Financial models for tight markets: the case of power markets, spikes and antispikes

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EXTENDED ABSTRACT

Electricity price time series show peculiar patterns very different from those found in the more studied stock and bond price series. Standard top-down models try to capture electricity prices behavior in path and distributional properties only, whereas hybrid models try to include in a few degrees of freedom physical and organizational features coming from the intrinsic network structure of power markets. In such markets agents coordinate strategically through partially competitive market insitutions about electricity delivery through the physical power grid. Consequently, network congestions and other tight market conditions can show up in price series as strong nonlinearities and threshold effects, and often appear as price spikes. Effective models should be able to relate microeconomics with complex financial behavior.

In this talk, two threshold nonlinear hybrid models for electricity prices will be presented, that use a Hopf critical point stochastic dynamics to generate price spikes, and that are able to include in their degrees of freedom typical and more general tight markets behavior. The models will be presented in the frame of TARX (Threshold AutoRegressive eXternally driven) and switching regime modelling of electricity time series [1] and they will be shown to take into account some basic but essential microeconomic features of real power markets.

The major microeconomic feature considered is the presence in the electricity market of some factors, like capacity constraints and the effect of power grid congestions [2], that can act on prices at varying levels of demand. These factors introduce a demand threshold in the price formation mechanism. Below the threshold prices react smoothly to demand variations, above the threshold prices can react in a non-smooth way, with spike-like patterns.

The following continuous-time model

$$\epsilon \dot{x} = g^S(x;a) - y \tag{1a}$$

$$\dot{y} = x - \gamma_b\, y + b + f(t) + \sigma(d)\,\xi(t), \tag{1b}$$

where

$$g^S(x;a) = \begin{cases} -x, & -\infty < x \leq a/2, \quad R = R_1 \\ x - a, & a/2 < x < (1+a)/2, \quad R = R_2 \\ 1 - x, & (1+a)/2 \leq x < +\infty, \quad R = R_3, \end{cases} \tag{2}$$

when discretized, turns to be a SETARX (Self-Excited TARX) model in the sense of Ref.[1] with thresholds $a/2$ and (1+a)/2, and stochastic electricity demand $f(t) + \sigma(d)\,\xi(t)$. It can accommodate spike phenomenology when used in an unusual parametric region. In Eqs. 1 and Eq. 2, x and y belong to the support of the stochastic processes $X(t)$ and $Y(t)$, $\xi(t) = \frac{dW}{dt}$ - where $W(t)$ is a Wiener process - is a stochastic driver, the parameters $\epsilon > 0$, $\gamma_b > 0$, b, $\sigma(d) = \sqrt{2d}$, and a are constants that don't change at regime changes, $f(t)$ is a deterministic function (an exogenous driver representing electricity demand, the X in TARX) that will be periodic. R_1, R_2, R_3 label the SETARX regimes.

In this three-regimes SETARX, whereas one ARX sector is set in the usual stable regime, two other sectors are set respectively in unstable and metastable regimes in a specific sequence. These two not-stable regimes together allow for nonlinear deviations from the stable regime, allowing of spikes. TARX self-excitation avoids linking the regime thresholds to data different from the prices themselves, and demand data are unnecessary to calibration as far as a sinusoidal driver is embedded in the model. If desired, real world demand data can be used as an external driver process in substitution of the embedded driver. Grid effects, capacity constraints and the presence of a forward market can be assumed at the origin of threshold effect, spikes and antispikes.

An extension of Eq. 2 allows of the inclusion of antispikes, changing $g^S(x;a)$ into

$$g_R^{SAS}(x;C_L,C_R) = \begin{cases} -\alpha_L(x+C_L), & -\infty < x \leq -C_L, \\ & R=R_1 \\ \beta_L(x+C_L), & -C_L < x < -D_L = \frac{\beta_L}{\gamma_0+\beta_L}C_L, \\ & R=R_2 \\ -\gamma_0 x, & -D_L \leq x \leq D_R = \frac{\beta_R}{\gamma_0+\beta_R}C_R, \\ & R=R_3 \\ \beta_R(x-D_R), & D_R < x < C_R, \\ & R=R_4 \\ -\alpha_R(x-D_R), & C_R \leq x < +\infty \\ & R=R_5. \end{cases} \tag{3}$$

(SAS stand for spike-antispike). In Eq. 3 all parameters are positive and the system has five regimes, two of them unstable (regimes R_2 and R_4), two left thresholds $-C_L$, $-D_L$ and two right thresholds D_R, C_R. An example of the dynamics that results combining Eqs. 1 with Eq. 3 can be seen in Fig.1. In the upper panel of Fig.1 the coordinate $x(t)$ is shown, which can be interpreted as an *electricity logprice*. The four horizontal lines mark the four thresholds, antispikes belong to regimes R_1 and R_2, spikes belong to regimes R_4 and R_5, the normal (non-tight) dynamics is confined in regime R_3. In the middle panel the graph has in its abscissa the indication about in which regime R (with $R_1=-2, R_2=1, R_3=0, R_4=1, R_5=2$) the systems finds itself at time t. In the lower panel the price $p(t)=e^{x(t)}$ is shown. Examples of the interesting effect of including a sinusoidal f will be shown. The models proposed

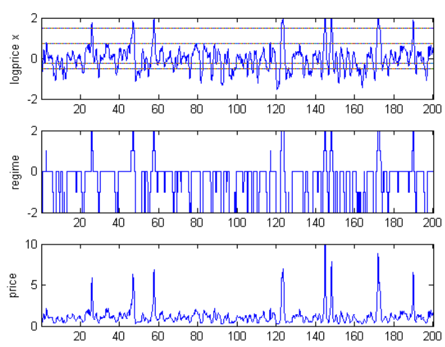

Fig. 1. SAS McKean model for $f=0$. Other parameters: $\epsilon=0.3$, $s=0.4$, $\gamma_b=1$, $\alpha_L=\alpha_R=1$, $\beta_L=\beta_R=1$, $\gamma_0=1$, $C_L=1/2$, $C_R=3/2$, $b=-1/2$. See text.

here, born as continuous-time models, can be used either in continuous-time or discrete-time contexts, for example to model assets underlying to derivative contracts on electricity or other commodities that show seasonally and irregularly peaking behavior, like gas. All this flexibility makes them useful for reduced-form top-down econometric approaches to electricity prices modelling, that want to include also some bottom-up perspective in a hybrid way.

References

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