Correlation of financial markets in times of crisis

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Abstract

Using the eigenvalues and eigenvectors of correlations matrices of some of the main financial market indices in the world, we show that high volatility of markets is directly linked with strong correlations between them. This means that markets tend to behave as one during great crashes. In order to do so, we investigate several financial markets that occurred in the years 1987 (Black Monday), 1989 (Russian crisis), 2001 (September 11), and 2008 (subprime mortgage crisis), which mark some of the largest downturns of financial markets in the last three decades.

1 Introduction

The study of why many world financial markets crash simultaneously is of central importance, particularly after the recent worldwide downturn of the major markets in 2007 and 2008. Economists have been studying the reasons why markets crash, and why there is propagation of volatility from one market to another, since a long time. After the crash of 1987, many studies have been published on transmision of volatility (contagion) between markets using econometric models [1]-[8], on how the correlation between world markets change with time [9]-[11], and how correlation tends to increase in times of high volatility [12]-[24]. This issue is of particular importance if one wishes to build portfolios of international assets which can withstand times of crisis [25]-[31]. Many models have been proposed by both economists and physicists in order to explain the correlation of international financial markets [32]-[50], which is considered a complex system with many relations which are difficult to identify and quantify.

One tool that was first developed in nuclear phyisics for studying complex systems with unknown correlation structure is random matrix theory [51]-[54], which confronts the results obtained for the eigenvalues of the correlation matrix of a real system with those of the correlation matrix obtained from a pure random matrix. This approach has been successfully applied to a large number of financial markets [55]-[78], and also to the relation between world markets [79]. This approach has also been used in the construction of hierarchical structures between different assets of financial markets [80]-[92].

Our work uses the tools of random matrix theory to analyze the correlation of world financial markets in times of crisis. In order to do so, we use data from some of the largest worldwide crashes since 1985, namely the 1987 Black Monday, the 1998 Russian Crisis, the Burst of the dot-com bubble of 2001, the shock after September, 11, 2001, and the USA subprime mortgage crisis of 2008. We start by defining a global financial crisis (section 2) based on evidence of some financial markets chosen from diverse parts of the world. Then, we discuss some of the main theoretical results on Random Matrix Theory (section 3). On the next three sections we study the correlation matrices between the log-returns of a number of financial market indices chosen so as to represent many geographical parts of the world and a diversity of economies.

In each section, we calculate the eigenvalues of the correlation matrix of the chosen indices and then study the eigenvector that corresponds to its largest eigenvalue, which is usually related with a *market mode*, which

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is a co-movement of all indices. We also use the correlation matrix in order to build hierarchycal structures that closely resemble the real relationships between world markets. In order to show those relationships, we adopt both a planar graph and a three dimensional one, which represents the "distances" between markets more faithfully.

We then calculate correlation matrices in running windows and compare the average correlation between markets with the volatility of the market mode obtained previously, showing that times of large volatility are strongly linked with strong correlations between world financial indices. We also perform a number of statistical tests in order to verify the validity of our hipoteses.

We finish by discussing recent methods for studying random matrices obtained from non-Gaussian distributions, shuch as t-Student distributions, which represent more closely the probability density distributions obtained from financial data [93]-[96], and the possibility of using other measures of co-movement of financial indices that are more appropriate for systems with very strong correlations, like it happens in times of financial crises [97]-[98].

2 Defining a global financial crisis

Before studying periods of financial crisis, we must make it clear what we consider to be a global crash of the financial markets. In order to adopt a more precise definition, we considered the time series of a certain number of financial markets representing different regions of the world from the beginning of 1985 until the end of 2008. Looking at the closing indices of every day in which there was negotiation, we considered the log-returns, given by

$$S_t = \ln(P_t) - \ln(P_{t-1}) \approx \frac{P_t - P_{t-1}}{P_t} \ , \tag{1}$$

which makes it easier to compare the variations of the many indices. After that, a distribution of frequencies was built for each index, and the 10 most negative variations were chosen. We then searched for events where variations greater or equal to the tenth most negative variation occured.

In order to illustrate the procedure, we consider the Dow Jones index of the New York Stock Exchange (NYSE). Figure 1 shows the log-density distribution for this index with data from 01/02/1985 to 12/31/2008. The log-density, defined as

$$\text{log-density} = \ln(1 + \text{density}) \ , \tag{2}$$

is used instead of simple density in order to better visualize the most extreme points.

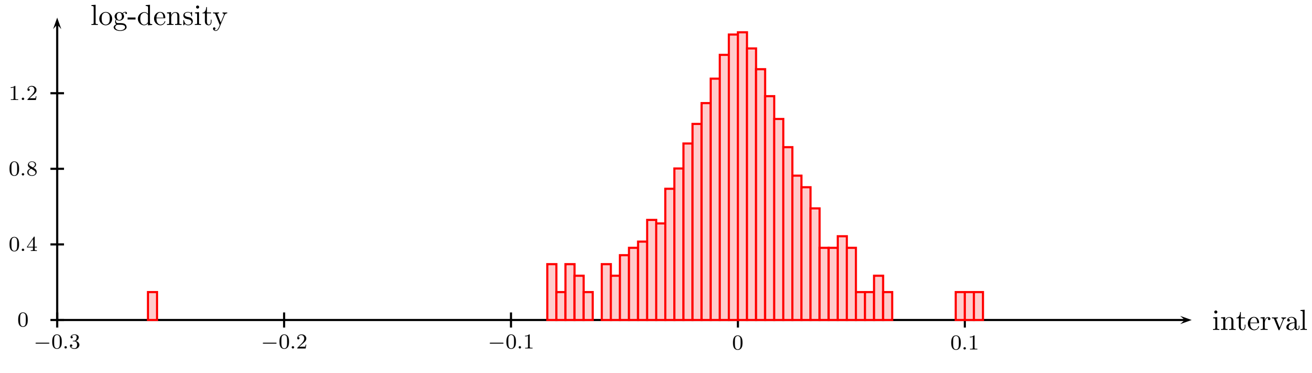

Figure 1: log-density distribution of the Dow Jones index of the NYSE, from 01/02/1985 to 12/31/2008.

The ten most negative values of the log-returns are bellow −0.07. These events occured in the following occasions: 10/19/1987 (22.61%), 10/26/1987 (8.04%), 01/08/1988 (6.85%), 10/13/1989 (6.90%), 10/27/1997 (7.18%), 09/17/2001 (7.13%), 09/29/2008 (6.98%), 10/09/2008 (7.33%), 10/15/2008 (7.87%), and 12/01/2008 (7.70%). These dates include the 1987 Black Monday, part of the Asian Crisis of 1997, the 1998 Russian Crisis, the aftermath of September 11th, 2001, and the subprime mortgage crisis of 2008.

The same technique was used for the Nasdaq (USA), FTSE (UK), Dax (West Germany), Nikkei (Japan), Hang Seng (Hong Kong), Ibovespa (Brazil), and IPC (Mexico). The next table displays the years of major drops (between the beginning of 1985 and the end of 2008) and the quantity of markets which presented those falls. When a market drops substantially more than once in the same year, these are counted more than once as well, in order to gauge the depth of the shocks. This helps to identity the times where there were major crashes around the world.

Year	1985	1987	1988	1989	1990	1991	1992	1997	1998	2000	2001	2008
Occurences	1	17	2	6	4	1	1	6	3	3	7	29

Table 1: number of occurences per year of major drops in eight diverse stock markets in the world.

It is possible to pinpoint two major crises in 1987 and 2008, and minor crises in 1989, 1990, 1997, 1998, 2000, and in 2001. The crisis of 1987 corresponds to the so called Black Monday, the one in 1989 is the USA saving and loan crisis, 1997 is the Asian financial crisis, 1998 is the Russian crisis, 2000 and 2001 corresponds to the Burst of the dot-com bubble, and 2008 corresponds to the subprime mortgage crisis in the USA. There were also documented crises in 1990 (Japanese asset price bubble and Scandinavian banking crisis), 1992 (Black Wednesday), and 1994 (Mexican crisis), nearly all of them spotlighted in our graph.

3 Random matrix theory

Random matrix theory had its origins in 1953, in the work of the German physicist Eugene Wigner [51] [52]. He was studying the energy levels of complex atomic nuclei, such as uranium, and had no means of calculating the distance between those levels. He then assumed that those distances were random, and arranged the random number in a matrix which expressed the connections between the many energy levels. Surprisingly, he could then be able to make sensible predictions about how the energy levels related to one another.

This method also found connections with the study of the Riemann zeta function, which is of primordial importance to the study of prime numbers, used for coding and decoding information, for example. The theory was later developed, with many and surprising results arising. Today, random matrix theory is applied to quantum physics, nanotechnology, quantum gravity, the study of the structure of crystals, and may have applications in ecology, linguistics, and many other fields where a large amount of apparently unrelated information may be understood as being somehow connected (for a recent book on the subject, see [53]). The theory has also been applied to finance in a series of works dealing with the correlation matrices of stock prices, and also to risk management in portfolios [54]-[60] (for recent reviews on the subject, see [61] and [62]).

In this section, we shall focus on the results that are most important to the present work, which is studying the correlations between world financial markets in times of crisis. The first result of the theory that we shall mention is that, given an $L\times N$ matrix with random numbers built on a Gaussian distribution with average zero and standard deviation σ, then, in the limit $L\to\infty$ and $N\to\infty$ such that $Q=L/N$ remains finite and greater than 1, the eigenvalues λ of such a matrix will have the following probability density function, called a Marčenku-Pastur distribution [63]:

$$\rho(\lambda)=\frac{Q}{2\pi\sigma^2}\frac{\sqrt{(\lambda_+-\lambda)(\lambda-\lambda_-)}}{\lambda}\ . \tag{3}$$

where

$$\lambda_-=\sigma^2\left(1+\frac{1}{Q}-2\sqrt{\frac{1}{Q}}\right)\quad,\quad\lambda_+=\sigma^2\left(1+\frac{1}{Q}+2\sqrt{\frac{1}{Q}}\right)\ , \tag{4}$$

and λ is restricted to the interval $[\lambda_-,\lambda_+]$.

Since the distribution (3) is only valid for the limit $L\to\infty$ and $N\to\infty$, finite distributions will present differences from this behavior. In figure 2, we compare the theoretical distribution for $Q=10$ and $\sigma=1$ to distributions of the eigenvalues of three correlation matrices generated from finite $L\times N$ matrices such that $Q=L/M=10$, and the elements of the matrices are random numbers with mean zero and standard deviation 1.

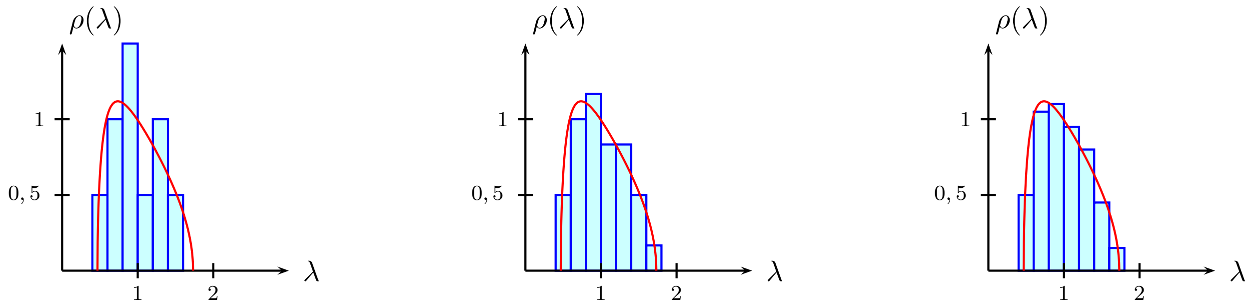

Figure 2: theoretical and sample finite distributions for a random matrix with $N = 10$ and $L = 100$, $N = 30$ and $L = 300$, and $N = 100$ and $L = 1000$.

Consequently, real data will deviate from the theoretic probability distribution. Nevertheless, the theoretical result may serve as a parameter to the results obtained experimentally.

Another source of deviations is the fact that financial time series are better described by non-Gaussian distributions, such as t Student or Tsallis distribution. This can be seen from figure 1: a Gaussian distribution would be represented by a parabola, what is clearly not the case.

Recent studies [66]-[69] developed part of the theoretical framework in which finite series and series with fat tales, as is the case of financial time series of returns, can be studied.

Since random matrix theory is based on random matrices with a single standard deviation σ, we must compensate the data obtained from the many indices so that all series have average zero and the same standard deviation, which we chose to be equal to one. This can be done using the formula

$$X_t = \frac{S_t - \langle S \rangle}{\sigma} \ , \tag{5}$$

where $\langle S \rangle$ is the average of the time series used, and σ is its standard deviation.

In what follows, we shall do an analysis for some of the years in which we detected global financial crises.

4 1987, the Black Monday

In 1987, the financial world lived a time of panic, much like the one of the great crash of 1920. In a matter of 3 days, most of the stock markets in the world lost about 30% of their value and trillions of dollars evaporated, leaving a trace of destruction that affected what is reffered to as real economy for many years. The day the first and major collapse occured was later called the *Black Monday*.

In order to analyze that crisis, we consider now the correlation matrix between the Dow Jones (DJ), S&P 500 (S&P), Nasdaq (Nasd), all from the USA, FTSE (UK), Dax from West Germany (WG), IGBM from Spain (Spa), OMX from Sweden (Swe), Ibovespa from Brazil (Bra), IPC from Mexico (Mex), Nikkei from Japan (Jap), Hang Seng from Hong Kong (HK), Kuala Lumpur Composite from Malaysia (Mal), Kospi from South Korea (SK), and Asx from Australia (Aus). In order to build the correlation matrix, when one or few of the markets didn't open, we considered the index of the previous day. If just one or very few of the markets operated on a day, we removed that index from the data.

We are also considering the correlation of markets on the same date, although it is well known that the Asian markets, for example, operate at different hours from their Western counterparts. There is also some evidence [19] that the correlations of Asian with the USA indices increase when one considers the correlation of the USA indices with the next day indices of the Asian markets. We did some calculations with correlations of Western markets with the next day of Asian markets, and the result was an increase in some correlations, and a dicrease in others. The overall difference of results was not significant, and whenever there is a remarkable discrepancy in results, we shall comment them.

We shall use 1987 as an example for the other years, and because of that we shall be showing more details in the calculations for that year. Calculating the correlation matrix for the indices that are being considered, one obtains a 14×14 matrix. The average of the values of this correlation matrix is a good measure of the overall correlation between the many indices. For the present correlation matrix, the average is given by $\langle C \rangle = 0.3079$, with standard deviation $\sigma = 0.0360$. Since the correlation matrix is 14×14, and the number of days considered in calculating it is 252, we then have $Q = L/M = 252/14 = 18$ and the upper and lower bounds

$$\lambda_- = 0.584 \quad , \quad \lambda_+ = 1.527 \tag{6}$$

for the eigenvalues that constitute the bulk of the eigenvalue distribution due to noise.

A frequency distribution of the eigenvalues of the correlation matrix is shown in figure 3, with the theoretical distribution of an infinite random matrix for $Q = 18$ superimposed to it. In figure 4, the eigenvalues are plotted in order of magnitude. The shaded area indicates the region predicted by theory were the data related with a purely ramdom behavior of the normalized log-returns.

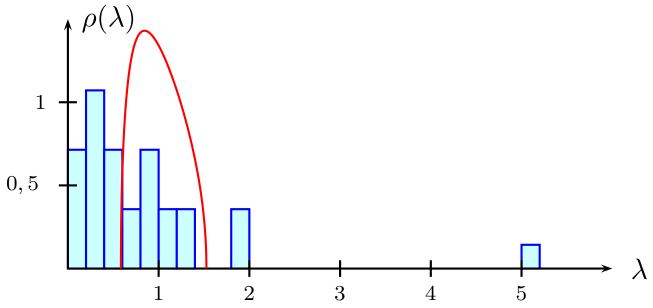

Figure 3: frequency distribution of the eigenvalues of the correlation matrix for 1987. The theoretical distribution is superimposed to it.

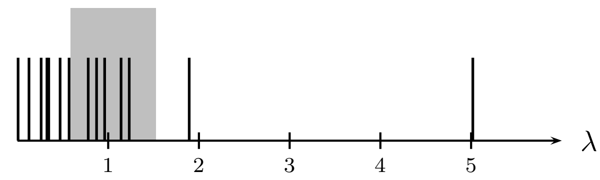

Figure 4: eigenvalues in order of magnitude. The shaded area corresponds to the eigenvalues predicted from a random matrix.

So, only about 36% of the eigenvectors fall inside the region predicted by the random matrix theory. Note that the highest eigenvalue stands out from all the others, being more than three times bigger than the uppermost limit λ_+ of the theoretical distribution. This is in aggreement with many other results, obtained for a great number of financial institutions to which this same formalism has been already applied [54]-[77]. It is believed that this eigenvalue corresponds to the action of a single market, which influences all the other members of the correlation matrix. Usually, for the correlation matrix of individual stocks in a single market, this eigenvalue is much larger, some times 25 larger, than the largest eigenvalue predicted for the correlation matrix of random time series. In our case, it responds for about 36% of the collective behavior of the time series being considered.

Figure 5 shows the contributions of the many indices which we are considering in our study in some of the eingenvectors of the correlation matrix. The blue bars represent positive values and the red bars represent negative ones.

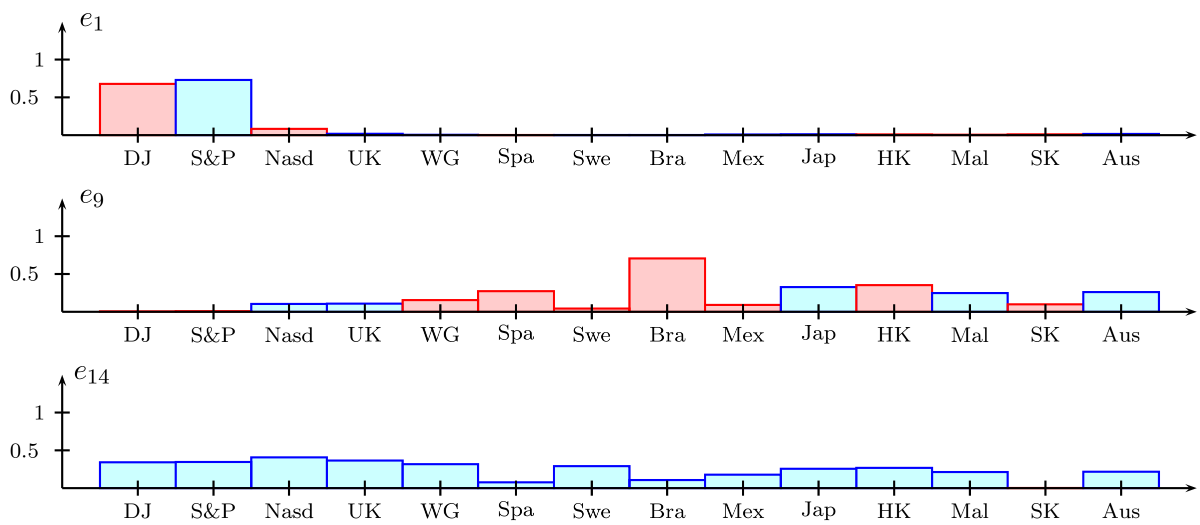

Figure 5: contributions of the stock market indices to eigenvectors e_1, e_9, and e_{14}. Blue bars indicate positive values, and red bars correspond to negative values.

One can see that the eigenvector corresponding to the largest eigenvalue is qualitatively different from the others. Nearly all markets (with the exception of South Korea) have positive representations. That is a compelling reason to believe that it represents a global market that is the result of the interactions of all local markets, or may also be the result of external news on the market as a whole. Figure 6 compares the time series of an index built using eigenvector 14 (in blue) with the world index calculated by the MSCI (Morgan Stanley Capital International), in red. Both indices are normalized so as to have zero mean and standard deviation one.

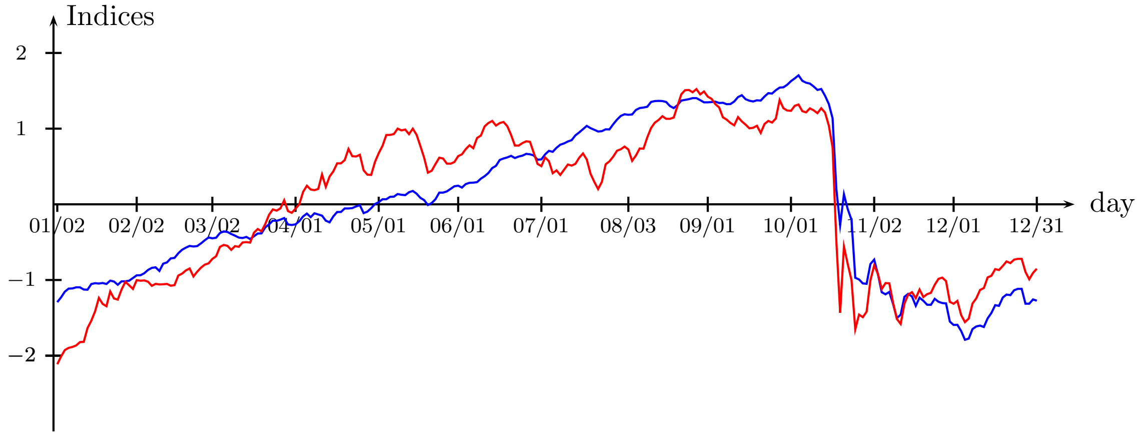

Figure 6: time series of the market mode calculated using the eigenvector related with the largest eingenvalue of the correlation matrix (blue) plotted against the world index calculated by the MSCI (red). Both indices are normalized so as to have mean zero and standard deviation one.

In terms of portfolio theory, as stated by Markovits' ideas [106], [107], the eigenvector corresponding to the largest eigenvalue represents the riskier portfolio one may build, as all the indices vary in the same way. In constrast, some of the smaller eigenvectors represent portfolios with less risk, as, for example, eigenvector e_1, which basically consists of "buying" S&P and "short-selling" Dow Jones, which are two indices that measure the same financial market (the New York Stock Exchange). Eigenvector e_9 corresponds to one of the eigenvectros that are within the region considered as noise, and should represent just a random combination of stock market indices.

More differences between eigenvector e_{14} and the other eigenvectors can be seen if we build frequency distribution graphs for the fourteen eigenvalues. All distributions, except the the one for eigenvalue e_{14}, resemble Gaussians around zero. Although most of the eigenvectors $e_1, \cdots, e_{13}$ have distributions which are not (they have excess curtosis far from 3), they all have average near zero and standard deviation around 2.5, and this is not the case for eigenvector e_{14}.

Some recent works have discussed how finite sized data and log-return distributions that are not Gaussian could affect the probability distribution of the eigenvalues of an empirical correlation matrix [66]-[69]. Some of the results imply that the usual Marĕnko-Pastur distribution acquires a fat tail in the direction of the largest eigenvalue.

A last analysis which shows the difference between the highest eigenvalues and the eigenvalues belonging to the range associated with noise may be done using the so called *Inverse Participation Ratio* (IPR),

$$I_k = \sum_{i=1}^{N} \left(e_k^i\right)^4 \quad , \tag{7}$$

where e_k^i is the i-th element of eigenvalue e_k, and N is the total number of eigenvectors. Its inverse gives the average number of stocks which contribute significantly to a portfolio built with such eigenvector. The next two figures show the IPR for the 14 eigenvectors, in ascending order from the left to the right (figure 7), and its inverse, $PR = 1/I$, for Participation Ratio (figure 8).

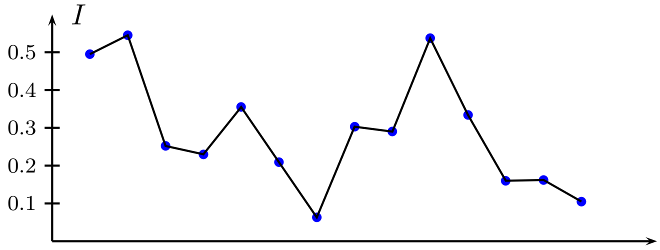

Figure 7: inverse participation ratio of the eigenvectors of the correlation matrix.

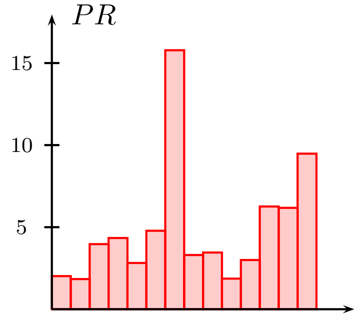

Figure 8: participation ratio of the eigenvectors of the correlation matrix.

Note that, for eigenvector e_{14}, the number of participating indices, with the exception of eigenvalue e_7, which is an aberration, is larger than the average, which is about 5. Most of the eigenvectors corresponding to noise fall around that average number, but this is not true for the eigenvectors corresponding to the lowest eigenvalues, which have a very small number of participating indices.

One important result of this theoretical treatment is that the largest eigenvalue, associated with a *market mode*, is like another matrix that is added to the true correlation matrix of the log-returns. In order to study the remaining eigenvalues, one must first clean the empirical correlation matrix from the market mode. The process is known as *single index model*, and is widely used by theoreticians and practitioners of financial markets in order to remove the market mode of stocks negotiated in the same stock exchange [106]. This is done in order to study clusters of stocks that move together as blocks in stock markets. Doing so for the stock market indices, we could not perceive any particular structure, and so we shall not expose our results here.

4.1 Hierarchical structure

The correlation matrix between log-returns of the indices may also be used in order to classify the markets from which it is made into clusters, or into "neighbors". For that, we use the process first developed in [70] and used in [71]-[74], which considers a suitable distance function between the markets based on the correlation function, and then uses the Minimum Spanning Tree technique [75] in order to establish links between them.

An Euclidean metric must fulfill the axioms $d_{ij} = 0 \Leftrightarrow i = j$, $d_{ij} = d_{ji}$, and $d_{ij} \leq d_{ik} + d_{kj}$. One such metric is

$$d_{ij} = \sqrt{2(1 - c_{ij})} \ , \tag{8}$$

where c_{ij} are elements of the correlation matrix.

Using the metric given by (8), one obtains a distance matrix, over which can be applied the Minimum Spanning Tree technique [75], which consists on choosing one of the indices (nodes) and finding the next node which is closest to that one and to link them. One then considers the cluster formed by those two nodes and finds the next node that is closest to any of the members of that cluster and links it to the node it is closest to inside the cluster. The process then goes on until there are no free nodes left.

As an example, starting from the Dow Jones index, we can see that the S&P is the closest index to it, what makes sense, since they measure the same market. So, we establish a link between them and look for the next node (index) that is closest to one of the two indices in the cluster. It turns out to be Nasdaq, which is closest to the node S&P. So, we establish a link between Nasdaq and S&P. Following this procedure, one obtains the following planar graph that shows relationships between the markets that are being considered.

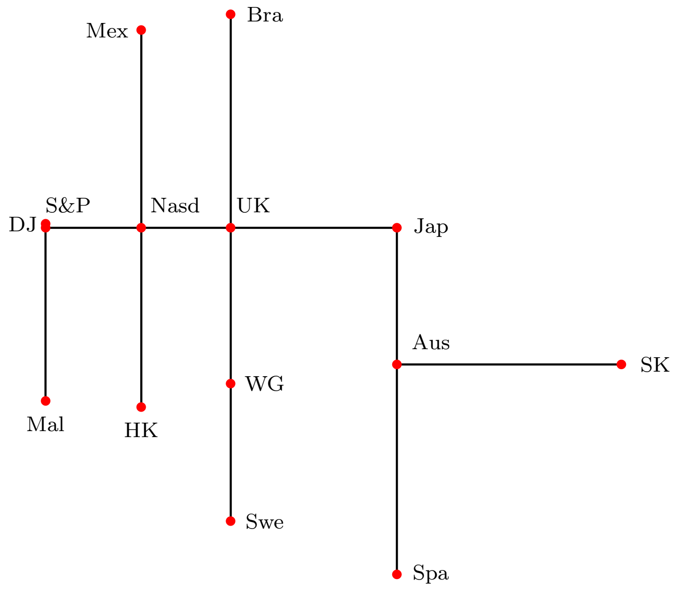

Figure 9: hierarchical structure of stock market indices obtained from the correlation matrix between them.

Note that the graphic captures features of reality, as the indices Dow Jones and S&P are nearly merged, the three USA indices are close together, and closely linked with the British index, the Mexican, and Malaysian one. The UK serves as a hub for connections with Brazil, Japan, West Germany and Sweden, and Australia and South Korea are more connected with Japan. Spain is strangely connected with Australia, but that is because, for markets which are "far" from all the others, like Spain or South Korea, the relative distances to all other markets becomes blurred.

Note that this minimum spanning tree only establishes relations between nearest neighbors. As an example, the index Dow Jones is closer to the FTSE (UK) than the figure shows. One procedure that turns the distances between the many markets graphically more real is to represent the nodes (markets) and their connections in three dimensions, in a way that their distances are the closest to the real ones as possible. Figure 10 shows a three dimensional projection of the approximate distances between the stock market indices as seen from two differents points of view, with just the strong relationships between the indices drawn.

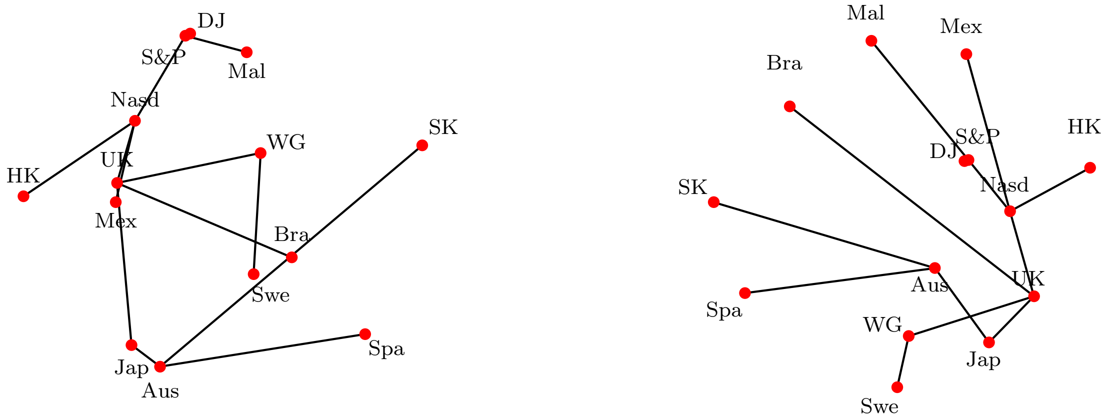

Figure 10: two different views of the three dimensional hierarchical structure between the stock market indices.

4.2 Correlations in times of crisis

We now measure the average of the correlation matrices in moving windows of 25 days, changing one day at a time. The results are displayed in figure 11, where the average correlation is plotted togehter with the volatility of the market mode, which we consider here as the absolute value of S_t, where S_t is a linear combination of all indices with the elements of eigenvalue e_{14} as the coefficients. Here, we work with the variables without normalizing for unit standard deviation. The reason is we want the volatility of the market to be clear. The plot represents the average correlation of each window as a function of the last day of the window. This is done in order to prevent that effects that will still happen influence the measure of the average correlation. In order to best compare the average volatility with the average correlation, both are normalized so as to have average zero and standard deviation 1. The normalized average volatility is in blue, and the normalized average correlation is in red.

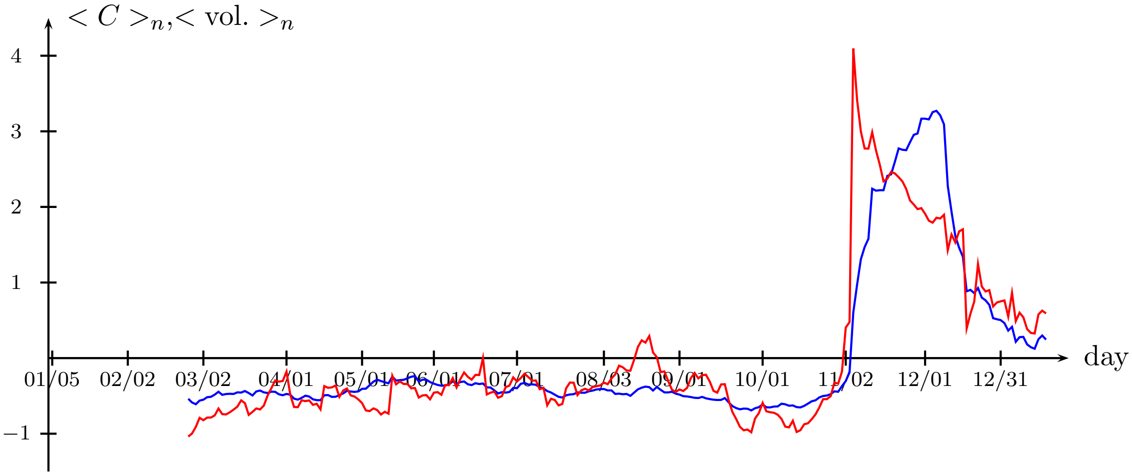

Figure 11: average volatility (blue) and average correlation (red) based on the log-returns for 1987, both calculated in moving windows of 25 days and normalized so as to have mean zero and standard deviation one.

It is quite clear there is a strong correspondence between global market volatility and the correlation of the market indices. The correlation between the two variables along this period is 0.59. One can also note that markets are much more correlated after the period of crisis, and this behavior tends to endure for some time after the crash [76]. Figure 12 shows the evolution of the covariance between volatility and $< C >$ in time, calculated in moving windows of 25 days, starting from 06/02/1987 (the first day we assign an average correlation). A clear peak can be seen on the days of greatest volatility (we plot the correlation at the end of the time interval considered for each calculation).

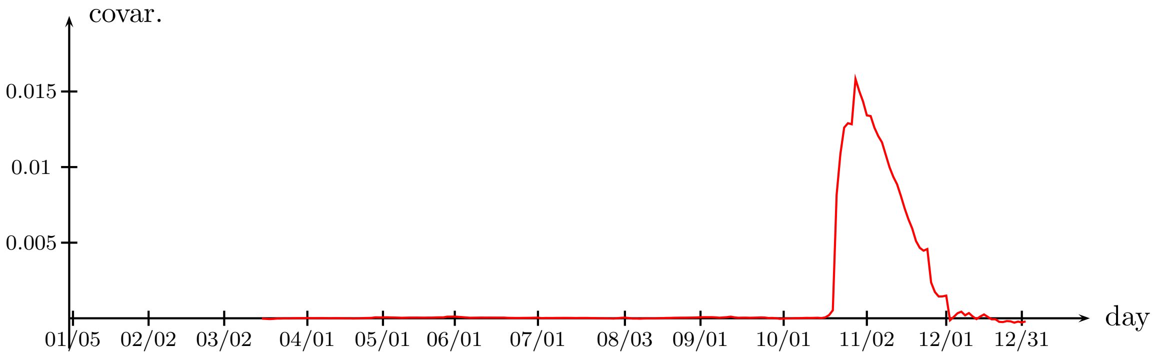

Figure 12: covariance between volatility and average correlation as a function of time.

5 1998, Russian crisis

By May, 1998, the Russian economy showed signs of a recession, and brought most of the world's financial markets down, since many countries had a good amount of money invested in that country. In order to analyze this crisis, we considered the Dow Jones and S&P 500, of the New York Stocks Exchange, the Nasdaq (USA), 8 indices of European countries - Germany, France, UK, Italy, Switzerland, Spain, Sweden, and Netherlands - 6 indices from East Asia - Japan, Hong Kong, Malaysia, Thailand, and South Korea - 4 indices from Latin America - Mexico, Brazil, Argentina, and Chile - and indices from Australia, Russia, Israel, Turkey, and South

Africa. This offers a good degree of diversification, and includes Russia, which was of paramount importance in this particular crisis.

Using the modified log-returns (5) based on the closing indices from 01/02/1998 to 12/30/1998, we built a 25×25 correlation matrix between those. This matrix (which we shall not reproduce here) has an average correlation $< C >= 0.3797$, standard deviation $\sigma = 0.0293$, and is based on $L = 247$ days for the $M = 25$ indices, which gives $Q = L/M = 247/25 = 9.88$.

The upper and lower bounds of the eigenvalues of the Marĕnko-Pastur distribution (3) are

$$\lambda_- = 0.4649 \quad \text{and} \quad \lambda_+ = 1.7375 \ . \tag{9}$$

The frequency distribution of the eigenvalues is displayed bellow, plotted against the theoretical Marĕnko-Pastur distribution were it an infinite random matrix with mean zero and standard deviation 1.

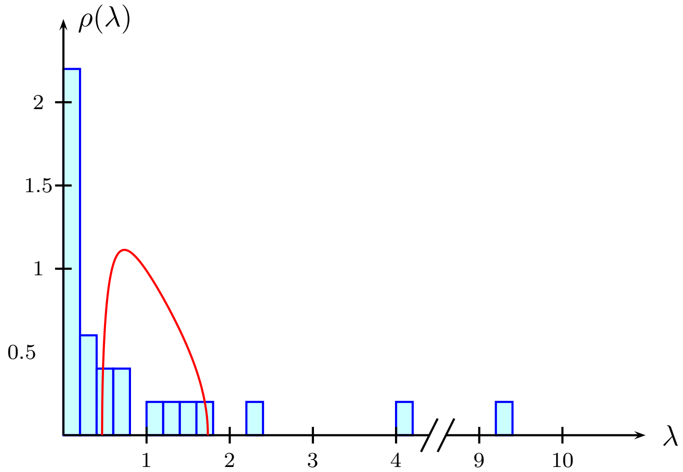

Figure 13: frequency distribution of the eigenvalues of the correlation matrix for 1987. The theoretical distribution is superimposed to it.

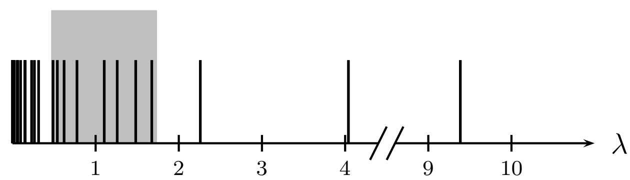

Figure 14: eigenvalues in order of magnitude. The shaded area corresponds to the eigenvalues predicted from a random matrix.

Note that the largest eigenvalue is completely out of scale. We also have several eigenvalues that are bellow the minimum theoretical eingenvalue. The next picture shows eigenvalue e_{25}, which corresponds to a combination of all indices in a market movement that explains about 38% of the collective movement of all indices.

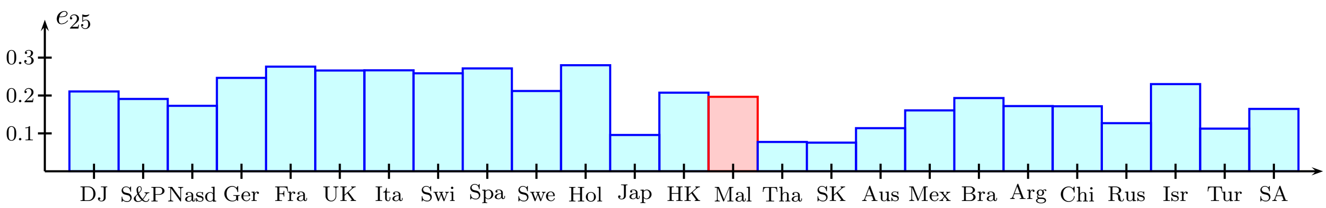

Figure 15: contributions of the stock market indices to eigenvector e_{25}, corresponding with the largest eigenvalue of the correlation matrix. Blue bars indicate positive values, and red bars correspond to negative values.

Note that most indices have similar participation in the index, with the USA and European indices appearing with the largest components for the eigenvector. The exception is the Malysian index, which has negative participation, what indicates that it went against the market on that particular year or, at least, is out of phase with it.

Figure 16 shows the Participation Rate (PR) for the 25 eigenvectors, in ascending order of eigenvalues from the left to the right.

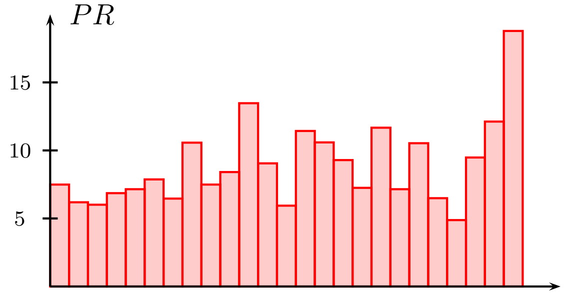

Figure 16: Participation Rate (PR) for the 25 eigenvectors, in ascending order of eigenvalues from the left to the right.

Again, the participation ratio for the largest eigenvalue is larger than the usual, indicating that most indices participate significantly in the corresponding eigenvector. Most of the eigenvectors corresponding to noise fall around the average number 9. For the eigenvectors corresponding to the lowest eigenvalues, the participation ratio is smaller than the average.

Next, we analyze the hierarchical structure that is revealed to us by the correlation matrix using definition (8) for a metric that establishes "distances" between the markets. Using the minimum spanning tree technique on the distance matrix so obtained, one can make the following diagram.

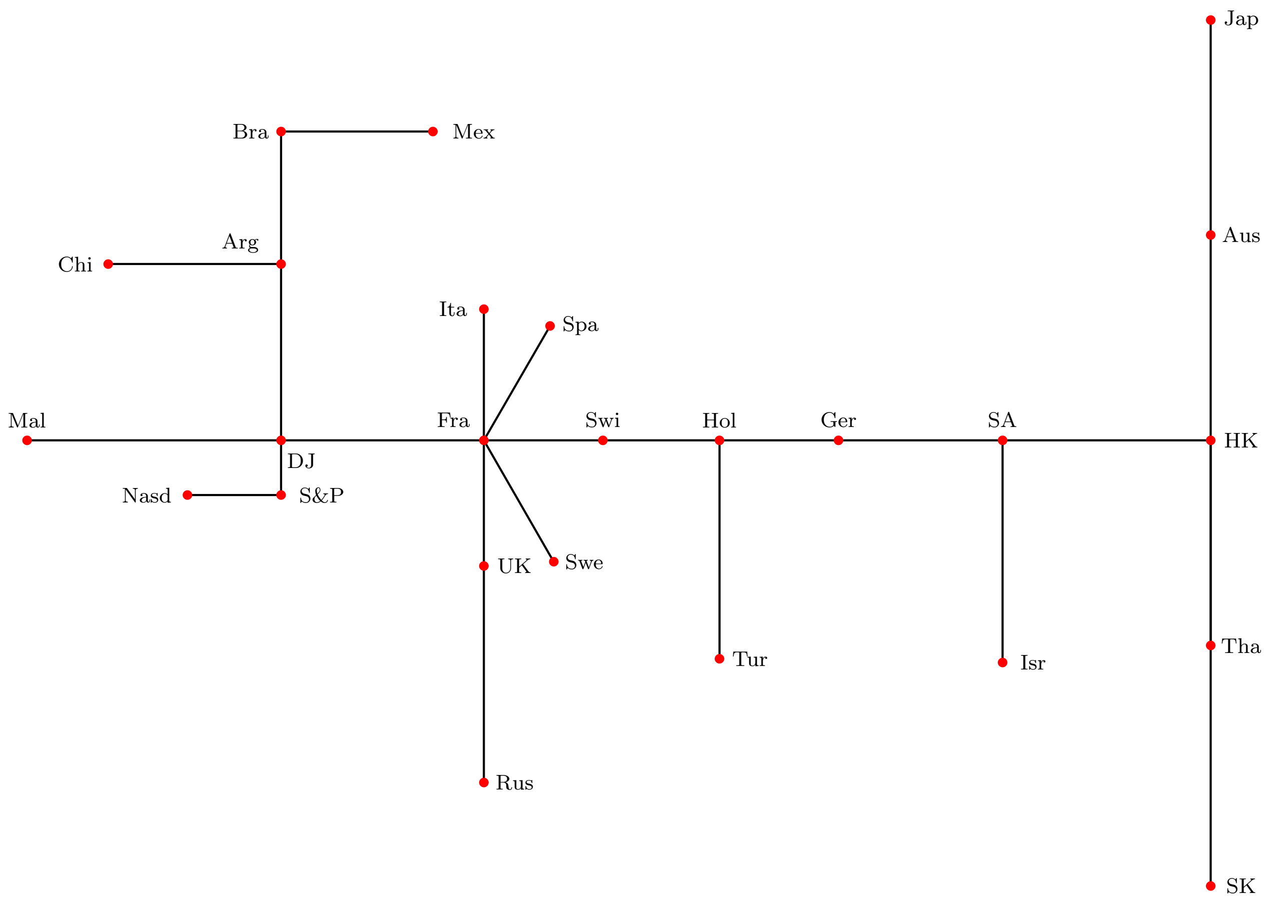

Figure 17: hierarchical structure of stock market indices obtained from the correlation matrix between them.

We next represent the distances between the many indices using a three dimensional map from two points of view.

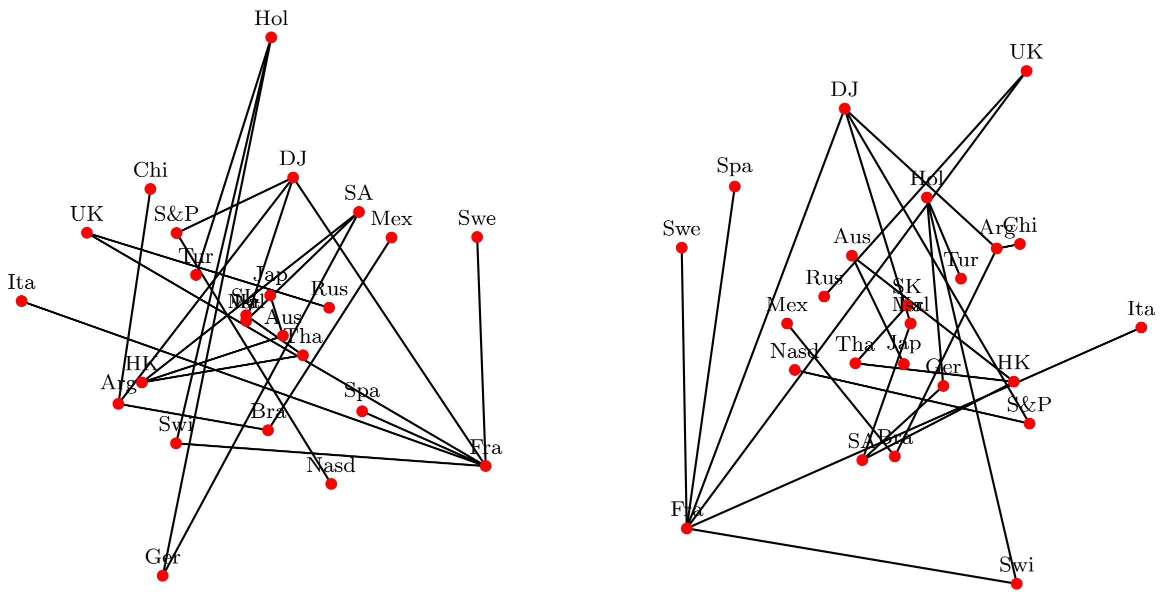

Figure 18: two different views of the three dimensional hierarchical structure between the stock market indices.

The clusters are now much more compact than in the previous case (1987), so it is difficult to separate some of the indices. Nevertheless, using the figures with the connections, one can see a cluster of the European countries, centered around France, the usual cluster of the indices from the USA, another cluster of Eastern Asian countries, and a cluster of Latin American countries.

Figure 19 plots the market volatility, together with the average correlation between the indices for the year of 1998, using windows of 30 days, and representing the average correlation of each window as a correlation of the last day of that window. Both are normalized so as to have mean zero and standard deviation one. The normalized average volatility is in blue, and the normalized average correlation is in red.

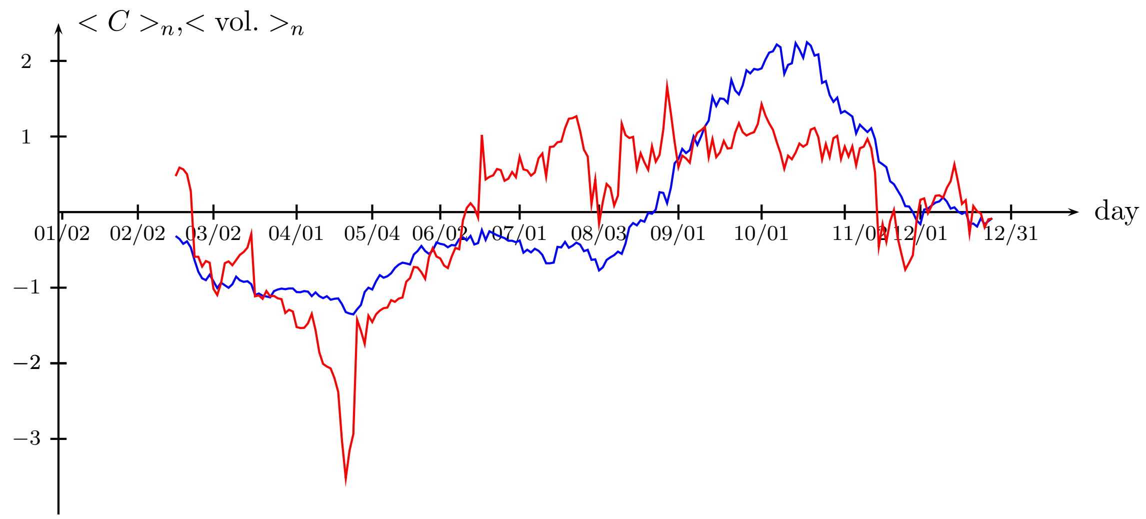

Figure 19: average volatility (blue) and average correlation (red) based on the log-returns for 1998, both calculated in moving windows of 30 days and normalized so as to have mean zero and standard deviation one.

The covariance between the volatility and the average correlation in moving window of 25 days is plotted in figure 20. One can verify that the covariance between them increases during and after the Russian crisis.

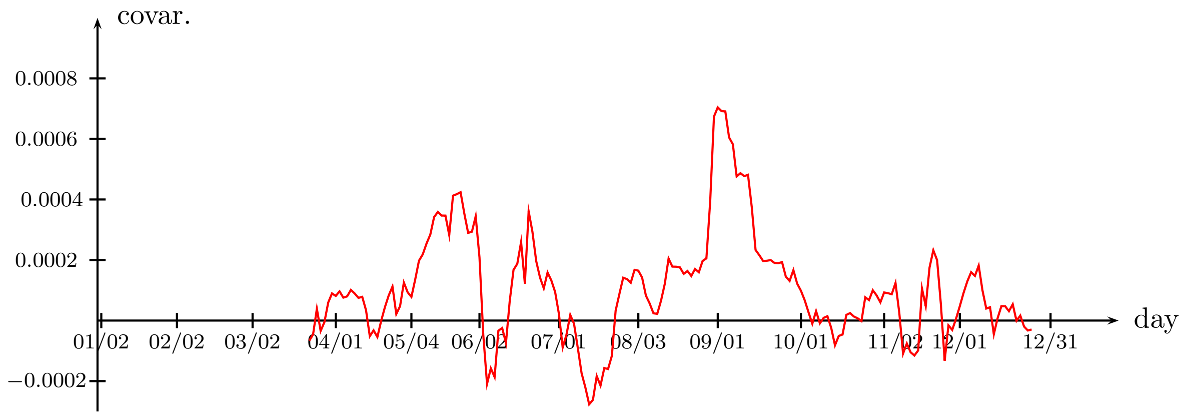

Figure 20: covariance between volatility and average correlation as a function of time.

6 2001, burst of the dot-com bubble and September 11th

On September, 11th, 2001, the world was shocked, as the biggest terrorist atack in human history was perpetrated against the USA. The death toll was close to 3,000, when two hijacked airplanes were thrown into the Twin Towers of the World Trade Center, in New York, one hit the Pentagon, in Virginia, and another fell in Pennsylvania. Financial markets all over the world plummeted, in an uncertainty crisis that lasted a few days.

On that same year, closer to March, there was the end of a financial bubble centered on internet-based companies, the so-called burst of the dot-com companies. That crash affected most markets in the world and is believed to be a result of an escalation of speculation with companies whose true values were much bellow the prices their stocks were being negotiated with.

Here we analyze these two crises, one (September 11th) which is a good exemple of a crisis which is caused by a completely exogenous cause, and the other (burst of the dot-com bubble) which is the result of high speculation on stock prices. We start by analyzing the log-returns of the 24 indices of markets all over the world. In our research, we used two indices of the same market, the Dow Jones and the S&P 500, of the New York Stock Exchange, the Nasdaq index (USA), DAX (Germany), FTSE (UK), CAC (France), FTSE Italia (Italy), IGBM (Spain), AEX (Netherlands), OMX Nordic (Sweden), (SMI) Switzerland, RTS (Russia), Nikkei (Japan), Hang Seng (Hong Kong), Shanghai Composite (China), KLCI (Malaysia), SET (Thailand), Ibovespa (Brazil), Merval (Argentina), IPC (Mexico), IPSA (Chile), TA-25 (Israel), ISE 100 (Turkey), and FTSE/JSE (South Africa). The indices were chosen so as to be representatives of a diversity of financial markets.

Using the modified log-returns (5) based on the closing indices from 01/02/2001 to 12/31/2001, we built a 24×24 correlation matrix between those. This matrix (which we shall not reproduce here) has an average correlation $< C >= 0.3488$, standard deviation $\sigma = 0.0402$, and is based on $L = 252$ days for the $M = 24$ indices, which gives $Q = L/M = 252/24 = 10.5$.

The upper and lower bounds of the eigenvalues of the Marĕnko-Pastur distribution (3) are

$$\lambda_- = 0.4780 \quad \text{and} \quad \lambda_+ = 1.7124 \ . \tag{10}$$

The frequency distribution of the eigenvalues is displayed bellow, plotted against the theoretical Marĕnko-Pastur distribution were it an infinite random matrix with mean zero and standard deviation 1.

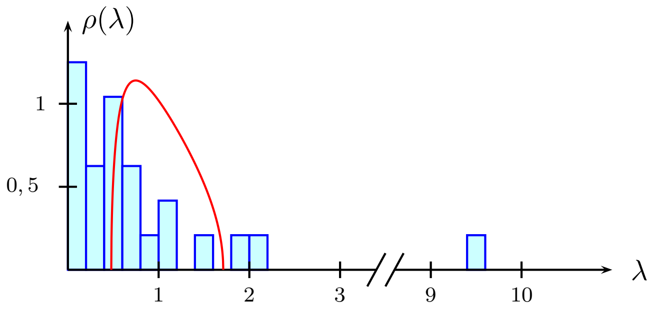

Figure 21: frequency distribution of the eigenvalues of the correlation matrix for 1987. The theoretical distribution is superimposed to it.

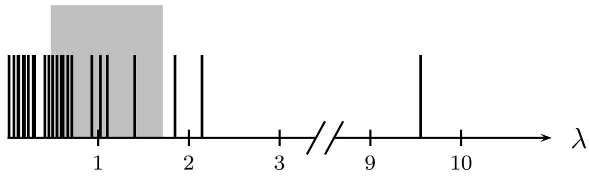

Figure 22: eigenvalues in order of magnitude. The shaded area corresponds to the eigenvalues predicted from a random matrix.

Note that the largest eigenvalue is completely out of scale. We also have several eigenvalues that are bellow the minimum theoretical eingenvalue. The next picture shows eigenvalue e_{24}, which corresponds to a combination of all indices in a market movement that explains about 40% of the collective movement of all indices.

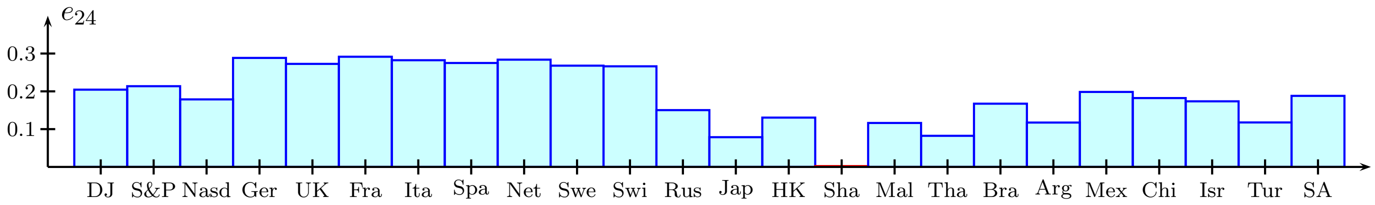

Figure 23: contributions of the stock market indices to eigenvector e_{25}, corresponding with the largest eigenvalue of the correlation matrix. Blue bars indicate positive values, and red bars correspond to negative values.

Note that most indices, with the exception of Shanghai, with participation virtually zero, have similar participation in the index, with the USA and European indices appearing with the largest components for the eigenvector.

Figure 24 show the Participation Ratio (PR) for the 24 eigenvectors, in ascending order from the left to the right.

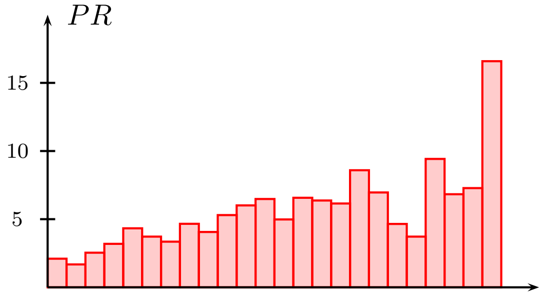

Figure 24: Participation Rate (PR) for the 25 eigenvectors, in ascending order of eigenvalues from the left to the right.

Again, the participation ratio for the largest eigenvalue is larger than usual, indicating that most indices participate significantly in the corresponding eigenvector. Most of the eigenvectors corresponding to noise fall around the average number 9. For the eigenvectors corresponding to the lowest eigenvalues, the participation ratio is smaller than the average.

Next, we analyze the hierarchical structure that is revealed to us by the correlation matrix using definition (8) for a metric that establishes "distances" between the markets. Using the minimum spanning tree technique on the distance matrix so obtained, one can make the following diagram.

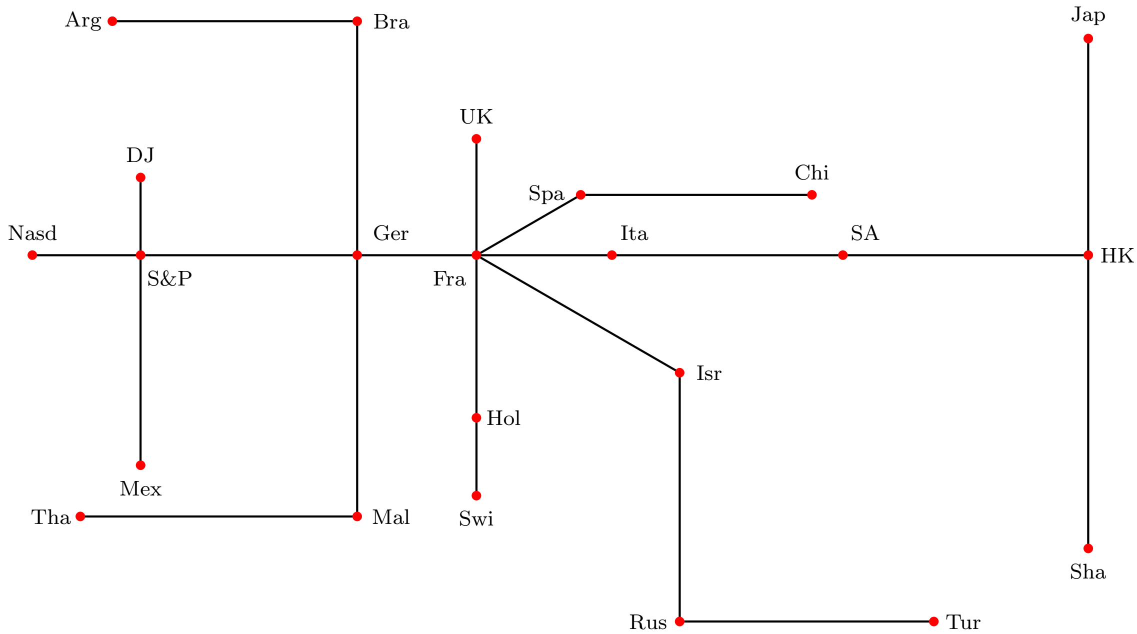

Figure 25: hierarchical structure of stock market indices obtained from the correlation matrix between them.

We next represent the distances between the many indices using a three dimensional map from two points of view.

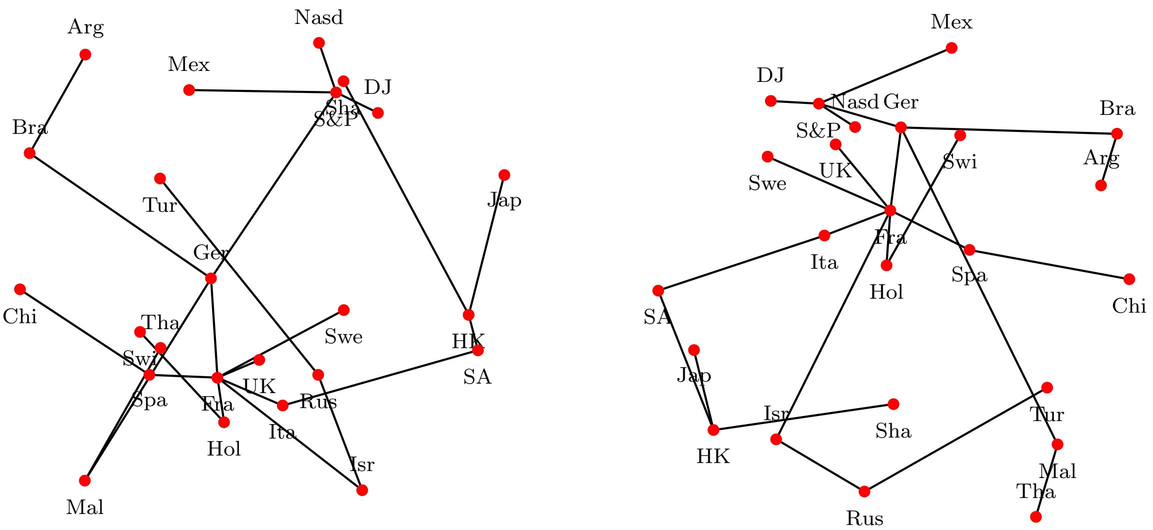

Figure 26: two different views of the three dimensional hierarchical structure between the stock market indices.

A cluster comprised of the three major indices of the USA's financial market (Dow Jones, S&P, and Nasdaq), together with the IPC (Mexico), is clearly visible. The indices of the European financial markets form another cluster, and one can also see another cluster whose components are the Nikkei (Japan), the Hang Seng (Hong Kong), and the SH Composite (Shanghai, China). Argentina is connected with Brazil, Malaysia is connected with Thailand, and Russia, Turkey, and Israel seem to be connected as well. There are some strange connections, like Spain and Chile, and South Africa connecting Italy and Hong Kong. Most of the connections relative to larger distances cannot be trusted, although the three dimensional graphic represents the approximate distance between all indices rather faithfully.

Figure 27 shows the average correlation and the average volatility, both calculated on windows of 25 days, normalized so as to have average zero and standard deviation 1. The correlation between both measures becomes more transparent in this framework. The normalized average volatility is in blue, and the normalized average correlation is in red.

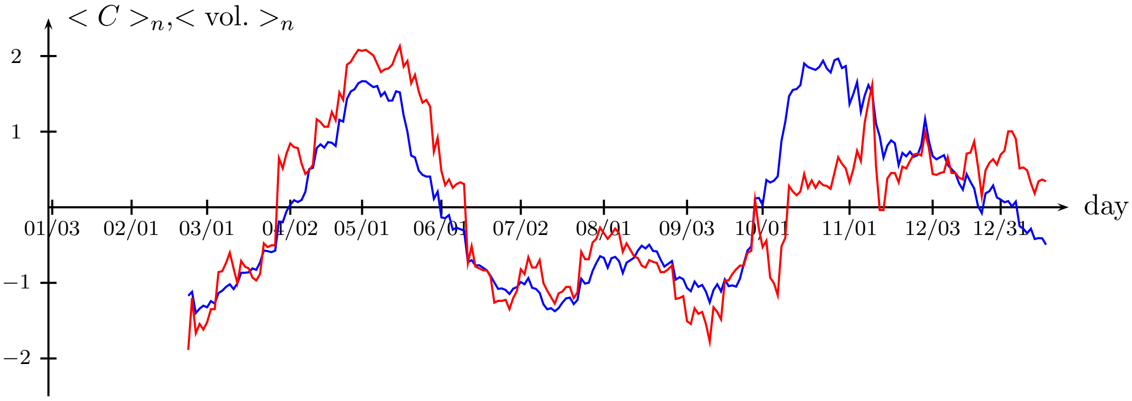

Figure 27: average volatility (blue) and average correlation (red) based on the log-returns for 2001, both calculated in moving windows of 25 days and normalized so as to have mean zero and standard deviation one.

The covariance between the volatility and the average correlation is plotted in the figure 28. One can readily identify peaks around the time of the burst of the dot-com bubble and September 11th.

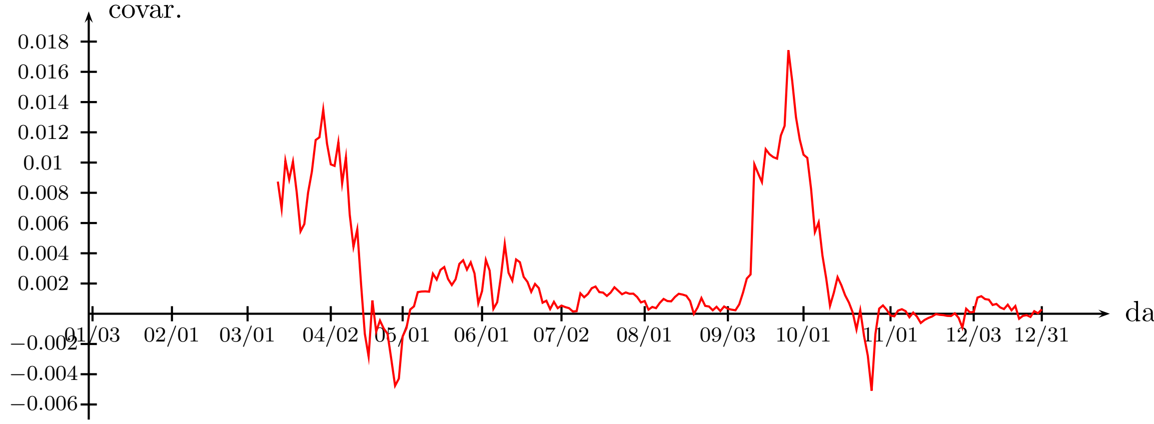

Figure 28: covariance between volatility and average correlation as a function of time.

7 2008, Subprime mortgage crisis

The last large financial crisis began in 2007, reached its peak in 2008, and is happening until now. This crisis was triggered by the default of a large number of mortgages in the USA. Subprimes are loans to borrowers who have low credit stores. Most of them had a small initial interest rate, adjustable for future payments, which led to many home foreclosures after the rates climbed substantially. Meanwhile, the loans were transformed in pools that were then resold to interested investors. Since the returns of such investments were high, a financial bubble was created, inflating the subprime mortgage market until the defaults started to pop up.

Because of their underestimation of risk, financial institutions worldwide lost trillions of dollars, and many of them declared banckrupcy. Because of that, credit lines tightened around the world, taking the financial crisis to the so called real economy. The world is yet to recover from this crisis, and many institutions are still to lose a good part of their assets in the following years due to this crisis.

Here we analyze the year 2008, which is considered the time when the subprime crisis reached its peak, marked by events like the Lehman Brothers' announcement of banckrupcy, and the liquidation of three of the largest investment banks in the USA. In our research, we used three indices for USA (Dow Jones and S%P, both related with the NYSE, and the Nasdaq), two more indices for North America (Canada and Mexico), 5 indices for South America (Brazil, Argentina, Chile, Colombia, and Peru), 11 indices for Europe (Germany, UK, France, Italy, Netherlands, Sweden, Switzerland, Spain, Portugal, Greece, and Russia), 10 indices for Asia (Japan, Hong Kong, China, South Korea, Malaysia, Singapore, Taiwan, Thailand, India, and Israel), 2 indices for Oceania (Australia and New Zealand), and 3 indices for Africa (South Africa, Nigeria, and Ghana).

Using the modified log-returns (5) based on the closing indices from 01/02/2008 to 12/31/2008, we built a 36×36 correlation matrix between those. This matrix (which we shall not reproduce here) has an average correlation $< C >= 0.4619$, standard deviation $\sigma = 0.2588$, and is based on $L = 254$ days for the $M = 36$ indices, which gives $Q = L/M = 254/36 = 7.1$.

The upper and lower bounds of the eigenvalues of the Marĕnko-Pastur distribution (3) are

$$\lambda_- = 0.3888 \quad \text{and} \quad \lambda_+ = 1.8947 \ . \tag{11}$$

The frequency distribution of the eigenvalues is displayed bellow, plotted against the theoretical Marĕnko-Pastur distribution were it an infinite random matrix with mean zero and standard deviation 1.

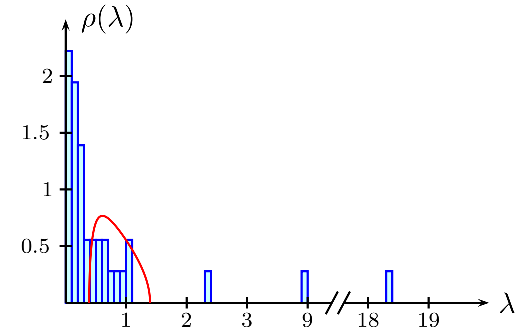

Figure 29: frequency distribution of the eigenvalues of the correlation matrix for 1987. The theoretical distribution is superimposed to it.

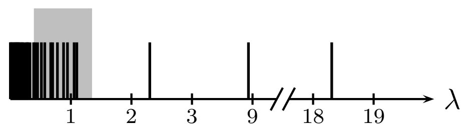

Figure 30: eigenvalues in order of magnitude. The shaded area corresponds to the eigenvalues predicted from a random matrix.

Note that the largest eigenvalue is, once more, completely out of scale. We also have several eigenvalues that are bellow the minimum theoretical eingenvalue. The next picture shows eigenvalue e_{36}, which corresponds to a combination of all indices in a market movement that explains about 51% of the collective movement of all indices.

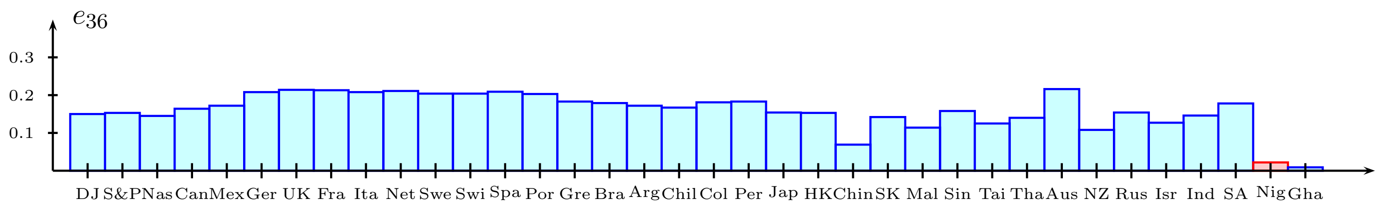

Figure 31: contributions of the stock market indices to eigenvector e_{36}, corresponding with the largest eigenvalue of the correlation matrix. Blue bars indicate positive values, and red bars correspond to negative values.

Most indices, with the exception of Nigeria, with a small negative participation, and China and Nigeria, with small values for the eigenvector, have similar participation in the index, with the USA and European indices appearing with the largest components for the eigenvector.

Figure 32 shows Participation Ratio (PR) for the 36 eigenvectors, in ascending order from the left to the right.

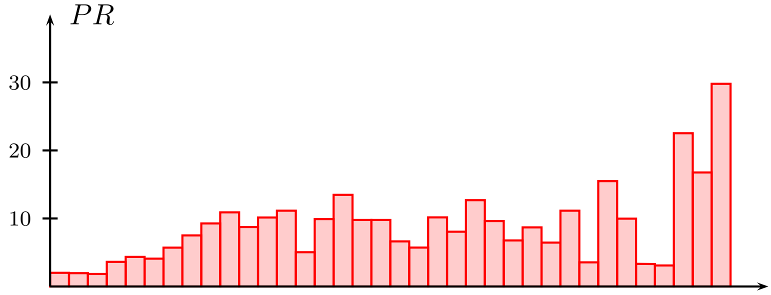

Figure 32: Participation Rate (PR) for the 25 eigenvectors, in ascending order of eigenvalues from the left to the right.

One can see that, for the largest eigenvalue, the participation ratio is almost equal to the number of indices we are using. Most of the eigenvectors corresponding to noise fall around the average number 8.

The hierarchical structure that is revealed to us by the correlation matrix is analyzed next using definition (8) for a metric that establishes "distances" between the markets. Using the minimum spanning tree technique on the distance matrix so obtained, one can make the following diagram.

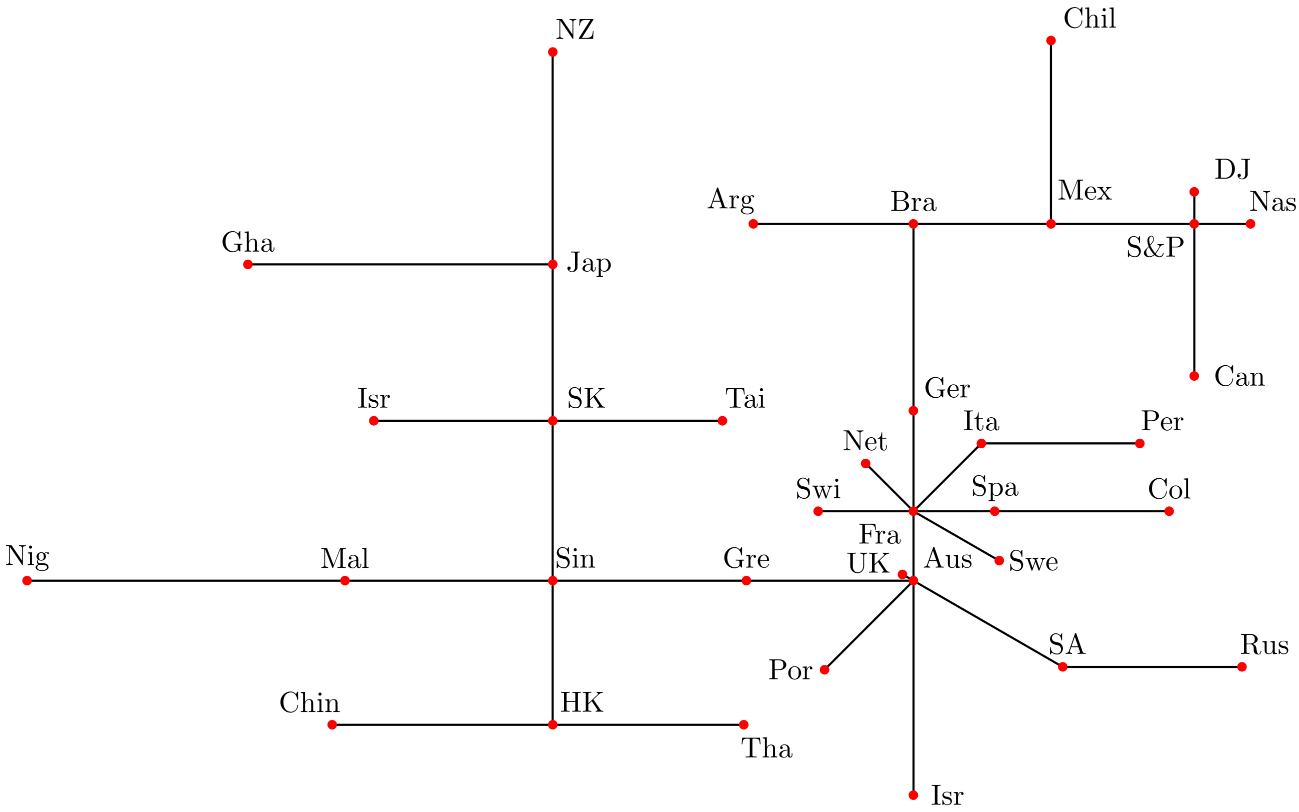

Figure 33: hierarchical structure of stock market indices obtained from the correlation matrix between them.

We next represent the distances between the many indices using a three dimensional map from two points of view.

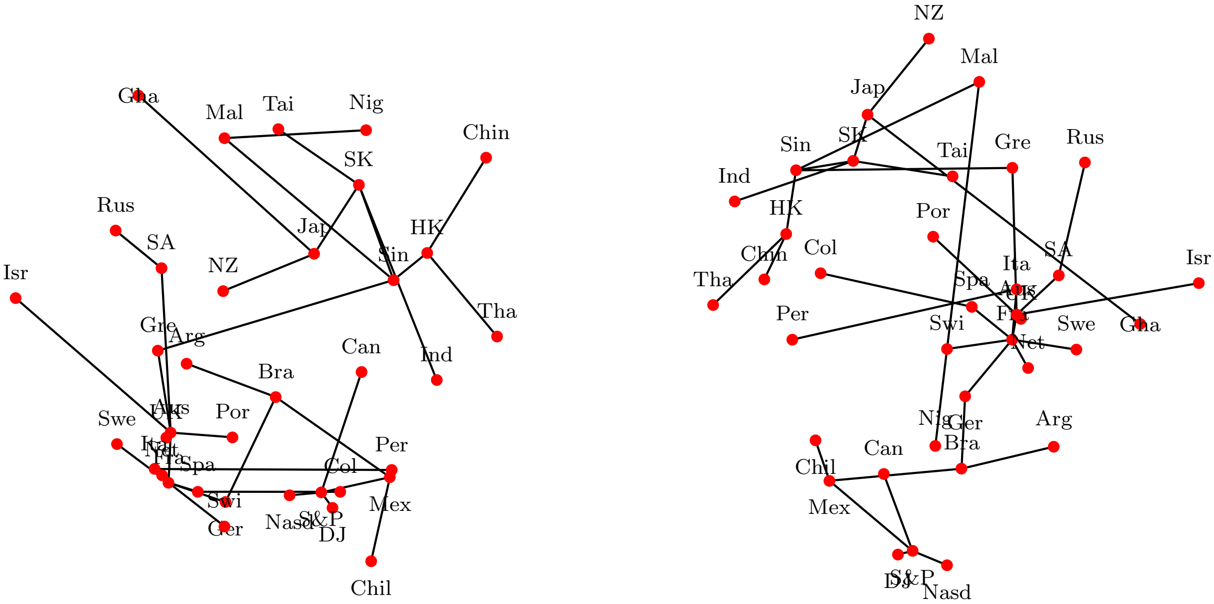

Figure 34: two different views of the three dimensional hierarchical structure between the stock market indices.

One can see clear cluster comprised of the North American indices (USA, Canada, and Mexico), connected with South America through Brazil. There is a second cluster of European markets centered around France, and the East Asian markets, more loosely connected, are in another cluster. Markets that are not strongly connected with any particular cluster occupy random positions in the outskirts of the maps.

The largest eigenvalue of the correlation matrix can be used in order to create a "market index" which expresses the average behavior of the international financial market as a whole. The following graphic shows the mean of the correlation matrix calculated in moving windows of 50 days (in red), plotted against the averages of the volatilites (absolute value of the standard deviation) of the market index, also in moving windows of 50 days (in blue). Both sets of data are normalized in order to have mean zero and standard deviation one.

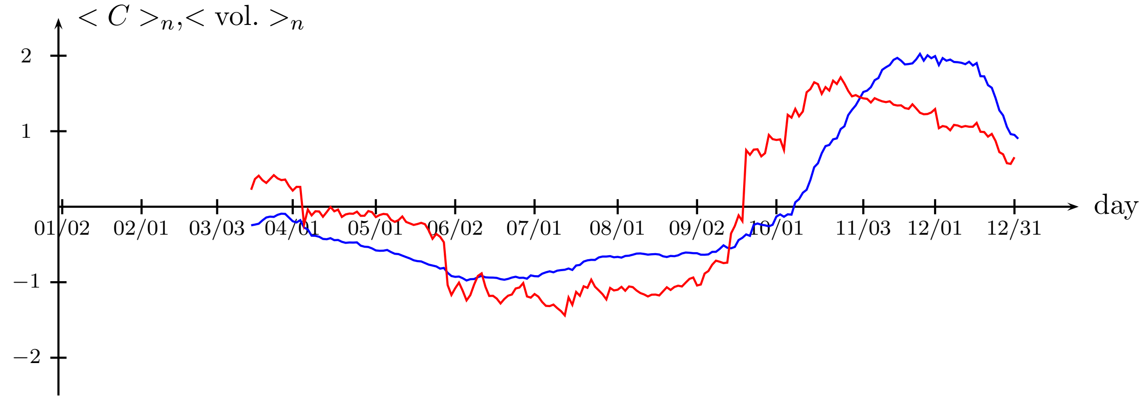

Figure 35: average volatility (blue) and average correlation (red) based on the log-returns for 2008, both calculated in moving windows of 50 days and normalized so as to have mean zero and standard deviation one.

One can see that the period of high volatiltiy seems to be preceded by a period of high correlation between the stock marckets of the world. The next picture shows the evolution of the covariance between the mean correlation and the mean volatility, calculated in moving windows of 25 days.

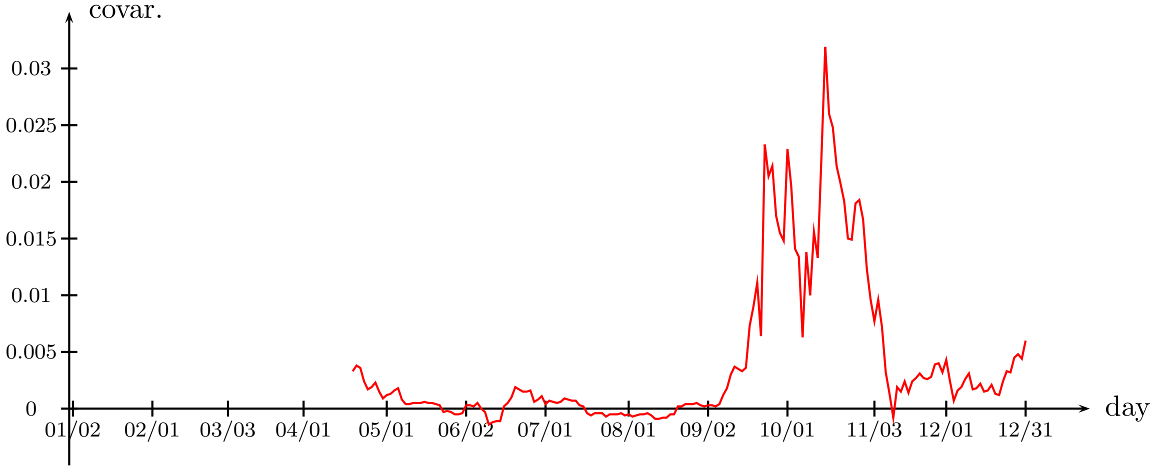

Figure 36: covariance between volatility and average correlation as a function of time.

8 Conclusion and future research

Using the correlation matrices of the log-returns of a diversity of market indices during times of crisis, we showed that markets tend to behave similarly during times of high volatility. We also showed that the correlation between markets has been growing for the past decades, and that one can devise a hierarchical structure between them that has remained quite stable during the period of time we studied them.

Some direction for future research is to use in our studies other correlation measures other than the Pearson correlation coefficient, such as Spearman's rank correlation and the Kendall tau correlation, which are more appropriate for non-linear correlations between variables. Another research topic is to analyze how the techniques used in this work are modified if we consider that the frequency distributions of the log-returns are not Gaussian. Some of the results obtained here shall also be used in our studies of financial markets as coupled damped harmonic oscillators subject to stochastic perturbations [108].

Acknowledgements

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