Is There a Link between Intraday Volatility and Long Term Dependence? Evidence from the Nikkei

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Abstract
Over a long time horizon financial time series, while appearing random, usually display complex non-linear dynamics, which can have significant economic consequences if not addressed. For example, the presence of either chaos or fractality in a financial series can affect the ability to manage the risk of portfolios of assets, where the statistical apparatus relies on concepts such as the ability to scale volatility, have stationary returns and stable volatility. In this paper we specifically investigate one aspect of complexity, the presence of fractality as measured by the Hurst rescaled range statistic. We investigate fractality in one of the world's most famous stock indices- the Nikkei 225 Stock Index (henceforth simply the Nikkei). While this series has been calculated by the Nihon Keizai Shimbun (Nikkei) newspaper since September 7, 1950, we rely on a more recent time series (from January 4, 1984 to May 28, 2010). Our objective is to assess the effects of interday volume and *intraday* volatility on a local measure of long term dependence. The local Hurst is measured over varying subsamples from 10-days to 500-days, which allows a stream of Hurst statistics to be computed.

While researchers have shown that time-varying interday volatility and mean non-stationarity will have an effect on the Hurst statistic, so far the effect of *intraday* volatility and volume has not been considered. Volume is an unstable correlate with asset market volatility, but is theoretically linked to information asymmetries and market inefficiency in the finance literature. Intraday volatility will also influence interday volume, which is typically measured as the variance or standard deviation of asset returns.

We first measure the daily returns of the Nikkei Index as $\Delta P_t = P_t - P_{t-1}$, where we prefilter returns using a simple AR(2) filter of the following form to accommodate the potential effects of short term dependence due to illiquidity and information effects:

$$\Delta P_t = \alpha_0 + \beta_1 \Delta P_{t-1} + \beta_2 \Delta P_{t-2} + \psi_t \qquad (1)$$

For each ψ_t the classical rescaled adjusted range $(R/\sigma)_n$ is then calculated as

$$(R/\sigma)_n = (1/\sigma_n)\left[\underset{1 \le k \le n}{\text{Max}} \sum_{j=1}^{k}(X_j - \overline{X_n}) - \underset{1 \le k \le n}{\text{Min}} \sum_{j=1}^{k}(X_j - \overline{X_n})\right] \quad (2)$$

where $\overline{X}_n$ is the sample mean and σ_n is the standard deviation of ψ_t over n

$$\sigma_n = \left[1/n\sum_{j=1}^{n}(X_j - \overline{X_n})^2\right]^{0.5} \quad (3)$$

In order to capture the time-varying nature of dependence in ψ_t this study employs a local measure of the Hurst exponent (h) calculated as

$$h_n = \frac{\log(R/\sigma)_n}{\log n} \quad (4)$$

where n is set to varying intervals from 10 to 500 days (which is equivalent to two standard weeks and two years respectively). While it is commonplace to measure asset volatility based the $\sigma_n(\Delta P_t)$ across a regular time interval n, we instead utilise a more complex measure, the Garman and Klass estimator (GKe), which measures volatility based on differences between the open, close, high and low prices *within* a particular time interval, which in this instance is 1-day:

$$GKe_t = \sigma^2 = 0.511(H-L)^2 - 0.019(C-0)(H+L-2C)(l-C) - 0.383(C-O)^2 1 \quad (5)$$

where H = log of interval high, L = log of interval low, O = log of interval open, C= log of interval close.

The results may be summarised as follows: *First*, a phase diagram of the Nikkei returns does not reveal any noticeable compass rose structure, which has been identified in other markets. That is, the series appears random, although standard statistical tests reveal slight negative skewness ($\equiv$-0.22) and positive kurtosis ($\equiv$8.4)in both the filtered and unfiltered series. The Nikkei return series requires an AR(2) filter to accommodate interday illiquidity and information effects (Equation 1, $\alpha \equiv 0$, $\beta_1 \equiv -0.02$, p=0.04, $\beta_2 \equiv -0.05$, p=0.00). Interday volatility measured as $\sigma_n(\Delta P_t)$ is time varying, which is well known, and so too is the GKe_t, which is not. The correlation (ρ) between GKe_t and volume is also time-varying but generally positive (ρ_n = -0.41→0.90 for n = 10), which is consistent with finance theory linking changes in volatility to price discovery by market participants. The local measure of the Hurst statistic is also time-varying and oscillates irregularly (between 0.48 and 0.92 for n = 100). As the estimation period of n increases from 10 to 500 the average Hurst declines from 0.72 (n = 10) to 0.63 (n = 500). Thus the series remains positively dependent and therefore tending to be non-mean reverting. When the daily GKe is correlated with the daily series of local Hurst statistics (n = 10→500) the results are slightly positive for $n \le 50$ ($\rho \equiv$ 0.10) and negative for $n \ge 50 \le 500$ ($\rho \equiv -0.10$). We interpret these results as volatility shocks linked to price discovery persisting for intervals of 50 trading days. Thereafter, the reverse appears to occur, but over long time intervals: negative shocks have a negative effect on the Hurst coefficient. This is consistent with mean reverting volatility affects. We conclude that fractality in the series may be due to the effects of mean reverting volatility.