, in general, significant positive correlation between the rate of mean return and the change in the intensity of market crowd's trading conditioning from April, 2007 to April, 2009 (reference to line A in Table).

Let us first to explain disposition effect and herd behavior briefly in stock market. Schifrin and Statman (1985) termed “disposition effect” that investors have a desire to realize gains by selling stocks that have appreciated, but to delay the realization of losses. Later, Odean (1998) tested and demonstrated disposition effect among investors by analyzing trading records for 10,000 accounts at a large discount brokerage house. Weber and Caterer (1998) designed experiments to test disposition effect and explained it by prospect theory (Tversky and Kahneman, 1992). Grinblatt and Keloharju (2001) found evidence that investors are reluctant to realize losses using a unique data set in Finnish stock market. Disposition effect is abnormal behavior in selling stock.

Herd behavior exists widely in our social activities (Shiller, 1985; Bannered, 1992). It has been theoretically linked to many economic activities. It is often said to occur when many people take the same action (Graham, 1999). There are many herd behaviors in stock market (Hirshleifer and Teoh, 2009). Lux (1995) formalized herd behavior to explain the emergence of bubbles.

In this paper, we study trading conditioning herd behavior in stock buying with expectancy on return (reinforcement and punishment) by analyzing correlation between the rate of mean return and the change in the intensity of trading conditioning. We defined it as “stimulated discriminatively by information on price volatility and return, market crowd buy stock with others with expectancy on positive return (reinforcement) or be onlookers with others with expectancy on negative return (punishment)”. It is a kind of herd behavior in stock buying.

The larger the positive mean return is, the more the intensity of trading conditioning increases. The transaction volume increases. The market crowd sell more. And, the more the mean return losses, the more the intensity of trading conditioning decreases. The transaction volume decreases. The market crowd sell less. It is disposition effect for stock sellers.

The larger the positive mean return is, the more the intensity of trading conditioning increases. The transaction volume increases. The market crowd buy more. It is herd behavior for stock buyers who buy stock with others with expectancy on positive return (reinforcement). And, the more the mean return losses, the more the intensity of trading conditioning decreases. The transaction volume decreases. The market crowd buy less. It is herd behavior for stock buyers who are onlookers with others with expectancy on negative return (punishment) in the market.

Therefore, there is significant trading conditioning herd behavior in stock buying and disposition effect in stock selling simultaneously if there is a significant positive correlation between the rate of mean return and the change in the intensity of trading conditioning. The magnitude of the correlation coefficient represents the degree of significance. The larger the correlation coefficient is, the higher significant the two kinds of behavior.

Our empirical results show that there is, in general, significant positive correlation between the rate of mean return and the change in the intensity of trading conditioning (reference to line A in Table). Market crowd have been conditioned by money, asset, and return in commodity exchange society. It is natural to trade for market crowd with expectancy on return, stimulated discriminatively by information on price volatility and return in stock market. The positive (negative) rate of mean return significantly changes trading frequency, strengthens (weakens) market crowd expectancy on return, and shows reinforcement (punishment) value for them. They behave notably trading conditioning herd behavior in stock buying and disposition effect in stock selling with expectancy on return (reinforcement or punishment) in stock market.

In addition, the herd behavior in stock buying shows that they have an increased expectancy on price continuous rising or dropping (momentum effect). When price rises, the intensity of trading conditioning increases and the units of stock buying grow. Market crowd have stronger expectancy on price continuous rising (momentum) and market reward. In another word, they have less expectancy on price drop (reversal). When price drops, the intensity of trading conditioning decreases and the units of stock buying reduce. Market crowd have stronger expectancy on price continuous dropping (momentum) and market punishment. They have less expectancy on price rise (reversal). In a word, “the herd” have significant stronger expectancy on price momentum than its reversal. We term it as “momentum action”[①] effect.

In contrast, disposition effect indicates that market crowd in stock selling have an increased expectancy on price reversal. “The disposition” have significant expectancy on price reversal than its momentum. We term it as “reversal momentum action” effect.

Now, we further study market crowd’s learning and psychological behaviors in 5 subdivided time intervals by analyzing the correlation (reference to Table). There is advantage for it because we can study their learning and psychological behavior subject to return and its change with environment (and time).

We are going to discuss market crowd’s expectancy on return (reinforcement or punishment) in each time interval, separately, according to correlation between the rate of mean return and the change in the intensity of trading conditioning. We will explain their trading behavior by conditioning.

The correlation coefficients are 0.0729 and 0.1026 right before and just after bubble burst in 2007 in China, respectively. The positive correlations lack significance in two periods. Disposition effect and herd behavior are not significant (reference to line C and D in Table). Positive (negative) return does not produce a significant reinforcement (punishment) value for market crowd in two time intervals. Specifically, stock holders hesitated to realize gain when price rose in the time interval in line C. On one hand, they were excited to ride on bubble for maximizing return as they had been rewarded and conditioned by past positive return. On the other hand, they did have desire to realize gain because they were aware of high price and high risk. At the same time, cash holders hesitated to buy. From one point, they had strong desire to trade and greedy for higher return because there was tremendous resistance to extinction on trading conditioning and expectancy with positive return (reinforcement). From the other point, they want to keep money because they had known high risk at high price. Therefore, positive (negative) return did not produce a significant reinforcement (punishment) value.

Similarly, we can explain insignificant positive correlation in the time interval in line D. Positive (negative) return does not produce a significant reinforcement (punishment) value, too.

There was significant positive correlation between the rate of mean return and the change in the intensity of trading conditioning (the correlation coefficient is 0.1963) when SSE Composite Index continued dropping in the second half of time intervals after bubble burst in 2007 (reference to line E in Table). Market crowd behaved significant herd behavior in stock buying and disposition effect in stock selling. In comparison with the time interval at line D, when market went down further in the time interval in line E, stock holders loss more money and got stronger punishment. There was less volume to be sold. Disposition effect pronounced. At the same time, cash holders had been gradually conditioned by market drop. They were willing to be onlookers

[①] Price momentum means price continuous movement in a direction or price movement inertia in finance. Momentum is derived from action and critically defined in econophysics (Shi, 2006).

with others. The herd's behavior pronounced (Please compare the correlation coefficient in line D with line E).

There was very strong positive correlation in the market reversal time interval (the correlation coefficient is 0.4766) after it took one year to have price deep drop (reference to line F in Table). We can draw two conclusions from the result. First, market crowd have learned from market punishment for a long time and been conditioned with strong expectancy on negative return after a year deep drop in SSE Composite Index from top at 6124.04 points to bottom at 1664.04 points. Therefore, there was most pronounced disposition effect to realize gains in short term. Second, it is a necessary condition for market reversal and rising after a long time deep price drop momentum in the index that we need sustainable incremental amount of money to pair a large volume of shares that are sold by strong disposition effect in short term, set an occasion for stock holders with expectancy and confidence on positive return, and alter their expectancy from previous market punishment to reinforcement.

Now, we focus on a special case that there is a significant negative correlation between the rate of mean return and the change in the intensity of trading conditioning (the correlation coefficient is -0.2567) in the time interval in line B in Table.

SSE Composite Index kept steady rising from 998.23 points in July, 2005 to 3183.98 points in March, 2007, the beginning time in our sampling. Market crowd have been conditioned and rewarded by buying and holding behavior with expectancy on positive return (reinforcement) in the bull market.

When price rises, there is more buying volume than selling volume in the market. The intensity of trading conditioning is reduced because of negative correlation. Transaction volume dwindles and liquidity worsens. Although risk increases (higher price and lower liquidity) at the time in the market, stock holders still behave reluctant to sell, expect higher return, and have an increased expectancy on price rising momentum. They show overconfidence. By contrast, when risk increases, cash holders buy less volume than before. They have stronger expectancy on price drop and behave prudence in stock buying.

When price drops, there is more selling volume than buying volume in the market. The intensity of trading conditioning is increased because of negative correlation. Transaction volume enlarges and liquidity improves. Although risk decreases (lower price and better liquidity) at the time in the market, stock holders are eager to sell to be onlookers with others. They behave panic. To the contrary, when risk reduces, cash holders buy more. They behave confidence with expectancy on price rise in the future.

Therefore, stock holders behave significantly overconfidence or panic with expectancy on price momentum in the time interval while cash holders behave notably prudence or confidence with expectancy on price turnaround.

6.3 Potential Applications

There are many possible applications in our research. First, we study market crowd's learning and psychological behavior quantitatively in stock market by analyzing correlation between the rate of mean return and the change in the intensity of trading conditioning, using transaction volume probability in a transaction volume-price probability wave equation to measure the intensity of trading conditioning. It can help us to study market crowd's learning and psychological behavior, for example, acquisition and extinction in trading conditioning. Therefore, it is the best laboratory to investigate trading conditioning in stock market (Soros, 1987); second, it can help us to understand anomalies in financial market, for example, excessive price volatility (Shiller, 1981) and bubble etc. Third, it can help us to make out investment strategies, manage risk and asset, and gain much great higher return than a stock fundamental value brings to, taking advantage of excessive price volatility and market crowd's psychological behavior in stock market. Fourth, it can help us to capture major investment opportunity, avoid big systematical risk, and increase return by studying market reversal condition. Fifth, it can help financial authorities to make out reinforcement and punishment policies on financial market and regulate trading behavior so that we can prevent bubbles from growing too big or drying up, avoid or deal with large financial crisis, and reduce its tremendous damage to economy (Soros, 2010b; Xiao and Houser, 2005). Sixth, it can help us to provide training courses for investors in financial physiology and psychology so that their behavior and activity can best suit to trading in their job. Seventh, it is helpful for us to study (behavioral) finance and establish a unified and normative (behavioral) capital asset pricing theory, a multidisciplinary field with economics, physics, mathematics, psychology, physiology, cognitive science (Camerer, Loewenstain, and Prelec, 2005), biology (Camelia, 2010), medicine, and computer science etc., using the measurement of trading conditioning and the probability wave equation[①].

7. SUMMARIES AND CONCLUSIONS

From previous study, we know that the transaction volume-price behavior resembles a probability wave in stock market (Shi, 2006). It follows a normative transaction volume-price probability wave equation (6). The equation can describe and explain leptokurtosis (more peaked and heavy tailed), cluster, and scaling phenomena in return distribution. It can describe and explain market crowd's interaction and coherence in trading behavior and the extremely conditional behaviors of both rational and irrational in decision making. All of its characteristics capture real behaviors in stock market. Therefore, we attempt to use it further to study market crowd psychological behavior.

In this paper, we not only introduce a notion of trading conditioning for the first time and annotate market crowd's psychological behavior in the probability wave equation, but also measure their major behavioral characteristics. The measurement includes: (1) transaction volume probability in the equation represents the intensity of market crowd's trading conditioning; (2) eigenvalue indicates the magnitude of market crowd's interaction and coherence; and (3) the price corresponding to the maximum intensity of trading conditioning over a trading price range is a stationary equilibrium price, the most acceptable price at a time interval.

In empirical test, we study market crowd's learning and psychological behavior in decision making by analyzing correlation between the rate of mean return and the change in the intensity of trading conditioning, using every trading high frequency data in China stock market from its bubble growth, burst, and shrink until market reversal again, a whole course that is paralleled with the collapse of sub-prime bubble in the United States. We use the amplitude of stationary equilibrium price jump to stand for the rate of price volatility mean return in any two consecutive

[①] Thaler (1999) predicted that in the not-too-distant future, the term "behavioral finance" will be correctly viewed as a redundant phrase. Economists will routinely incorporate as much "behavior" into their models as they observe in the real world.

trading days and the rate of change in total transaction volume to represent the change in the intensity of market crowd trading conditioning subject to mean return, based on a stationary equilibrium price jump model and a set of equations.

We get several results as follows: First, there is, in general, significant positive correlation between them in any two consecutive trading days. It evidences that the positive (negative) rate of mean return significantly changes trading frequency, strengthens (weakens) market crowd expectancy on return, and has a trading conditioning reinforcement (punishment) value for market crowd. They behave notably disposition effect in selling and herd behavior in buying with expectancy on return in stock market. Specifically, “the herd” have significant stronger expectancy on price momentum than its reversal, a “momentum action” effect, and “the disposition” have significant stronger expectancy on price reversal than its momentum, a “reversal momentum action” effect. Second, a new trading conditioning is formed for market crowd if stock holding behavior is associated with positive return and market award consistently in steady bull market. There is a significant negative correlation between the rate of mean return and the change in the intensity of trading conditioning. Stock holders behave significantly overconfidence or panic with expectancy on price momentum while cash holders behave notably prudence or confidence with expectancy on price turnaround. Third, there is very strong positive correlation when price reversal and rising takes place after a long time deep price drop. Market crowd have learned from market punishment for a long time and been conditioned with strong expectancy on negative return. They behaved the strongest disposition effect in short term. Therefore, it is a necessary condition for market reversal after a long time deep drop momentum in the index that we need sustainable incremental amount of money to pair a large volume of shares that are sold by strong disposition effect in short term, set an occasion for stock holders with confidence and expectancy on market reward, and alter their expectancy from previous market punishment to its reinforcement. Last, the positive correlation lacks significant if positive (negative) return does not produce a significant reinforcement (punishment) value for market crowd right before and just after bubble burst.

In conclusion, the transaction volume-price probability wave equation can describe market crowd’s psychological behavior. We use transaction volume probability to measure the intensity of their trading conditioning and study their learning and psychological behavior by analyzing correlation between the rate of mean return (reinforcement and punishment) and the change in the intensity of trading conditioning in stock market. It can help us understand market behaviors, explain its anomalies, for examples, excessive price volatility and bubble, and manage risk and investment in stock market. On one hand, investors can gain much great higher return than a stock fundamental value brings to, taking advantage of excessive price volatility and market crowd’s trading conditioning behavior in stock market. On the other hand, financial authorities can apply reinforcement and punishment into practices properly to regulate trading behavior effectively, prevent bubbles from growing too big or drying up, avoid or deal with financial crisis efficiently, and reduce its tremendous damage to our economy.

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