Relationship between bid-ask spreads and fluctuations in market prices

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Abstract We investigate the impact of bid-ask spreads on the financial market fluctuations by defining an analytical model for time evolution of stock share prices. The defined model is similar to the GARCH class of models, but can additionally exhibit bimodal behaviour. Moreover, it differs from existing Ising-type models. It turns out that the constructed model is a solution of a thermodynamic limit of a Gibbs probability measure when the number of traders and the number of stock shares approaches the infinity. The energy functional of the Gibbs probability measure is derived from the Nash equilibrium of the underlying game.

Keywords: Complex systems; Financial markets; Equilibrium states; Bid-ask spread.

1. Introduction

The motivation for this paper is to provide insight into the relationship between bid-ask spreads and stock price fluctuations in the financial markets. Most financial markets have nowadays become fully electronic [1]. In these markets, any market participant can post limit orders — propositions to sell or buy a certain volume of shares at a fixed minimum or maximum price. With that market participant actually offer other traders the opportunity to trade at the posted price. At a given moment of time, the best offer on the sell side, the ask price, is higher than the best price on the buy side, the bid price, so no transaction occurs. For a transaction to happen, an market participant must issue a market order to buy or to sell a certain number of shares. The amount by which the ask price exceeds the bid is usually called the bid-ask spread. This is essentially the difference in price between the highest price that a buyer is willing to pay for an asset and the lowest price for which a seller is willing to sell it [2]. The bid-ask spread partially characterize the liquidity of the market, which is one of the most important attribute of financial markets. The focus of recent research has been to estimate the bid-ask spread, and its components, using transaction returns [3, 4]. In his seminal paper, Roll (1984) derives an implicit spread estimator in the equity market [5]. He uses a relationship between transaction price changes to estimate indirectly the effective spread in an efficient market [6], and his method only requires the transaction prices themselves.

Corwin and Schultz (2009) derive an estimator for the bid-ask spread based on daily high and low prices [7]. To demonstrate the applicability of the high-low estimator to non-U.S. markets, they estimate high-low spreads for individual stocks in Hong Kong and India using daily high and low prices [8]. A question of both theoretical and practical crucial importance is to know what impact have the spread on the stock price fluctuation [1, 9, 10, 11, 12]. Roll (1984) [5] and French and Roll (1986) [10] show that serial correlation and variance of observed price changes are both affected by spreads . Here, we will develop a model of stock price evolution, which would exhibit statistical properties very close to the empirical findings [13, 14, 15]. Based on the developed model and on the two aforementioned spread estimators, we will show how bid-ask spread implies the positive autocorrelation in the absolute stock price returns. Also, we will show that our research suggests that bid-ask spread is responsible for the two-phase behaviour [15, 16] of the financial markets.

2. Model

Let p_t be the price of a given stock. We define the stock price return as $r_t = \ln \frac{p_t}{p_{t-\triangle t}}$, where $\triangle t$ is a given time interval. Without loss of generality we will focus on a one-period (one time step) case. A multi-period case will be considered later. In one-period the stock price at time $t = 0$ is p_0 and all market participants know that. Based on available information [17, 18, 19] they estimate the expected return at the time $t = \triangle t$ in an interval form $(\mu - \varepsilon, \mu + \varepsilon)$, where μ is the estimated expected return, and ε is the estimation uncertainty.

2.1. *Probability space*

In order to define the probability space Ω_N, we consider a company with M issued stock shares on a market with N agents (market participants). Every agent in every time step can buy or sell $\{0, \dots, M\}$ shares. Thus, agent $i \in \{1, \dots, N\}$ at every time step plays a strategy $\omega_i \in \{-M, \dots, M\}$. This defines the probability (configuration) space as $\Omega_N = \{-M, \dots, M\}^N$.

2.2. *Personal preferences and interactions*

Further, based on the assumptions, personal preferences of agents and interactions between them are postulated. For the sake of simplicity, we first focus on a case with no uncertainty, $\varepsilon = 0$. The difference between the supply and the demand is[20] $\sum_{i=1}^{N} \omega_i$. Accordingly, the return is usually defined as[21, 22],

$$r = \frac{1}{\lambda} \frac{\sum_{i=1}^{N} \omega_i}{N}, \tag{1}$$

where λ is the market depth, i.e., the excess demand needed to move the price by one unit. If we assume that all trading is done at the return r, the gain of agent i is

$$g_i = -\omega_i \left(\frac{1}{\lambda N} \sum_{i=1}^{N} \omega_i - \mu \right) \quad i = 1, \dots, N. \tag{2}$$

For example, if agent i played strategy $\omega_i = 10$ (i.e. she wants to buy 10 shares) and if others played such strategies that $\frac{1}{\lambda N}\sum_{i=1}^{N}\omega_i > \mu$, then the agent has bought shares at a higher price than a fair one and has achieved a negative gain.

If $\triangle t$ is large enough, it can be assumed that between two discrete time steps every agent's behaviour converges to Nash equilibrium. In a Nash equilibrium, every agent is doing the very best she can, given the actions of all others. It is evident that when all agents have reached such a point, none has any incentive to change unilaterally what she is doing, so the situation is regarded as an equilibrium. If $(\omega_1^*, \ldots, \omega_N^*)$ is to be a Nash equilibrium, each agent maximises its own gain given the other agents' choices. Thus, $(\omega_1^*, \ldots, \omega_N^*)$ must satisfy the first-order conditions,

$$\frac{\partial g_i(\omega_1^*, \ldots, \omega_N^*)}{\partial \omega_i} = 0, \ i = 1, \ldots, N. \tag{3}$$

Here, for the purpose of differentiation we have extended the set of strategies, $\omega_i \in \mathbb{R}, i \in \{1, \ldots, N\}$. From Eqs. (2) and (3) Nash equilibrium can be easily calculated,

$$\omega_1^* = \omega_2^* = \ldots = \omega_N^* = \frac{\lambda\mu}{1 + \frac{1}{N}}. \tag{4}$$

Evidently, for large N all agents prefer to play the strategies close to $\lambda\mu$. In a variational form, each of them minimises $(\omega_i - \lambda\mu)^2, \ i = 1, \ldots, N$. Given this, the personal preferences of every agent can be modelled as

$$w_1(\omega_i) = \frac{\pi}{\lambda\sigma_1}(\omega_i - \lambda\mu)^2 + \pi\lambda\sigma_1, \tag{5}$$

where σ_1^{-1} is the agent risk aversion[23] (σ_1 is analogous to the temperature in physical systems) and π is a technical constant. The solution Eq. (4) is derived under the assumption that all agents are behaving optimally. Due to possible false or incomplete information, this assumption may not always be true[24]. Regardless of the others, the optimal strategy $\omega_i^* = \frac{1}{2}(\lambda\mu N - \sum_{j\neq i}\omega_j)$ for agent i is the solution of the equation, $\frac{\partial g_i(\omega_1, \ldots, \omega_i^*, \ldots, \omega_N)}{\partial \omega_i} = 0$. For large N, agent i minimises $\left(\frac{1}{N}\sum_{i=1}^{N}\omega_i - \lambda\mu\right)^2$. Analogous to the personal preferences, the interactions between agents can then be modelled with

$$w_2(\omega_1, \ldots, \omega_N) = \frac{\pi}{\lambda\sigma_2}\left(\frac{1}{N}\sum_{i=1}^{N}\omega_i - \lambda\mu\right)^2 + \pi\lambda\sigma_2, \tag{6}$$

where σ_2^{-1} is the agent regret aversion[23]. Thus, by minimising Eq. (5), the agents maximise their gain at minimum risk and at the same time, by minimising Eq. (6), they minimise their maximal regret (opportunity cost). According to that, a natural way to include uncertainty ε into the model is to replace Eq. (5) and Eq. (6) with $w_1(\omega_i) = \frac{\pi}{\lambda\sigma_1}(\omega_i - \lambda(\mu - \varepsilon))^2 + \pi\lambda\sigma_1$, and $w_2(\omega_1, \ldots, \omega_N) = \frac{\pi}{\lambda\sigma_2}\left(\frac{1}{N}\sum_{i=1}^{N}\omega_i - \lambda(\mu + \varepsilon)\right)^2 + \pi\lambda\sigma_2$, respectively. Finally, from behavioural finance it is known[25, 17, 26] that the agents choose from set of strategies close to equilibrium in a different way than from set of extreme strategies. The difference between a gain of 100 and a gain of 200 appears to be greater than the difference between a gain of 1100 and a gain of 1200. Mentioned fact is usually modelled with log utility function[26]. Therefore, instead of using w_1 and w_2, the personal preferences of each agent and the interactions between agents will be modelled with, $u_1(\omega_i) = \ln w_1(\omega_i),\ i \in \{1, \dots, N\}$ and $u_2(\omega_1, \dots, \omega_N) = \ln w_2(\omega_1, \dots, \omega_N)$.

2.3. *Equilibrium probability measure*

Personal preferences define what strategies agents prefer individually, and interactions define how the other agents react when one agent decides to play a certain strategy. Next, based on personal preferences and interactions, the equilibrium probability measure of the stock price return is derived. Summing the interactions over all agents defines the energy[27],

$$U_N(\omega_1, \dots, \omega_N) = \sum_{i=1}^{N} \ln w_1(\omega_i) + \ln w_2(\omega_1, \dots, \omega_N). \tag{7}$$

With the introduced probability space and the energy defined on it, it is easy to calculate the Gibbs probability measure[27, 28] on a set of N agents,

$$P_N(\omega_1, \dots, \omega_N) = \frac{\exp(-U_N)}{Z_N}, \tag{8}$$

where $Z_N = \sum_{\{\omega_1, \dots, \omega_N\} \in \Omega_N} \exp(-U_N)$ is the corresponding partition function [27, 28]. The Gibbs probability measure converges to a Gibbs state for large N. Due to the translation invariance of energy functional, the Gibbs state is an equilibrium state of the system[27]. Using the probability measure Eq. (8) of a random vector $(\omega_1, \dots, \omega_N)$, the distribution of the return r Eq. (1) can be easily determined. We are particularly interested in the case of weak convergence[29]. For that purpose we want to know the expectation of the arbitrary continuous function $h \in \mathcal{C}(\mathbb{R})$ of random variable r,

$$\frac{\int_\Omega h\left(\frac{1}{\lambda N}\sum_{i=1}^{N}\omega_i\right) \frac{1}{\left(\frac{1}{N}\sum_{i=1}^{N}\omega_i - \lambda(\mu+\varepsilon)\right)^2 + (\lambda\sigma_2)^2} P_\rho(d\omega)}{\int_\Omega \frac{1}{\left(\frac{1}{N}\sum_{i=1}^{N}\omega_i - \lambda(\mu+\varepsilon)\right)^2 + (\lambda\sigma_2)^2} P_\rho(d\omega)}. \tag{9}$$

P_ρ denotes the product measure on $\Omega_N = \{-M, \dots, M\}^N$ with identical one-dimensional marginal distributions[29] $\rho = \sum_{i=-M}^{M} \frac{1}{\gamma w_1(i)} \delta_i$, where δ_i is the Kronecker delta function and $\gamma = \sum_{k=-M}^{M} \frac{1}{w_1(k)}$ is a positive constant. With respect to P_ρ, the coordinates $\{\omega_i\}$ are i.i.d. The "obvious" step is to apply generalised central limit theorem[30, 31, 32], which states that a sum of i.i.d. random variables, when properly centred and scaled belongs to the domain of attraction of a stable distribution[31, 32, 33]. Because of the properties that ρ inherits from $w_1(\omega_i)$, when the number of stocks M and the number of agents N approach the infinity (thermodynamic limit), the distribution P_ρ converges to a Levy stable distribution with index (1,0), i.e. to the Cauchy distribution[30, 31]. As a consequence, the whole expression converges to

$$\frac{\int_{\mathbb{R}} h(x)\, \frac{1}{(x-(\mu+\varepsilon))^2+\sigma_2^2} f_C(x;\mu-\varepsilon,\frac{\sigma_1}{\gamma})dx}{\int_{\mathbb{R}} \frac{1}{(x-(\mu+\varepsilon))^2+\sigma_2^2} f_C(x;\mu-\varepsilon,\frac{\sigma_1}{\gamma})dx} =$$
$$= \frac{\int_{\mathbb{R}} h(x)\, \frac{1}{(x-(\mu+\varepsilon))^2+\sigma_2^2} \frac{1}{(x-(\mu-\varepsilon))^2+(\sigma_1/\gamma)^2} dx}{\int_{\mathbb{R}} \frac{1}{(x-(\mu+\varepsilon))^2+\sigma_2^2} \frac{1}{(x-(\mu-\varepsilon))^2+(\sigma_1/\gamma)^2} dx}. \tag{10}$$

If the regret aversion and the risk aversion are in a specific relation $\sigma_2 = \sigma_1/\gamma = \sigma$, then the following symmetric distribution is obtained

$$r = \frac{1}{\lambda}\frac{\sum_{i=1}^{N}\omega_i}{N} \xrightarrow{\mathcal{D}} C\frac{1}{(x-(\mu+\varepsilon))^2+\sigma^2}\frac{1}{(x-(\mu-\varepsilon))^2+\sigma^2}, \tag{11}$$

where C is normalisation constant. According to the theory of truncated Levy distributions[30] the same result can be obtained for large, but finite M and N. The equation (11) can be rearranged in the following manner,

$$f(x \mid \varepsilon) = C\frac{1}{1+\frac{(x-\mu)^2}{\varepsilon^2+\frac{\sigma^2}{2}}+\frac{((x-\mu)^2-\varepsilon^2)^2}{2\sigma^2\varepsilon^2+\sigma^4}}. \tag{12}$$

We now move from the one-period to the multi-period setting. For this purpose the time dynamics of μ, σ and ε need to be determined. In the first approximation μ and σ can be modelled as constants, since they primarily depend on the growth rate of a company and the agents risk aversions. In the first approximation, the uncertainty ε in estimation of the expected return μ depends on the difference between the realised returns and the estimated expected return. If the realised returns, in a few steps of a sequence, turn out greater or lower than the estimated expected returns due to stochastic mechanisms, then the agents' uncertainty in the estimation of the expected return becomes greater. In line with that, local deviations of the realised returns from the estimated expected returns are modelled with the variable y, whose distance from zero represents the agent's uncertainty, $\varepsilon = |y|$. Time dynamics of y will be defined under three natural assumptions. First, $y = 0$ should be the only equilibrium point of the mentioned dynamics. Second, return to equilibrium in terms of percentile change of uncertainty should be greater if uncertainty is greater. Third, the return to equilibrium should not depend on the sign of y. One simple deterministic system with the above prescribed properties is[30],

$$\frac{y_t - y_{t-\triangle t}}{y_{t-\triangle t}} = -\alpha y_{t-\triangle t}^2, \tag{13}$$

where α is a constant that determines how fast the market returns to equilibrium. Pursuant to the above, the deviation of the realised return from the estimated expected return serves as the input to Eq. (13). Finally, the following model is proposed for the time evolution of the random variable r_t,

a) $r_0 = \mu, y_0 = 0,$

b) $y_t = y_{t-\triangle t} - \alpha y^3_{t-\triangle t} + \frac{1}{\alpha}(r_{t-\triangle t} - \mu)\mathbf{1}_{\left\{|y_{t-\triangle t} - \alpha y^3_{t-\triangle t} + \frac{1}{\alpha}(r_{t-\triangle t}-\mu)|<1\right\}}$,

c) $f(x_t \mid y_t) = C(y_t)\frac{1}{1+\frac{(x_t-\mu)^2}{y_t^2+\frac{\sigma^2}{2}}+\frac{((x_t-\mu)^2-y_t^2)^2}{2\sigma^2 y_t^2+\sigma^4}}$.

where $f(x_t|y_t)$ is the probability density function of the random variable r_t and $\mathbf{1}_A$ is the indicator function, a function of a technical condition. The indicator function ensures that the discrete dynamical system Eq. (13) is in a region where zero is the only equilibrium point. The obtained distribution has fat tails and it is unimodal for low values of uncertainty, whereas for higher values of uncertainty, the shape of the distribution is bimodal. This is shown in Fig. 1.

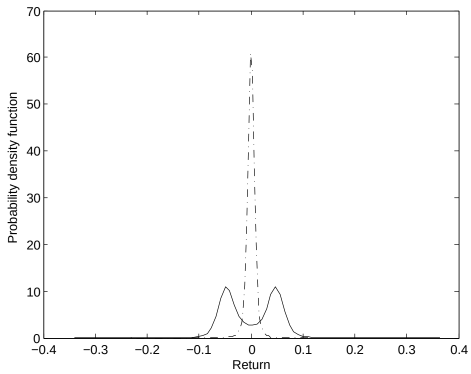


Fig. 1: Probability density function. The solid line is for $f(x_t|y_t = 0.05)$, and the dash-dot line is for $f(x_t|y_t = 0)$.

3. Validation

A validation is performed to show how defined model statistically matches the actual data. For this purpose, we have employed the daily returns for the S&P 500 index from January 3 1950 until September 3 2009. Using the Metropolis-Hastings algorithm[34], 50,000 samples from the proposed model with the parameters $\left\{\mu, \sigma^2, \alpha\right\} = \left\{2.75 \cdot 10^{-4}, 0.33 \cdot 10^{-4}, 15\right\}$ have been generated. In Fig. 2, the similarities between the distributions of the two data sets are examined.

From the above, it can be inferred that the estimated probability density functions from S&P 500 returns and the returns from the model show satisfactory similarity. Furthermore, the time dependence of the two time series has been explored with the autocorrelation functions, Fig. 3. The concurrence between them is found to be very good and consistent with the stylized facts[35, 32, 13] relating to time dependence of returns, zero autocorrelation for returns, and positive autocorrelation for absolute returns.

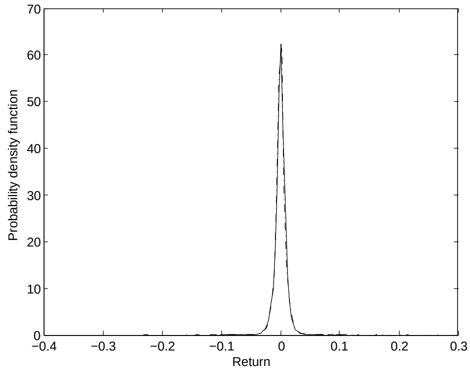
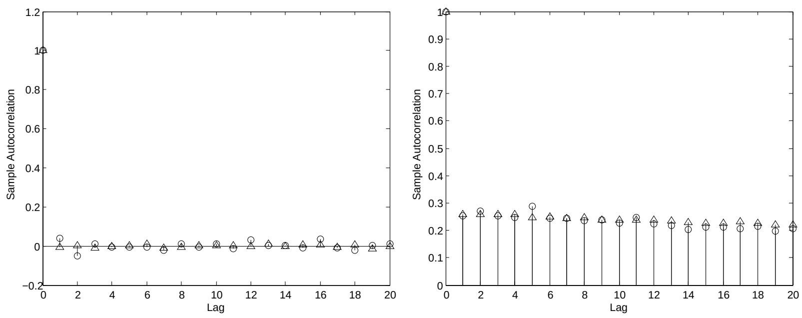


Fig. 2: Comparison of probability density functions. Dash-dot line represents estimated pdf from the proposed model returns and solid line represents the estimated pdf from S&P 500 returns. The kernel based method was used for estimating the pdf from data.



(a) (b)

Fig. 3: Comparison of the autocorrelation functions. (a) '$\triangle$' - the autocorrelation function of S&P 500 daily returns, 'o' - the autocorrelation function of the returns from the model, (b) '$\triangle$' - the autocorrelation function of S&P 500 daily absolute returns, 'o' - the autocorrelation function of the absolute returns from the model.

4. Bid-ask spread modelling

One of the first models of the bid-ask spread is due to the Richard Roll and his paper, "A Simple Implicit Measure of the Effective Bid-Ask Spread in an Efficient Market" [5]. Under Rolls ideal market assumptions, transaction prices can only bounce either at the ask price or at the bid price. Based on that assumption and based on efficient market hypothesis [6] he derived the following effective spread estimator,

$$S = 2\sqrt{-Cov(\triangle P_t, \triangle P_{t+1})}. \tag{14}$$

Rolls measure is simple and easy to compute. Obviously positive values of the covariance price changes represents a problem for defined estimator. Several strategies exist in literature which resolves the problem [7]. The most common approach is to multiply the covariance by negative one, estimate the spread, and multiply the spread by negative one. That approach we follow here and we also deal with the returns, rather than with the price changes. Further, uncertainty y defined in Chapter 2 enters into the proposed model for stock return evolution with square. As our goal here is to connect dynamics of the uncertainty y with the bid-ask spread dynamics, we are primarily interested in the square values of the bid-ask spreads. According to that, in Fig. 4. we compare behaviour of the square values of the y multiplied by 10 (proportionality factor) and square values of the bid-ask spreads obtained with Roll's estimator.

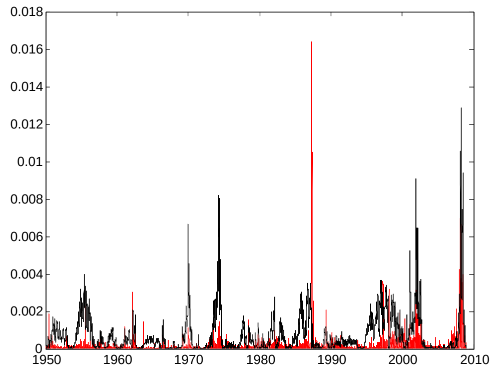


Fig. 4: Roll's estimator and uncertainty y (13). Red line represents a square of the bid-ask spread estimation obtained with Roll's estimator. Black line represents a square of the uncertainty estimation multiplied by 10.

Analysis is performed on the data set from Chapter 3. Besides mentioned, there exist several similar spread estimators in literature [3, 36]. One of the most recent bid-ask spread estimator is due to Corwin and Schultz (2009) [7]. They derive an estimator for the bid-ask spread based on daily high and low prices. Their estimator is based on two ideas. Firstly, daily high prices are almost always buy orders and daily low prices are almost always sell orders. Secondly, the component of the high-to-low price ratio that is due to volatility increases proportionately with the length of the trading interval, while the component due to bid-ask spreads is constant over different trading intervals. Based on that, they derive an estimate of a stocks bid-ask spread as a function of the high-to-low price ratio for a single two-day period and the high-to-low ratios for two consecutive single days. Here, we present their final result,

$$\beta = \left(\ln \frac{H_t}{L_t}\right)^2 + \left(\ln \frac{H_{t+1}}{L_{t+1}}\right)^2, \ \gamma = \left(\ln \frac{H_{t,t+1}}{L_{t,t+1}}\right)^2, \tag{15}$$

where, H_t (L_t) is the high (low) price for day t, and $H_{t,t+1}$ ($L_{t,t+1}$), is the high (low) price for two day period. Further, from β and γ they calculate α as,

$$\alpha = \frac{\sqrt{2\beta} - \sqrt{\beta}}{3 - 2\sqrt{2}} - \sqrt{\frac{\gamma}{3 - 2\sqrt{2}}}. \tag{16}$$

Finally, they defined spread estimator as,

$$S = \frac{2\left(e^{\alpha} - 1\right)}{1 + e^{\alpha}}. \tag{17}$$

We present in Fig. 5. square values of the estimation of the bid-ask spread obtained with high-low spread estimator and square values of the y multiplied by 10 (proportionality factor).

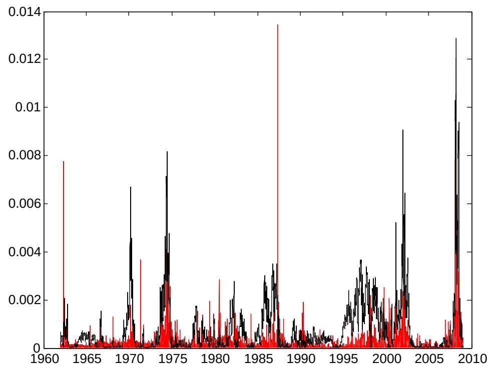


Fig. 5: High-low estimator and uncertainty y (13). Red line represents a square of the bid-ask spread estimation obtained with high-low estimator. Black line represents a square of the uncertainty estimation multiplied by 10.

Analysis is also performed on the data set from Chapter 3, but from January 3rd 1962 until September 3rd 2009. This is because daily high and low prices needed by the high-low estimator are available from January 3rd 1962. Obviously, the results on the Fig. 5. and 6. are similar. Both bid-ask spread estimates are consistent with uncertainty dynamics. We can conclude that our definition of uncertainty is actually very similar to the bid-ask spread. Finally, in first approximation we can state that bid-ask spread is good measure of the uncertainty in the expected return estimation.

5. Conclusion

In this paper, the model for time evolution of stock returns was derived based on a micro-level description of the financial market. The defined model is similar to the GARCH class of models, but can additionally exhibit bimodal behaviour. Moreover, it differs from existing Ising-type models. In many physical systems, despite the fact that microscopic description is stochastic, macroscopic behaviour is deterministic due to the law of large numbers, e.g. magnetisation in materials. Surprisingly, in the case of a financial market, we have found that the macroscopic variable, stock return, is a random variable. Particularly, we have found that positive autocorrelation of absolute returns is consequence of the existence of the bid-ask spread. This findings are completely in agreement with findings of Stoll [36], who state that in an informationally efficient market, the spread is the only possible cause for the serial covariance of price changes. Finally, it seems that the two-phase behaviour of the financial markets is primarily caused by the appearance of extremely large bid-ask spread.

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