Detecting and analyzing nonstationarity in foreign exchange market

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Financial time series have been studied by using variety of methods in econophysics. Most of studies assume implicitly stationarity of the time series, but this is not an obvious assumption. Therefore, it is important to check whether a time series is stationary or not and detect stationary regions in an observational record. To this end, we analyzed the dollar-yen exchange rate from January 1998 to December 2008, provided by ICAP EBS with a recording frequency of every one second [1, 2]. We focused on the time series of 1-tick price changes of mid-quote.

Formally, a stochastic process X_t is called (strongly) stationary if for any set of times $t_1, t_2, \cdots, t_n$ and any k the joint probability distributions of $\{X_{t_1}, X_{t_2}, \cdots, X_{t_n}\}$ and of $\{X_{t_1+k}, X_{t_2+k}, \cdots, X_{t_n+k}\}$ coincide. Based on this definition, we divide the whole time series into some sub-series with one-day

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period, and then test whether these sub-series follow the same probability distribution. For the comparison of the probability distribution of the i th and the $(i+t)$ th day, the two-sample Kolmogorov–Smirnov test is applied. This test has the advantage of being nonparametric, and without making assumptions about the distribution function of the data, it returns the probability P_{it} that two sets of data are drawn from the same distribution. We find that the probability P_{it} monotonically decreases as the lag t increases.

We also analyze the statistical properties of the recurrence intervals τ between the days nonstationarity occurs. The probability distribution of recurrence intervals follows an exponential distribution. The mean conditional interval $\langle \tau | \tau_0 \rangle$, which is defined as the mean of recurrence intervals τ conditioned on the preceding interval τ_0, is independent of τ_0. This indicate that there is no memory effect in recurrence intervals. Therefore, the recurrence intervals can be modeled as Poisson process and the probability of being nonstationary is constant in time. A possible interpretation of this results is that the nonstationarity is caused by external events and news occurring at random.

References

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