LARGE CORRELATIONS AS A SIGNAL OF INSTABILITY IN STOCK MARKET

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Abstract
We analyze the cross-correlation between stock returns of the constituent issues of FTSE 100 index listed on London Stock Exchange for the two period: a) The period from May-2007 to Jan-2009, which includes the period that we have experienced the drastic price change, due to the US sub-prime crisis, and b) the period from July-2004 to December-2004 by way of comparison. As a result of the day-to-day principal component analysis of the time series sampled at the 1minute time interval during the continuous auction of the daytime, we find the long range up to a couple of month auto-correlation of the maximum eigenvalue of the correlation matrix. It also correlates with the drawdowns of each issue, which are the cumulative values of successive losses. Using those results, we propose, as a risk measurement, the probability of large drawdowns conditioned on the maximum eigenvalue threshold as a market signal which notices the intensification of the herding behavior of the prices.

Keywords
cross-correlation, stock market, market risk, US sub-prime crisis, principal component analysis

1 Introduction

Recently, many stock markets in the world underwent heavy falls in the stock prices, due to the ripple effects of the US sub-prime crisis. In some situation, a piece of exogenous information can trigger such a financial crisis as this case. The precursor and the dynamics of the rising of market risks are valuable to study for financial risk control.

Risks are conventionally measured based on the historical probability distribution of each asset return, e. g. , historical volatility, VaR and so on, where the dependence between assets or portfolios the dependence upon the historical values are ignored. Historical probability distributions are momentarily updated by the accumulation of coming data. However, the change is very slow. As in the case of recent economic crisis, in which unexpected large fluctuations of asset price was triggered by the affairs happened outside the relevant market, the conventional risk managements cannot catch up with the structural change of markets. In addition, it is known that the drastic price change as in abrupt rise or crash has sometimes different statistical characteristics from that of ordinary price changes[1]. Those are outliers[2], and it is not appropriate to extrapolate them from the accumulated past data.

What causes such unexpected drastic price changes? According to the theory of efficient market hypothesis of the mainstream attitude of economics, the price moves due to news which come as a surprise to market participants. However, many previous works suggest that this picture of price movement is not empirically acceptable[3][4]. The framework presented in this paper is entirely different from the conventional picture of price movement. We regard news as merely a trigger of the subsequent large price change, as in the case of the emergence of spontaneous collective moves triggered by endogenous fluctuation. In this picture, the sensitivity of the market to exogenous or endogenous fluctuations is supposed to be crucial.

The cross-correlations of returns among various stocks carry the signals about market risks. If we take an example out of statistical physics, the cross-correlation coefficient indicates the sensitivity of the stock price to external forces according to the fluctuation-response theorem[5]. In this paper, We analyze the cross-correlation between stock returns of the constituent issues of FTSE 100 index listed on London Stock Exchange for the two period:

a) The period from May-2007 to Jan-2009, which includes the period that we have experienced the drastic price change, due to the US sub-prime crisis.
b) The period from July-2004 to December-2004 by way of comparison.

As a result of the day-to-day principal component analysis of the time series sampled at the 1minute time interval during the continuous auction of the daytime, we find the long range up to a couple of month autocorrelation of the maximum eigenvalue of the correlation matrix. t also correlates with the drawdowns of each issue, which are the cumulative values of successive losses. Using those results, we propose , as a risk measurement, the probability of large price movement conditional on the maximum eigenvalue as a market signal threshold which notices the intensification of the herding behavior of the prices.

2 The first principal component

First of all, we show the histogram of the off-diagonal components (fig.1) and the empirical probability distribution function of eigenvalues (fig.2) of the cross-correlation matrix of log returns sampled at the 1minute time interval during the continuous auction of the daytime of July 2007 which is included in the period a).

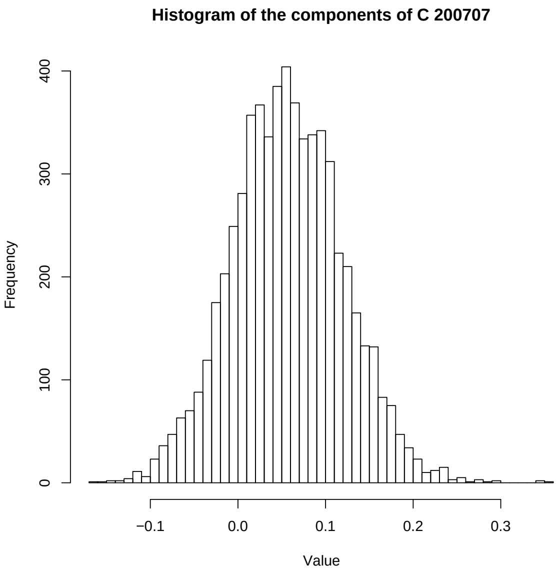

Figure 1: The histogram of the off-diagonal components of the cross-correlation matrix of log returns sampled at the 1minute time interval during the continuous auction of the daytime of July 2007.

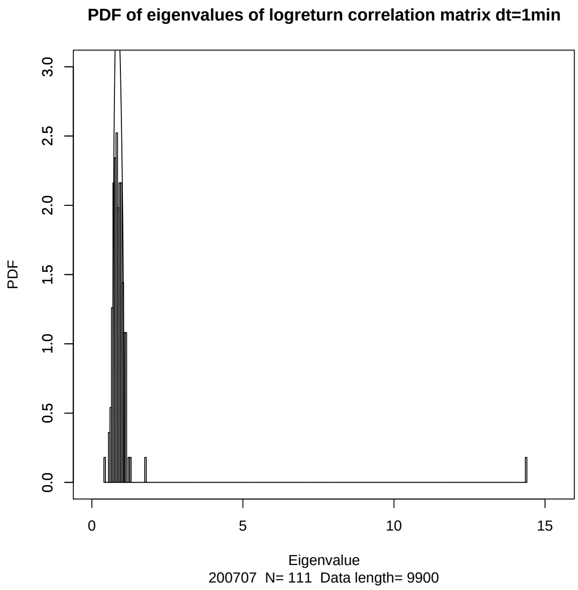

Figure 2: The empirical probability distribution function of eigenvalues of the same matrix of fig. 1. The solid line presents the prediction by the random matrix theory.

The null hypothesis making the point that each time series is independent of others is presented by solid line, which is predicted by the random matrix theory[6]. Most eigenvalues are restricted inside the closed region. Several eigenvalues are located separately from the region. The maximum eigenvalue for this month is about 14, which varies month by month, and even day by day. It means that the first principal component, which is called market mode because the weight of the mode are uniformly distributed among all issues of FTSE 100 index, is $114/111 \times 100 = 13\%$ dominant to the whole amplitude of all time series. In fig. 3, we show the time evolution of the maximum eigenvalue of the cross-correlation matrix and FTSE100 index for the period a).

3 The dependence of the maximum eigenvalue and a new risk measurement

The maximum eigenvalue seems to increases when the stock price heavily changes. The cross-correlation between the drawdowns and the maximum eigenvalue for the dataset a) is indeed statistically significant (fig.4), while it is not the case for the dataset b)(fig.5).

Moreover, the maximum eigenvalue for the dataset a) has long memory up to a couple of month(fig.5), while it is not the case for the dataset b) (fig.6) nor the daily return for the both datasets.

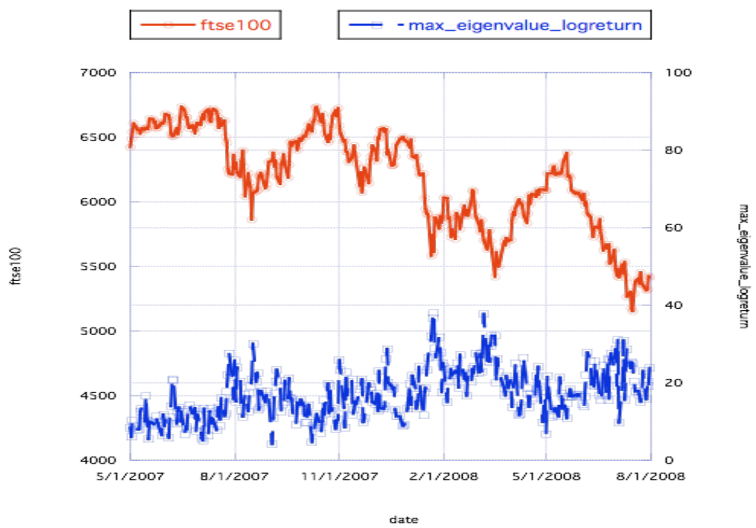

Figure 3: Time evolution of the maximum eigenvalue of the cross-correlation matrix and FTSE100 index for the period a).

Using those results, we will propose , as a risk measurement, the probability of large drawdowns conditional on the maximum eigenvalue λ_{max} as a market signal which notices the intensification of the herding behavior of the prices. We analyze the conditional probability of the amplitude of daily return $P(The\, largest\, drawdown\, of\, the\, day > s|\lambda_{max} > t)$ for the dataset a). For the issues with high factor loading, the probability tends to increase according to the raise of the threshold. In fig.8 , the conditional probability $P(The\ largest\ drawdown\ of\ the\ day > s|\lambda_{max} > t)$ for the issue HSBA is shown as an typical case. We recognize from the figure that the drawdowns over 0.5%, which occur with unconditional probability 0.3 or less, occur with probability 0.8 or more conditioned on the maximum eigenvalues over 30.

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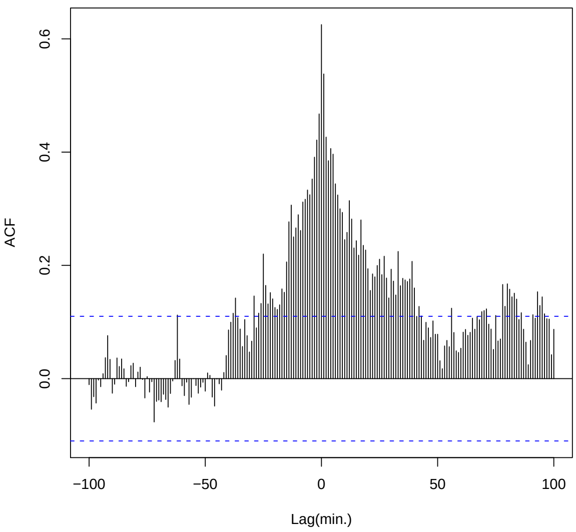


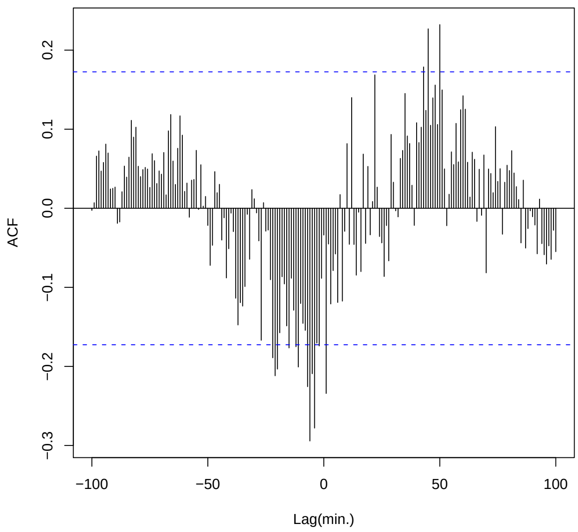
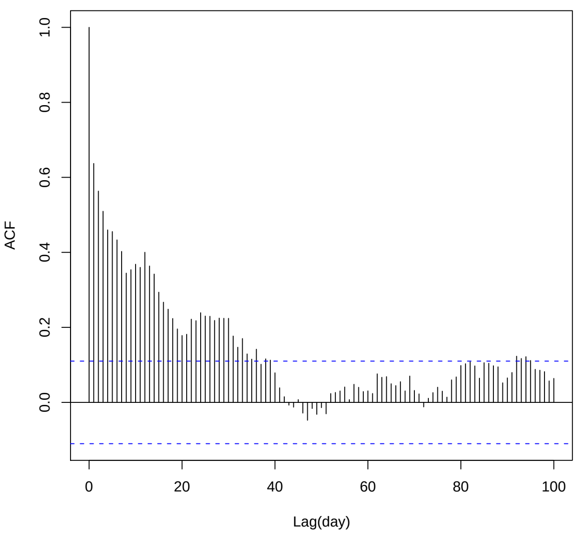


Figure 4: Cross-correlation between the drawdown of the issue HSBA and the maximum eigenvalue for the dataset a).

Figure 5: Cross-correlation between the drawdown of the issue HSBA and the maximum eigenvalue for the dataset b).



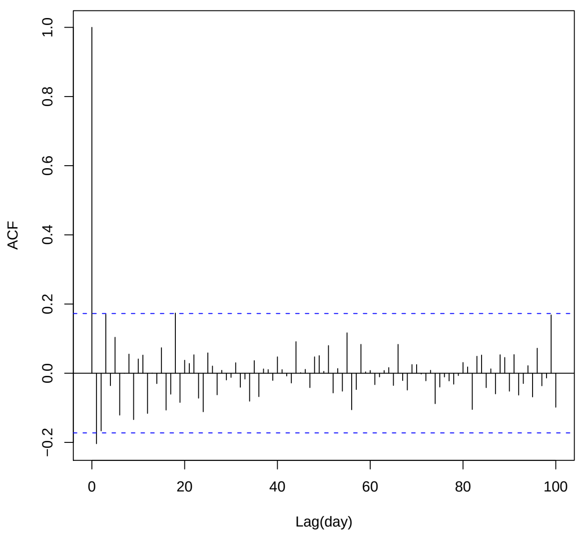


Figure 6: Autocorrelation function of the maximum eigenvalue for the dataset a).

Figure 7: Autocorrelation function of the maximum eigenvalue for the dataset b).

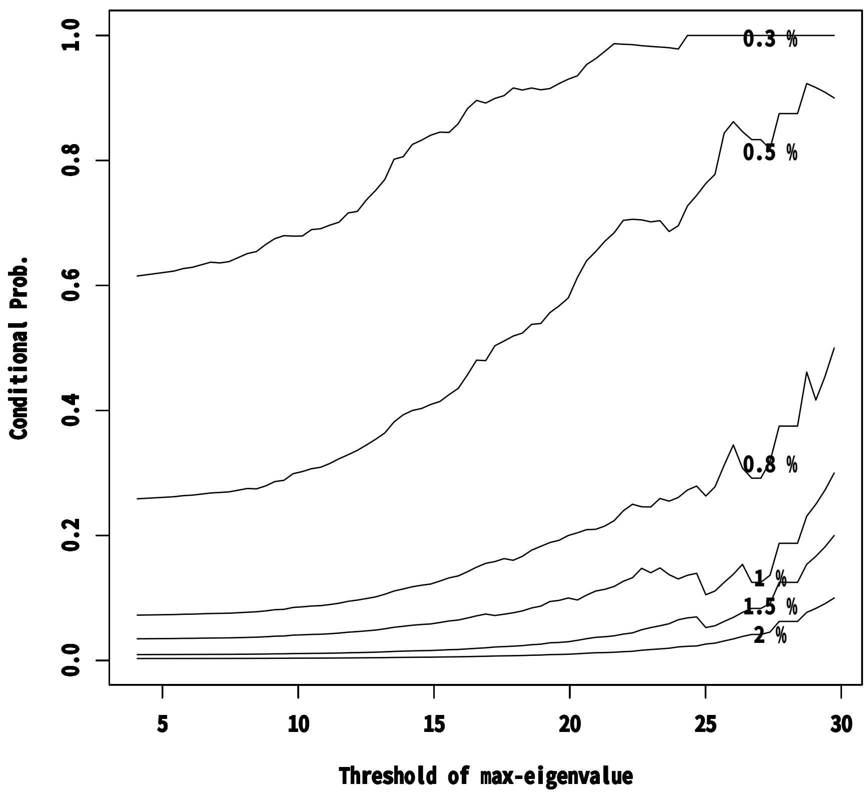


Figure 8: Conditional probability of the amplitude of the largest drawdown of the day $P(The\ largest\ drawdown\ of\ the\ day > s|\lambda_{max} > t)$ for the dataset a). Issue:HSBA. The horizontal axis is threshold t of maximum eigenvalue λ_{max}. The value attached to each line is the threshold of the largest drawdown of the day.