Lagrangian mechanics of a simple economic model

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Abstract

Economics is often considered as an exact natural science. Most definitely this was the idea of the developers of neoclassical economics like William Stanley Jevons, Alfred Marshall et al. at 19$^{\text{th}}$ century. However, unlike physics, even today economics lacks a "general framework of modeling" that can be used to model changes in economic quantities. In some cases these changes can be drastical, like stock market crashes or depressions. The "dynamic" theory of current main stream economics is mainly static with some added features that are interpreted as "dynamics".

We propose a novel approach to economics. The basic idea is that economic agents are modeled like moving particles in a potential. The "free will" of an economic agent is restricted by the principle that the agent tries to improve its degree of utility, or decrease its degree of disutility. In physics, before Newton (most notably in Aristotelian physics) particles were considered to have "purposes" (known as *telos*) to move according to certain observed "laws". For example, an unsupported stone in the air has *telos* to fall into ground. Similarly, Aristotelian teleology was applied to human behavior. Newton explained that *telos* is a result of the forces acting on a particle. In this article, we do a similar abstraction in economics as Newton made in physics; we assume that *telos* for an economic agent to do something is the result of economic forces acting on the agent. These forces are caused by an economic potential.

With these principles, we are able to construct a Lagrangian of a simple economic system that consists of K consumers and I firms that produce and

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consume J different products. From the Lagrangian we are able to derive the economic forces acting upon productions of the firms, and consumptions of the consumers. Moreover, price dynamics of the products is a direct result of the Lagrangian. We demonstrate the behavior of the system by computer simulations with different parameter values. In order to be a viable model, our model should be able to produce "well-known" results.

It is a huge leap in economics from in principle static models to Lagrangian dynamics. We can ask whether this is necessary, or would Newtonian economics be enough, because Lagrangian dynamics produces the Newtonian equations of motion as well. There are some fundamental reasons for this extra work, however. First, Lagrangian (and Hamiltonian) dynamics are general formulations that are easy to use and have a powerful formalism with many applications. Second, our present work is only classical deterministic economic dynamics. Applying Lagrangian dynamics opens many possible ways for economics to follow physics in the future, e.g., statistical physics (ensembles and probabilities) and quantum physics, which both apply Lagrangian and Hamiltonian formalism.

What could "quantum economic dynamics" possibly be? Different consumption (or production) possibilities would have probability distributions, and a "quantum hypothesis" can be made, i.e., it is not possible to consume 0.03 of a car, to pay by 0.0001 euro for a fraction of another product etc. Products are produced, consumed, and paid as multiples of unit of "quantum". Lagrangian mechanics provides a good initial point for this generalization; if the functions in a Lagrangian are treated like operators, then one can quite straightforwardly apply the formalism of quantum physics. These extensions are our future goals, and in this paper we present the Lagrangian formalism.